A STUDY ON THE DEVELOPMENT OF THE CHARACTERISTIC CURVES OF AN ELECTRORHEOLOGICAL DAMPER FOR PASSENGER CARS

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ABSTRACT

Semi-active suspension systems are increasingly applied in the automotive industry due to their balance between performance and manufacturing cost. Among them, electrorheological (ER) dampers stand out thanks to their fast response time, low energy consumption, and flexible adjustability of damping force through controlled voltage. This paper develops a mathematical model of an ER damper based on the Bingham plastic constitutive law, from which the force-displacement and force–velocity characteristics are established. Simulation results indicate that the damping force exhibits nonlinear dependence on both piston displacement and velocity, and increases significantly with higher control voltages. Furthermore, the study highlights the influence of PWM levels on the variation of damping force, providing a scientific basis for designing adaptive controllers and optimizing the performance of semi-active suspension systems for passenger cars.

***Keywords*:** *Electrorheological damper; ER fluid; Bingham model; Characteristic curves; Semi-active suspension.*

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1. INTRODUCTION

The suspension system in automobiles plays a crucial role in ensuring ride comfort and driving safety. By combining springs and dampers, the suspension helps reduce road-induced vibrations, maintain tire-road contact, and improve vehicle stability at high speeds or on uneven surfaces.

Suspension systems can be classified into three main categories: passive, semi-active, and active. Passive suspensions have a simple structure, typically using springs and conventional hydraulic dampers. Their advantages are low cost and high reliability; however, they lack the flexibility to adapt to changing road conditions [1]. Active suspensions employ actuators to directly control the applied forces, thereby significantly improving ride comfort and safety. Nevertheless, their drawbacks include high energy consumption, complex structure, and high cost [2]. Semi-active suspensions are considered a compromise between performance and production cost. In this type, damping characteristics are adjusted using smart fluids such as magnetorheological (MR) or electrorheological (ER) fluids. As a result, semi-active systems improve dynamic performance while consuming less energy compared to active suspensions, and thus have been widely adopted in many modern passenger cars [3].

Within a semi-active suspension system, the damper is a key component that determines the system's ability to control vibrations. Intelligent dampers often employ MR or ER fluids, which exhibit variable viscosity or yield stress under the influence of a magnetic or electric field [4]. MR dampers have a high yield stress range and have been applied in the automotive industry; however, they still entail high manufacturing costs [5]. In contrast, ER dampers offer distinct advantages such as extremely fast response time, low energy consumption, and precise damping force adjustability through voltage control. These features make ER dampers particularly suitable for applications requiring rapid response, such as passenger car suspensions [6].

From a modeling perspective, numerous studies have developed mathematical models for ER dampers. The traditional Bingham plastic model describes the relationship between damping force and piston motion with two main parameters: yield stress and post-yield viscosity [3, 7]. The Eyring-plastic model enables smooth transition between pre-yield and post-yield states, providing a better reflection of experimental ER fluid behavior under electric fields [8]. Nonlinear viscoelastic–plastic models employ smooth switching functions around the yield region, allowing for more flexible simulations under varying operating conditions [9]. Among these, the Bingham plastic model remains one of the most widely used, as it is both simple and effective in describing the rheological properties of ER fluids within practical applications [10], while also being well-suited for control and design purposes [3, 7].

Recent studies have shown that developing damping force characteristic curves of ER dampers with respect to displacement and velocity provides valuable data for suspension controller design and optimization [7, 11]. However, the influence of control voltage on these characteristic curves has not yet been fully investigated, despite being a critical factor for practical applications of ER dampers [3, 12].

Based on these considerations, this paper focuses on studying the damping force characteristics of an ER damper in a semi-active suspension system for passenger cars, modeled using the Bingham plastic approach. The investigation analyzes the damping force with respect to displacement and velocity, while also examining the effect of voltage on the variation of these characteristic curves.

2. DEVELOPMENT OF THE ELECTRORHEOLOGICAL DAMPER MODEL

The schematic diagram of the electrorheological (ER) damper is shown in Figure 1. The structure consists of an inner and an outer cylinder, with the piston dividing the volume into upper and lower chambers, both filled with ER fluid. When the piston moves, the ER fluid flows through the flow channel connecting the two chambers. A high voltage source is applied between the inner cylinder (positive electrode) and the outer cylinder (negative electrode), while a gas chamber is arranged outside the lower chamber to serve as an accumulator.

