SURVIVABILITY ANALYSIS OF UAVS WITH CYLINDRICAL RANDOM TRAJECTORY UNDER DIRECT-FIRE THREATS: A PROBABILISTIC APPROACH

Nguyen Hoang Viet[1],
Pham Thi Phuong Anh[1,*], Nguyen Vu[2]

DOI: https://doi.org/10.57001/huih5804.2026.112

ABSTRACT

This paper presents a probabilistic model for evaluating the survivability of unmanned aerial vehicles (UAVs) employing a cylindrical random trajectory (CRT) relative to straight-line flight under direct-fire threats. The per-shot hit probability is derived analytically using a Gaussian projectile dispersion model and the error function (erf). For the CRT, the UAV's lateral position is uniformly distributed over the upper semicircle of a cylinder surrounding the nominal flight axis; the expected hit probability is obtained by integrating the conditional hit probability over the angular distribution. Survival probability over a given engagement range is computed by compounding independent per-shot survival factors. With representative parameters cylinder radius $R = 20$m, gunner dispersion $\sigma = 6$m, and shot spacing $d = 30$m the CRT reduces the per-shot hit probability from approximately 5.3% to 0.9%, yielding survival probabilities of 54.7% versus 2.9% at a 1 km engagement range. The CRT thus offers a substantial and quantifiable survivability advantage. The model provides closed-form sensitivity expressions and identifies optimal design parameters for trajectory planning in contested environments.

Keywords: *UAV survivability; random trajectory; cylindrical evasion maneuver; Gaussian hit probability; direct-fire threat; error function; trajectory optimization.*

[1]Control, Automation in Production and Improvement of Technology Institute (CAPITI), Academy of Military Science and Technology (AMST), Vietnam
[2]Academy of Military Science and Technology (AMST), Vietnam
[*]Email: anh.phamphuonganh@gmail.com
Received: 07/3/2026
Revised: 07/5/2026
Accepted: 25/5/2026

1. INTRODUCTION

The increasing operational use of unmanned aerial vehicles in contested environments has driven significant research interest in survivability enhancement techniques. UAVs operating at low altitude face threats from direct-fire weapons, including small-caliber automatic weapons, heavy machine guns, and man-portable air-defense systems. Unlike maneuvering aircraft, small tactical UAVs have limited speed and payload capacity for active countermeasures, making trajectory design a primary passive survivability measure.

Classical survivability theory [1, 2] defines the probability that a platform survives an engagement as the complementary cumulative product of per-shot hit probabilities. For a target following a predictable trajectory, the gunner can aim with minimal lead compensation, maximizing hit probability. Trajectory unpredictability is therefore a direct survivability asset.

Several approaches to UAV trajectory randomization have been proposed in the literature, including sinusoidal lateral weaving [3], stochastic waypoint selection [4], and three-dimensional random walk models [5]. However, few studies provide closed-form analytical models that directly link trajectory geometry parameters to survivability metrics in a form amenable to system design.

This paper analyzes a specific trajectory construction method: the cylindrical random trajectory (CRT), in which waypoints are distributed randomly on the upper half of a cylinder surrounding the reference axis. This construction ensures the UAV maintains non-negative altitude offset while executing unpredictable lateral excursions. We derive the hit probability analytically, validate it through numerical integration, and present parametric sensitivity results that guide practical trajectory design.

2. MATERIALS AND METHODS

2.1. Trajectory Model

Let the reference (nominal) flight axis coincide with the *Z*-axis, connecting the UAV departure point to its designated target. A cylinder of radius *R* is defined coaxially with this reference axis. The CRT is constructed by the following algorithm:

• Projection points $\{z_i\}$ are placed at uniform intervals d along the Z-axis: $z_i = i*d$, for i = 1, 2,…, N.

• For each projection point, an angle θ_i is drawn independently and uniformly from [0, 2π).

• If $\theta_i \in (\pi, 2\pi)$, it is reflected as $\theta_i \leftarrow -\theta_i$ (equivalently mapped to the upper semicircle, i.e., [0, π]).

• The waypoint is placed at $(x_i, y_i, z_i) = (R*\cos\theta_i, R*\sin\theta_i, z_i)$, ensuring $y_i \geq 0$ at all waypoints.

• The trajectory is the piecewise-linear path connecting consecutive waypoints.

After the reflection operation, the effective distribution of θ_i is uniform on [0, π], corresponding to the upper semicircle. The lateral displacement projected onto a horizontal aim-point plane is $r = R*\sin\theta$, which follows the distribution of the absolute value of a sinusoidally weighted random variable on that interval. The *straight-line trajectory* is

2.2. Threat Model

The gunner fires at the predicted position of the UAV along its trajectory. For a straight-line UAV, the prediction is exact: the aim point coincides with the UAV's actual position at each shot. For the CRT UAV, the gunner observes only the reference axis and aims at $z = z_i$ on that axis, unaware of the instantaneous lateral offset.

