

# ROBUST CONTROLLER DESIGN FOR ACTIVE SUSPENSION SYSTEMS IN CARS

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## ABSTRACT

The active suspension system plays an important role in enhancing ride comfort and road holding in automobiles. This paper focuses on designing a robust controller for an active suspension system to improve both these criteria. First, a half-car model in the longitudinal direction is established with active suspension implemented on the front and rear axles. The dynamic equations are transformed into the state-space representation, and a robust controller is designed with the objective of improving ride comfort while considering the road-holding capability of the wheels. Simulation results in both the frequency and time domains show that the active suspension system with the robust controller achieves an improvement of approximately 53.38% in ride comfort and about 77% in road holding compared to a conventional passive suspension system.

**Keywords:** Active suspension system, robust  $H_\infty$  control, ride comfort, road holding, vehicle dynamics and control.

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## 1. INTRODUCTION

Active suspension systems are an advanced technology in the field of mechanical engineering and automotive design, playing a crucial role in improving vehicle performance and driving experience. Unlike passive suspension systems, active suspensions employ active control mechanisms consisting of sensors, controllers, and actuators to adjust the actuator forces in real time. This not only optimizes ride comfort across different road surfaces but also enhances road-holding capability, stability, and safety when operating at high speeds or under poor road conditions. The development of active suspension systems reflects not only the

progress of control and sensing technologies but also represents a significant step forward in improving both performance and comfort for modern vehicles.

In the study of active suspension systems, mathematical models are employed to simulate vehicle dynamics and to design control strategies. Three commonly used models include: the quarter-car model, which considers a single corner of the vehicle, offering simplicity and suitability for rapid analysis; the half-car model, which considers two wheels and describes vertical vibrations as well as pitch or roll motion; and the full-car model, which incorporates all degrees of freedom for a comprehensive evaluation. The choice of model depends on the research objectives and the desired level of accuracy.

Many recent studies have focused on developing active suspension systems using various modern control methods. Zhao and Gu proposed an LQR controller optimized by a hybrid PSO-GA algorithm for a quarter-car model, demonstrating improved vibration reduction of the vehicle body [1]. Dif investigated the optimization of an LQR controller using the Firefly Algorithm for a quarter-car model, showing significant improvements compared to conventional LQR [2]. Ji et al. developed a fuzzy PID controller in which the parameters were optimized through a genetic algorithm, resulting in considerable enhancement of ride comfort [3]. Moreover, Esen and Metin conducted a comparative study of PID and fuzzy logic control on semi-active and passive suspension systems in a full-car model, highlighting that active and semi-active approaches outperform passive suspensions in terms of passenger comfort [4]. These studies have made important contributions to broadening control approaches, thereby improving the effectiveness and practical applicability of active suspension systems under real-world operating conditions.

The robust  $H_\infty$  control method is regarded as a powerful tool to enhance the performance of closed-loop systems within a specified frequency range. A critical step in its design process is the proper selection of weighting functions. However, this task often proves challenging due to the complex nature of suspension systems and the diversity of possible control structures.

The main contributions of this paper are as follows:

- A half-car longitudinal model is employed to evaluate the vibration characteristics of a vehicle with passive suspension and to design a controller for the active suspension system.
- The  $H_\infty$  control method is applied to design the controller with the objective of improving ride comfort and road holding.
- Simulation results in both the frequency and time domains are presented to compare the effectiveness of the  $H_\infty$  controller in the active suspension system against the passive suspension system. The results confirm that the  $H_\infty$  controller fully meets the design objectives.

The paper is organized as follows: Section 2 presents the half-car longitudinal model. Section 3 provides the theoretical background of the robust  $H_\infty$  controller for the active suspension system. Section 4 discusses the simulation results in both the time and frequency domains. Finally, conclusions are drawn in Section 5.