In the absence of an electric field, the damping force is mainly generated by the natural viscosity of the ER fluid. When an electric field is applied, the ER fluid exhibits yield stress, thereby increasing the damping force. This property allows the damping force to be continuously controlled by adjusting the electric field strength [3, 7].

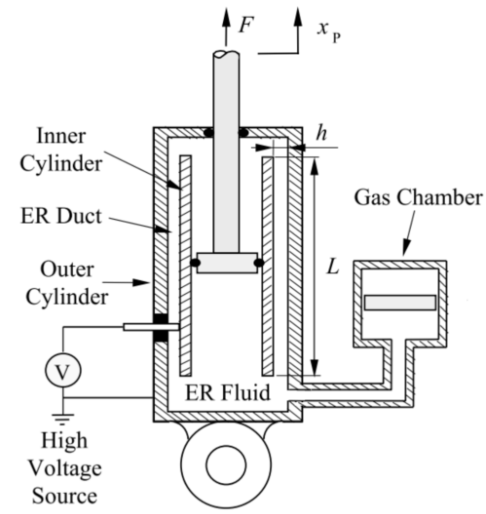

Figure 1. Schematic diagram of the electrorheological (ER) damper [13]

In this study, a quasi-static model based on the Bingham plastic constitutive law is employed. The total force acting on the piston can be expressed as:

$$F_d = P_a A_s + \Delta P_d (A_p - A_s) \tag{1}$$

where A_p is the piston area; A_s is the piston rod area; $P_a A_s$is the elastic force generated by the gas chamber, and $\Delta P_d = (A_p - A_s)$ is the viscous force from the fluid.

According to the Bingham model [14], the damping force is given by:

$$F_d = P_a A_s + \frac{6\mu L_d}{\pi . d^3 R_d} x_p' + (A_p - A_s)\frac{cL_d}{d}\alpha . E^\beta . sign(x_p') \tag{2}$$

where L_d is the electrode length; d is the electrode gap; E is the electric field strength, μ is the viscosity of the ER fluid; x_p' is the piston velocity, and α, β are material-dependent constants. The gas force component can be approximated by:

$$P_a A_s \approx k_0 x_p \tag{3}$$

where k_0 is the equivalent stiffness of the gas chamber.

When the electric field is controlled by a voltage $U(t)$, it can be expressed as:

$$E = \frac{U(t)}{d} = \frac{U_m}{d} u \tag{4}$$

where U_m is the maximum voltage; u is the duty ratio of the PWM control signal. Experimental studies show that when $u \geq 0.3$, the electric field reaches saturation.

After substitution, the damper force equation can be simplified as:

$$F_d = k_0 x_p + c_0 x_p' + \sigma . u^\beta . sign(x_p') \tag{5}$$

where c_0 is the viscous damping coefficient in the absence of an electric field, and σ is the yield stress coefficient.

However, the above model only describes the static behavior of the ER damper. To represent its dynamic behavior, the yield stress component is expanded by a first-order differential equation:

$$\tau(u)F'_{er} + F_{er} = \sigma u^{\beta} sign(x'_p) \quad (6)$$

where $\tau(u)$ is a time constant dependent on the control input. Experimental studies indicate that the dynamic behavior of ER fluids can be adequately approximated by a first-order system [12]. This parameter is calculated as follows [15]:

$$\tau(u) = 0.3643u^2 + 0.1124u + 0.002 \quad (7)$$

By combining the static equation (5) with the dynamic equation (6), the general model of the ER damper is expressed as:

$$\begin{cases} F_d = k_0 x_p + c_0 x'_p + F_{er} \\ \tau(u)F'_{er} + F_{er} = \sigma u^{\beta} sign(x'_p) \end{cases} \quad (8)$$

where the parameters k_0, c_0, σ, β are determined through compression–release experiments, as shown in Table 1. This model highlights the ability of the ER damper to adjust its damping force according to the PWM voltage control signal and is employed in this study for further analysis.