## 2. VEHICLE MODELING WITH AN ACTIVE SUSPENSION SYSTEM

### 2.1. Half-Car Model

The half-car model represents the longitudinal dynamics of one half of the vehicle, including the main components: the sprung mass ( $m_s$ ), which corresponds to the vehicle body, and the unsprung masses ( $m_{u1}$ ,  $m_{u2}$ ), which represent the wheels and the suspension systems of the front and rear axles. The system parameters consist of the suspension stiffness ( $k_{s1}$ ,  $k_{s2}$ ), damping coefficients ( $c_1$ ,  $c_2$ ), and the equivalent tire stiffnesses ( $k_{t1}$ ,  $k_{t2}$ ). The vertical displacements include the body displacement ( $Z_s$ ), the displacements of the body at the front and rear axles ( $Z_{s1}$ ,  $Z_{s2}$ ), as well as the wheel displacements ( $Z_{u1}$ ,  $Z_{u2}$ ) and the road inputs ( $q_1$ ,  $q_2$ ). The model also considers the distances from the vehicle's center of gravity to the front and rear axles ( $a$ ,  $b$ ), along with the pitch angle ( $\varphi$ ), to describe the rotational motion of the sprung mass about its center of gravity.

In this model,  $F_1$  and  $F_2$  denote the control forces applied to the active suspension system at the front and

rear axles, respectively. These forces are used to regulate the vehicle's dynamic response, thereby improving stability and ride comfort under various road conditions.

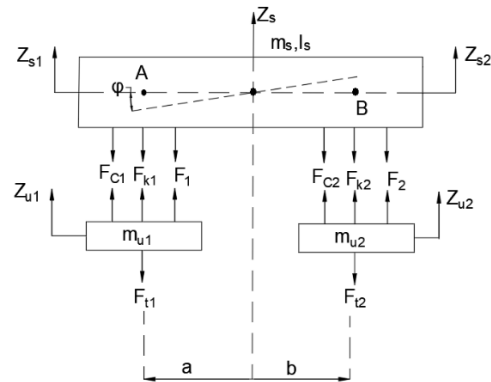


Figure 1. Longitudinal half-car active suspension model

The parameters of the investigated model are listed in Table 1.

Table 1. Parameters of the half-car model [8]

Symbol	Parameter	Value	Unit
$m_s$	Sprung mass	700	Kg
$m_{u1}, m_{u2}$	Unsprung mass	40	Kg
$a$	Distance from center of gravity to front axle	1.803	m
$b$	Distance from center of gravity to rear axle	1.011	m
$k_{s1}, k_{s2}$	Front/Rear spring stiffness	15000	N/m
$k_{t1}, k_{t2}$	Front/Rear tire stiffness	200000	N/m
$c_1, c_2$	Front/Rear damping coefficient	1290	Ns/m
$I_s$	Moment of inertia of sprung mass	2460	Kg.m <sup>2</sup>

The dynamic equations of the half-car longitudinal model are formulated as follows:

$$\begin{cases}
 m_s \ddot{Z}_s = (Z_s - a\varphi - Z_{u1})k_{s1} - (Z_s + b\varphi - Z_{u2})k_{s2} - (\dot{Z}_s - a\dot{\varphi} - \dot{Z}_{u1})c_1 - (\dot{Z}_s + b\dot{\varphi} - \dot{Z}_{u2})c_2 - F_1 - F_2 \\
 I_s \ddot{\varphi} = (Z_s - a\varphi - Z_{u1})k_{s1}a - (Z_s + b\varphi - Z_{u2})k_{s2}b + (\dot{Z}_s - a\dot{\varphi} - \dot{Z}_{u1})c_1a - (\dot{Z}_s + b\dot{\varphi} - \dot{Z}_{u2})c_2b + F_1a - F_2b \\
 m_{u1} \ddot{Z}_{u1} = (\dot{Z}_s - a\dot{\varphi} - \dot{Z}_{u1})c_1 + (Z_s - a\varphi - Z_{u1})k_{s1} - (Z_{u1} - q_1)k_{t1} + F_1 \\
 m_{u2} \ddot{Z}_{u2} = (\dot{Z}_s + b\dot{\varphi} - \dot{Z}_{u2})c_2 + (Z_s + b\varphi - Z_{u2})k_{s2} - (Z_{u2} - q_2)k_{t2} + F_2
 \end{cases} \quad (1)$$

The dynamic equations (1) can be rewritten in the state-space form as follows:

$$\begin{aligned}
 \dot{x} &= Ax + B_1w + B_2u \\
 y &= Cx + D_1w + D_2u
 \end{aligned} \quad (2)$$

In there, the control signals are  $u = [F_1 \ F_2]^T$ ; road disturbance inputs  $w = [q_1 \ q_2]^T$ . State vector và Output vector are defined as follows:

$$x = [\dot{Z}_s \ Z_s \ \dot{Z}_{u1} \ Z_{u1} \ \dot{Z}_{u2} \ Z_{u2} \ \dot{\phi} \ \phi]^T;$$

$$y = [\ddot{Z}_s \ (Z_{u1} - q_1) \ (Z_{u2} - q_2) \ \ddot{\phi}]^T$$

**2.2. Evaluation Criteria**

To evaluate the vibration model, the commonly used criteria are ride comfort and road holding. These are also the two main objectives adopted by the authors for analysis and evaluation in this paper.

**a) Ride comfort**

In this research, the authors use the body displacement acceleration ( $\ddot{Z}_s$ ) and pitch angle acceleration ( $\ddot{\phi}$ ) of the sprung mass to evaluate the ride comfort of the half-car model.

**b) Road holding**

In this research, the author uses the front wheel deflection ( $Z_{u1}-q_1$ ) denoted as  $X_{tf}$  and the deformation of the rear wheel tire ( $Z_{u2}-q_2$ ) denoted as  $X_{tr}$  of the unsprung mass to evaluate the road holding of the half-car model.

To evaluate the improvement of the active suspension system, the research employs the formula for calculating the mean square error of the vertical displacements, pitch angle, and accelerations of the vehicle body and wheels to provide a comparison before and after applying the controller, as given by the following formula:

$$RMS(x) = \sqrt{\frac{1}{N} \sum_1^N |x_n|^2} \tag{3}$$

**3. DESIGN OF H<sub>∞</sub> CONTROLLER FOR THE ACTIVE SUSPENSION SYSTEM**

**3.1. Theoretical background**

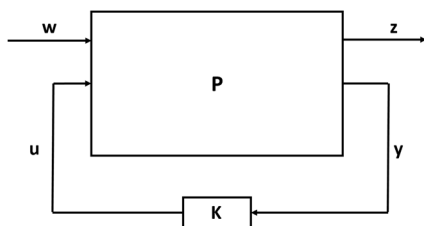


Figure 2. Standard structure of the H<sub>∞</sub> controller

The model P has two inputs: The disturbance input w, which includes the reference signal and noise, and the controlled variables u. It has two outputs: the performance signal z, which the research aims to minimize, and the measured variables y, which are used for controller. y is employed in K to compute the controlled variables u. Note that all these signals are typically vectors, while P and K are matrices.

**3.2. H<sub>∞</sub> Controller design for active suspension system**

In H<sub>∞</sub> synthesis algorithms, the design objectives are expressed as a single cost function that needs to be minimized. For the half-car model, the main control objectives are formulated based on passenger comfort and road handling capability. These objectives are related to the body acceleration ( $\ddot{Z}_s$ ), pitch angle acceleration ( $\ddot{\phi}$ ) and the tire deflection of the front and rear wheels. Other factors influencing the control design include:

- Characteristics of road disturbances
- Quality of sensor measurements for feedback
- Limitations of the available control force

Weighting functions are used to model external disturbances and quantify the design objectives, as illustrated in Figure 3.

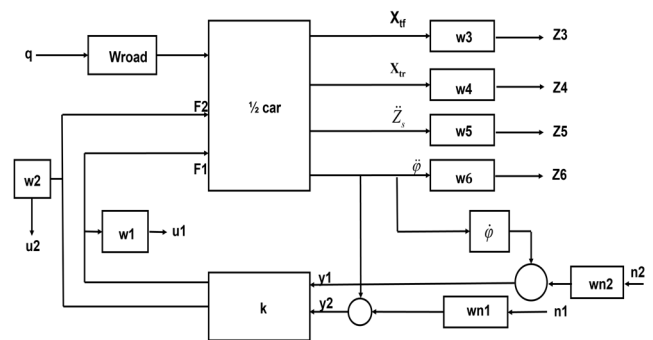


Figure 3. Structure of the H<sub>∞</sub> Controller for half car mode

**a) H<sub>∞</sub> Controller Synthesis**

The closed-loop configuration shown in Figure 3 comprises the half-car model, the controller K, and the weighting functions W<sub>ij</sub>. In this diagram, F<sub>1</sub> and F<sub>2</sub> are the control inputs, y<sub>1</sub> and y<sub>2</sub> are the measured signals, and n<sub>1</sub> and n<sub>2</sub> denote sensor-specific measurement noises. q represents the road bump profile and is considered the road excitation signal. The signals Z<sub>3</sub>, Z<sub>4</sub>, Z<sub>5</sub> and Z<sub>6</sub> correspond to the performance outputs. According to Figure 3, interconnecting the linear model (2) with the weighting functions yields the following state-space representation of P(s) [5]:

$$\begin{bmatrix} \dot{X} \\ Z \\ Y \end{bmatrix} = \begin{bmatrix} A & B1 & B2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} X \\ W \\ U \end{bmatrix}$$

The disturbance input  $W = [q_1 \ q_2 \ n_1 \ n_2]$ , input control signals:  $U = [F_1 \ F_2]^T$ , performance output:  $Z = [\ddot{Z}_s \ X_{tf} \ X_{tr} \ \ddot{\phi}]$ , measured output:  $Y = [\ddot{Z}_s \ \ddot{\phi}]$ .

**b) Selection of Weighting Functions**

The objective of the controller is to enhance ride comfort while considering driving safety through the wheel deflection relative to the road, also referred to as the dynamic wheel force. Therefore, the weighting functions are selected to achieve the optimal objectives, as shown in Table 2.

Table 2. Values of the weights  $W_{ij}$

Weight function	W1	W2	W3	W4	W5	W6	Wn1	Wn2
		$F_1$	$F_2$	$X_{tf}$	$X_{tr}$	$\ddot{z}_s$	$\ddot{\varphi}$	$n_1$
Value	0.002	0.002	0.7	0.7	3.0	0.9	0.02	0.005

**4. SIMULATION AND EVALUATION**

In this section, the simulation results are presented in both the frequency and time domains to evaluate the effectiveness of the proposed  $H_\infty$  controllers for the active suspension system. The model parameter values are given in Table 1. The road disturbance is considered as a harmonic input (sine wave). In the system of differential equations describing the oscillatory motion of a two-axle vehicle;  $q_1, q_2$  represent the road elevations at the contact points of the wheels with the road surface at the front and rear axles, respectively, when the vehicle is in motion. In the case where the road profile follows a harmonic function, the road elevation as a function of time can be expressed as follows:

$$q(t) = q_0 \times \sin \omega \times t = q_0 \times \sin \frac{2\pi}{T} \times x \tag{4}$$