Table 1. Parameters of the ER damper on the INOVE test rig based on the Bingham model [15]

Parameters	Compression	Rebound	Unit
k_0	263.1168	170.4045	N/m
c_0	64.6433	68.8289	N.s/m
σ	12.8157	17.0442	N
β	0.2373	0.3948	-

3. DEVELOPMENT OF THE CHARACTERISTIC CURVES OF THE ELECTRORHEOLOGICAL DAMPER IN THE BINGHAM MODEL

Based on the mathematical model of the ER damper presented in Section 2, the damping characteristic curves are constructed. The piston displacement is assumed in the form of a sinusoidal function $x_p = 0{,}025\, sin(5t)$, with the PWM duty ratio initially set at $u = 0.1$.

The simulation result of the force-displacement relationship is shown in Figure 2. The obtained curve has the form of a closed hysteresis loop, which represents the energy dissipation of the damper during one oscillation cycle. At this value of PWM, the maximum damping force is approximately ±18N, demonstrating the ability to generate significant resistance force even at a low control level.

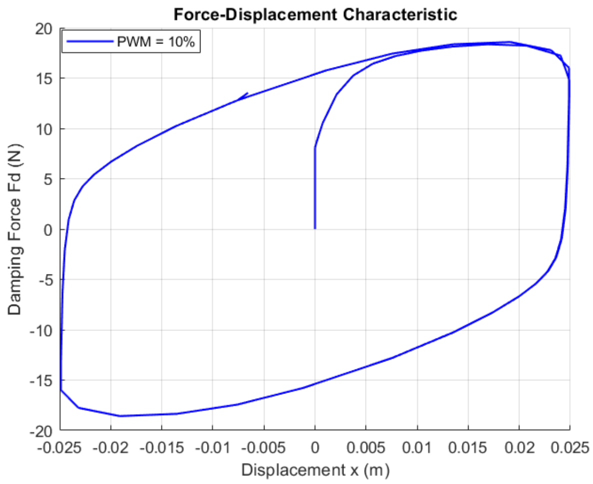

Figure 2. Relationship between damping force and piston displacement

The simulation result of the force–velocity relationship is shown in Figure 3. The curve also has a nonlinear form: the damping force remains nonzero even when the piston velocity is close to zero. This is a consequence of the yield stress characteristic of the ER fluid under the influence of an applied electric field [7, 11]. At this PWM level, the maximum damping force is also around ±18N corresponding to a piston velocity of about ±0.12m/s, which is consistent with the result from the force-displacement characteristic.

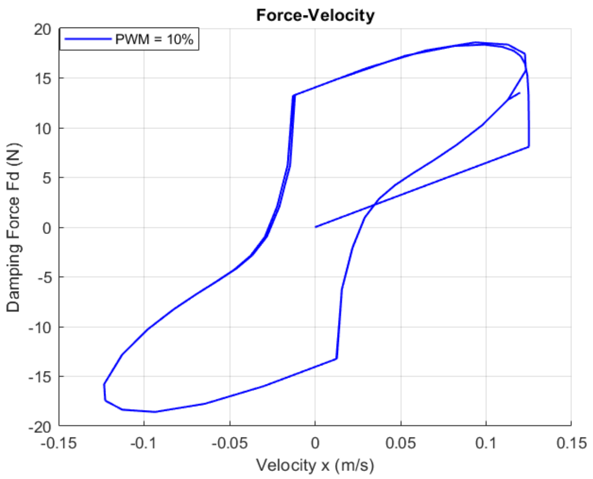

Figure 3. Relationship between damping force and piston velocity

Thus, the Bingham model not only accurately describes the nonlinear characteristics of the ER damper, but also allows the evaluation of its ability to adjust the damping force according to displacement amplitude and velocity. This result provides an important basis for the design of semi-active suspension controllers, while also confirming the feasibility of applying ER dampers under real operating conditions [3, 7, 12].