**4.1. Frequency-Domain simulation analysis**

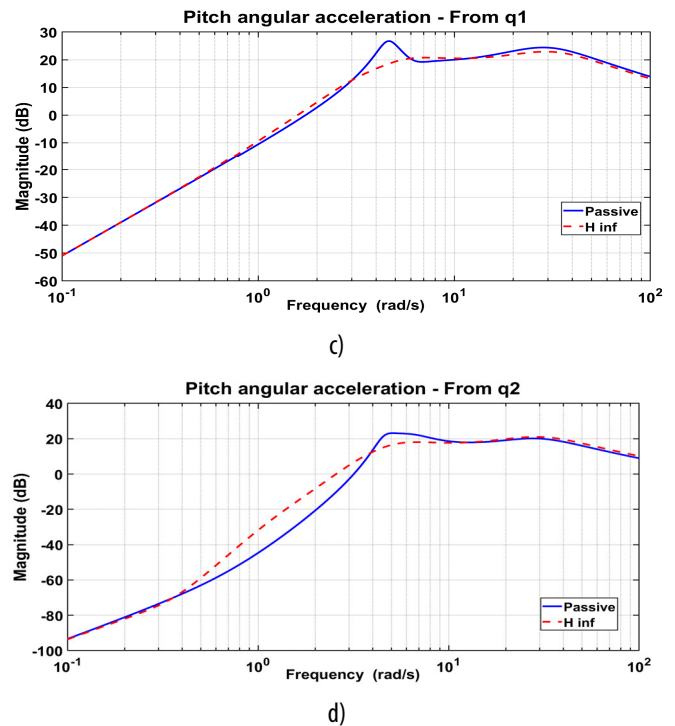
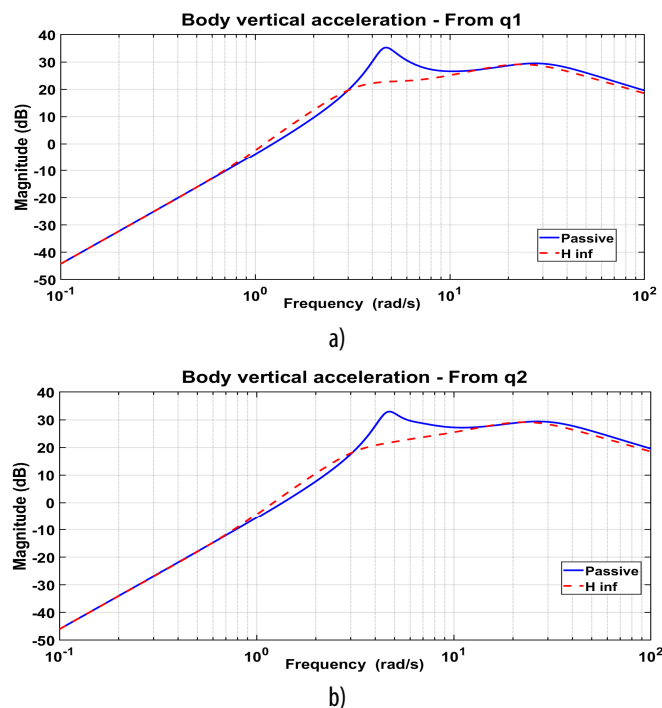
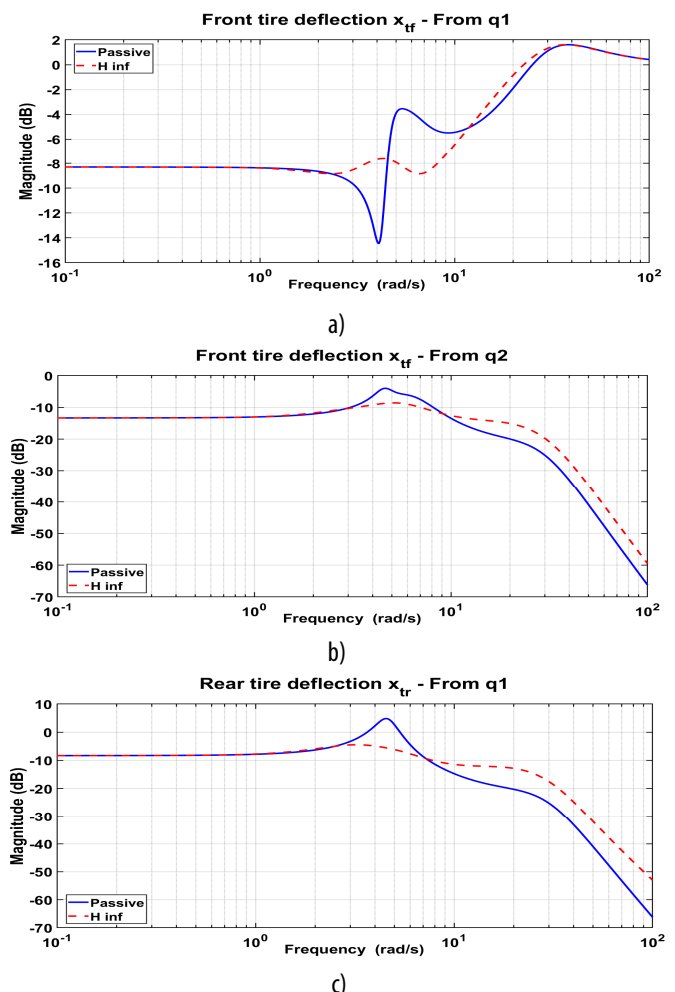


Figure 4. Transfer function from road inputs  $q_1, q_2$  to body displacement acceleration and pitch angular acceleration



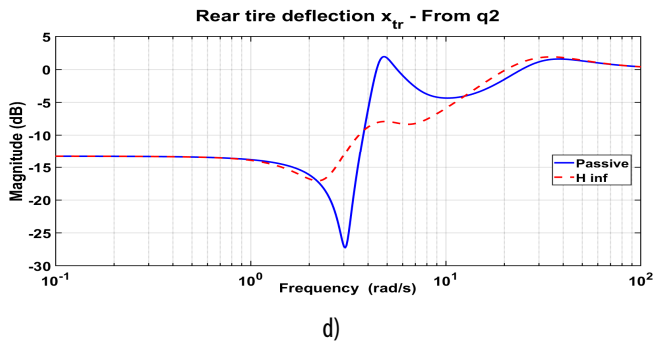


Figure 5. Transfer function from road inputs  $q_1, q_2$  to the front and rear wheel deflections

The plots in Figure 4 show the amplitude transfer functions of body displacement acceleration and pitch angular acceleration. The  $H_\infty$  controller demonstrates the best performance from around 3rad/s onwards, significantly reducing the resonance intensity of body displacement acceleration compared to the passive system. At low frequencies, the  $H_\infty$  controller has less effect than the passive system, but the impact in the 1 - 3rad/s range is minimal and negligible. Similarly, the amplitude transfer function of pitch angular acceleration shows effectiveness from approximately 3rad/s compared to the passive system. Therefore, the  $H_\infty$  controller fully achieves the objective of improving ride comfort.

The plots in Figure 5 illustrate the amplitude transfer functions of the front and rear wheel deflections. The robust controller successfully improves the driving safety criteria, with effective performance in the 5 - 10rad/s frequency range for  $q_1$  and 3 - 9rad/s for  $q_2$  at the front wheel. Similarly, the rear wheel performance is well enhanced in the 3 - 7rad/s range for  $q_1$  and 4 - 10rad/s for  $q_2$ .

**4.2. Time-Domain simulation analysis**

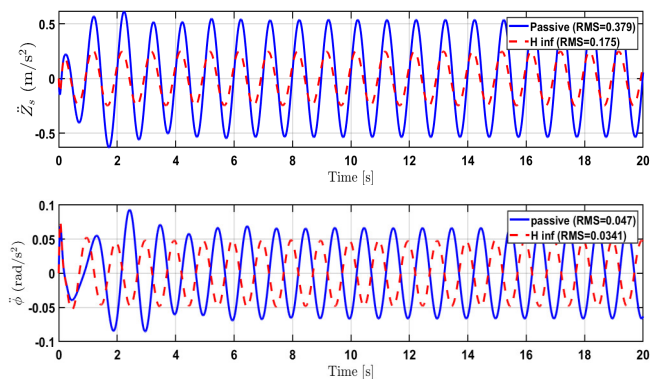


Figure 6. Time-domain simulation results of body displacement and pitch acceleration

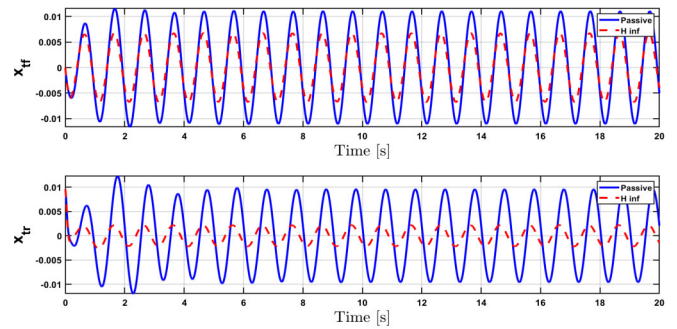


Figure 7. Time-domain simulation results of front and rear wheel deflections

With the  $H_\infty$  controller aiming to improve ride comfort, the time-domain plot indicated by the red dashed line demonstrates the best performance, significantly reducing the oscillation amplitude of the ride comfort evaluation criteria, such as body displacement acceleration ( $\ddot{z}_s$ ) and pitch angular acceleration ( $\ddot{\phi}$ ) compared to the passive system, while also significantly improving the safety criteria at both the front and rear axles. The results are presented in detail in Table 3 through the mean square error values.