4. INVESTIGATION OF THE INFLUENCE OF CHARACTERISTICS ON THE MODEL

In order to evaluate the influence of the control signal on the damping characteristics, simulations are performed with different PWM duty cycles, including u={0.1, 0.15, 0.2, 0.25, 0.3}. The piston displacement profile is kept in the sinusoidal form: $x_p = 0{,}025\,sin(5t)$.

The simulation results of the force-displacement relationship are shown in Figure 4. When the PWM value increases, the hysteresis loop area becomes larger, indicating that the dissipated energy per oscillation cycle also increases. This demonstrates that the control voltage has a direct effect on the apparent viscosity and the yield stress of the ER fluid, thereby increasing the damping force.

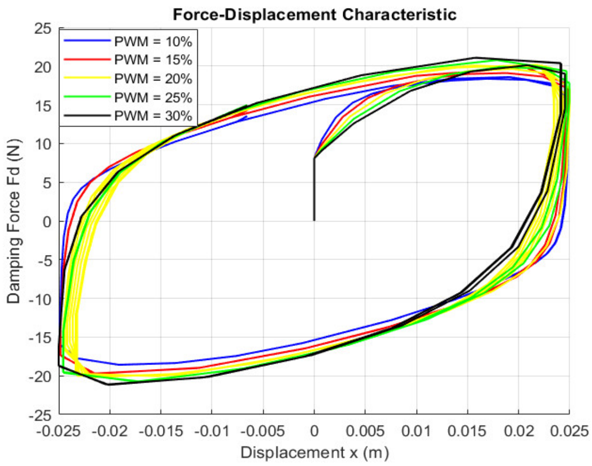

Figure 4. Relationship between damping force and piston displacement

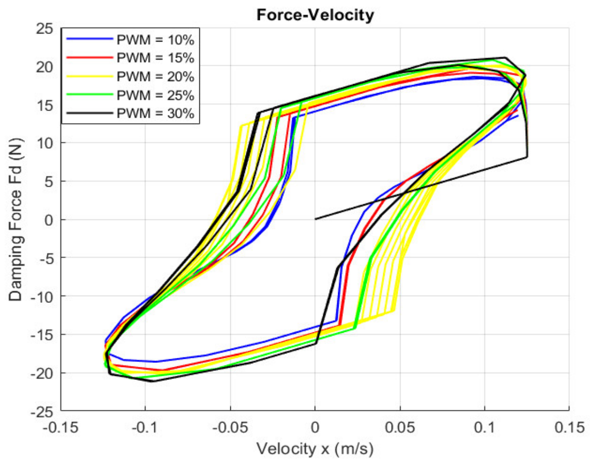

Figure 5. Relationship between damping force and piston velocity

The simulation results of the force-velocity relationship are presented in Figure 5. The obtained curves are clearly nonlinear: when the velocity increases, the damping force does not increase linearly but grows faster. This phenomenon is consistent with the rheological behavior of the ER fluid, which exhibits yield stress dependent on voltage and the ability to transition from a "soft" state to a "stiff" state as the electric field intensity increases [3, 12]. At the same time, the maximum force value obtained from the force-velocity curves is consistent with the result from the force-displacement characteristic

In general, when the PWM duty cycle changes from 0.1 to 0.3, the damping force increases significantly in both the force-displacement and force-velocity relationships. This proves that by adjusting the control signal, the damping characteristics of the system can be flexibly varied. The simulation results are consistent with previous studies on the application of ER fluids in semi-active suspension systems [7, 11, 12], and provide important data for the design of adaptive controllers for practical applications.

5. CONCLUSION

This paper has built a mathematical model of the ER damper based on the Bingham model and then used it to construct the force-displacement and force-velocity characteristic curves under different control conditions. The simulation results show that: The damping force has a nonlinear nature, depending on both the piston displacement and velocity; When the control voltage increases, the damping characteristics of the damper change from "soft" to "stiff," thereby allowing flexible adjustment of the vibration control capability; The force-displacement and force-velocity curves established from the model are consistent with theoretical analysis and experimental characteristics that have been published.

The results obtained in this study affirm the feasibility of applying ER dampers in semi-active suspension systems. At the same time, they provide a scientific basis for the design of adaptive control algorithms aimed at improving ride comfort, safety, and stability of passenger cars under real operating conditions.

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