Table 3. Comparison of the root mean square (RMS) values of the  $H_\infty$  controller

Evaluation criteria	$H_\infty$	Passive	Improvement
$\ddot{z}_s$	0.1765	0.3786	53,38%
$x_{tr}$	0.0049	0.0078	37,17%
$x_{tr}$	0.0015	0.006742	77,61%
$\ddot{\phi}$	0.03722	0.04696	20,74%

**5. CONCLUSION**

In this study, a half-car longitudinal model was developed as the basis for designing an  $H_\infty$  controller for an active suspension system. The modeling process included the formulation of dynamic equations and their transformation into state-space form, allowing for the implementation of robust control strategies. The  $H_\infty$  control method was employed to synthesize a controller with the objective of simultaneously improving ride comfort and road holding performance. The simulation analysis in both the frequency and time domains confirm that the proposed controller effectively achieves the design objectives. In the frequency domain, the  $H_\infty$  controller significantly attenuates the resonance amplitudes of body acceleration and pitch angular acceleration compared to the passive suspension system, thereby enhancing ride comfort. Similarly, improvements

in the front and rear wheel deflections demonstrate better road-holding capability across a wide frequency range. In the time domain, the active suspension system with  $H_\infty$  control substantially reduces vibration amplitudes of body acceleration and pitch motion while also decreasing wheel deflections. Quantitative evaluation through RMS values shows improvements of over 50% in ride comfort and more than 70% in road holding relative to the passive suspension system.

Future research will apply the  $H_\infty$  method to full-vehicle models with uncertain parameters and simultaneously compare it with other controllers such as PID and LQR to more accurately assess the proposed control performance.

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## REFERENCES

- [1]. Zhao W., Gu L., "Hybrid Particle Swarm Optimization - Genetic LQR Controller for Active Suspension," *Applied Sciences*, 13(14), 8204, 2023.
- [2]. Dif I., Dif N., "Firefly Algorithm Optimization-Based LQR Controller for 1/4 Vehicle Active Suspension System: Design and Performance Evaluation," *The Journal of Engineering and Exact Sciences*, 9(5), 15928-01e, 2023.
- [3]. Ji G., Zhang L., Shan M., Zhang J., "Enhanced variable universe fuzzy PID control of the active suspension based on expansion factor parameters adaption and genetic algorithm," *Engineering Research Express*, 5, 035007, 2023.
- [4]. Esen U. Y., Metin M., "Performance Analysis of PID and Fuzzy Logic Controlled Semi-Active and Passive Suspension Elements on Full Vehicle Model," *Orclever Proceedings of Research and Development*, 4(1), 59-72, 2024.
- [5]. Van Tan V., Senname O., Gaspar P., Do T. T., *Active anti-roll bar control design for heavy vehicles*. Berlin, Germany: Springer, 2024.
- [6]. Singh S., Chhabra V., Bhargal N., "Robust control of vehicle active suspension system using  $H_\infty$  and  $\mu$ -synthesis," *International Journal of Control and Automation*, 2016.
- [7]. Liu H., Zhang W., Chen Y., "Vehicle active suspension control considering parameter uncertainty: a robust finite-frequency  $H_\infty$  output feedback control strategy based on linear parameter varying models," *Vehicle System Dynamics*, 62(10), 1875-1894, 2024.
- [8]. Pham Tat Thang, Vu Van Tan, Dao Huy Hai, Ta Nhu Thuan, Do Trong Tu, "Research on active suspension control systems to improve the ride quality of passenger cars," *Transport Magazine*, 65 (751), 2025.
- [9]. Afshar A., Javadi S., "Constrained  $H_\infty$  control for a half-car model of an active suspension with input delay," *Journal of Vibration and Control*, 25(14), 2034-2046, 2019.
- [10]. P. Kousik, Pravin M Singru, Narayan Manjrekar, "Comparing PID and  $H_\infty$  Controllers on a 2-DoF Nonlinear Quarter Car Suspension System," *Journal of Vibro-Engineering*, 2014.
- [11]. Abdelkader M., El-Hefnawy A., "Semi-Active Suspension Control Using Modern Methodology: A Comparative Study with  $H_\infty$  Robust Control," *arXiv preprint arXiv:1411.3305*, 2014.
- [12]. Wei C., "Modelling and simulation of active half-vehicle suspension based on a new output-feedback  $H_\infty$  controller," *International Journal of Control, Automation and Systems*, 22, 775-784, 2024.