The projectile aim-point error in the lateral plane is modeled as a zero-mean Gaussian random variable with standard deviation σ:

$$X \sim N(0, \sigma^2) \quad (1)$$

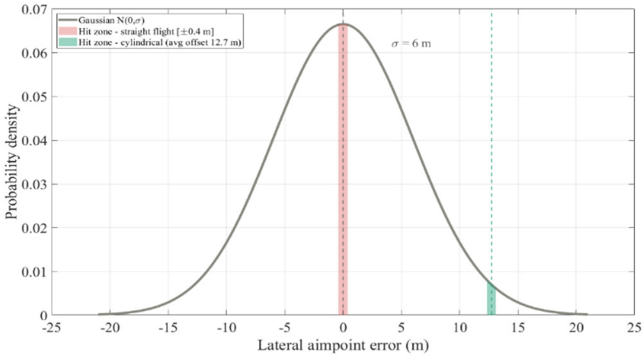

Figure 1. Gaussian projectile dispersion model and hit-zone definition

This Gaussian model (illustrated in Figure 1) is well-established in the ballistics literature [6, 7] and subsumes all lateral error sources weapon dispersion, aiming error, crosswind, and ballistic deviations into a single scalar parameter σ.

The UAV is modeled as presenting a rectangular cross-section with effective lateral half-width *a*. A *hit* is registered when the projectile lateral position falls within [−a, +a] relative to the UAV centerline.

2.3. Analytical hit-probability Model

2.3.1. Background: The Error Function

The error function is defined as:

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\, dt \quad (2)$$

It represents the probability that a standard normal variable lies within $[-x\sqrt{2}, +x\sqrt{2}]$. For a Gaussian variable $X \sim N(\mu, \sigma^2)$, the probability that $X \in [a, b]$ is given by:

$$P(a \leq X \leq b) = \frac{1}{2}\left[\mathrm{erf}\left(\frac{b-\mu}{\sigma\sqrt{2}}\right) - \mathrm{erf}\left(\frac{a-\mu}{\sigma\sqrt{2}}\right)\right] \quad (3)$$

This identity forms the analytical basis for all hit-probability expressions derived below.

2.3.2. Hit probability: Straight-Line Trajectory

For straight-line flight, the gunner aims directly at the UAV center (μ = 0). The per-shot hit probability is:

$$\mathrm{P_s} = \mathrm{erf}\left(\frac{a}{\sigma\sqrt{2}}\right) \quad (4)$$

2.3.3. Hit probability: Cylindrical Random Trajectory

When the UAV is displaced laterally by *r* from the gunner's aim point, the conditional hit probability is:

$$\mathrm{P(hit} \mid r) = \frac{\left[\mathrm{erf}\left(\frac{a+r}{\sigma\sqrt{2}}\right) - \mathrm{erf}\left(\frac{r-a}{\sigma\sqrt{2}}\right)\right]}{2} \quad (5)$$

Since $r = R*\sin\theta$ with θ uniformly distributed on [0, π], the expected per-shot hit probability for the CRT is:

$$\mathrm{P_C} = \frac{1}{\pi}\int_0^{\pi} \mathrm{P(hit} \mid R\sin\theta)\, \mathrm{d}\theta \quad (6)$$

Equation (6) does not admit a simple closed form but converges rapidly; numerical quadrature with $N = 500$ points achieves a relative error below 10^{-6} across all parameter combinations of practical interest. The **survivability gain ratio** is defined as:

$$G = \frac{\mathrm{P_s}}{\mathrm{P_C}} \quad (7)$$

and quantifies how many times more likely a hit is on the straight-line UAV compared to the CRT UAV, per shot.

2.3.4. Survival Probability Over Engagement Range

Assuming firing opportunities occur at uniform spacing *d* along the engagement axis, the total number of shots over range L is $N = \lfloor L/d \rfloor$. Treating successive shots as independent Bernoulli trials, the cumulative survival probabilities for the straight-line and CRT configurations are, respectively:

$$S_s(L) = (1 - P_s)^N \quad (8)$$

$$S_C(L) = (1 - P_C)^N \quad (9)$$

The ratio S_C/S_s grows *exponentially* with N, so the per-shot survivability advantage of the CRT is compounded and progressively amplified over extended engagements.

3. RESULTS AND DISCUSSION

3.1. Baseline configuration

Table 1 summarizes the baseline parameter set used throughout all numerical evaluations.

Table 1. Baseline parameters for numerical evaluation

Parameter	Symbol	Baseline Value
UAV effective lateral half-width	a	0.4m
Gunner lateral dispersion (1σ)	σ	6m
Shot spacing along range axis	d	30m
Cylinder radius	R	20m
Engagement range	L	500 - 2,000m

With these parameters, Eq. (4) yields $P_s \approx 5.3\%$ per shot, while numerical evaluation of Eq. (6) gives $Pc \approx 0.9\%$ per shot, corresponding to a survivability gain ratio of $G \approx 5.9$.

3.2. Survival probability vs. engagement range

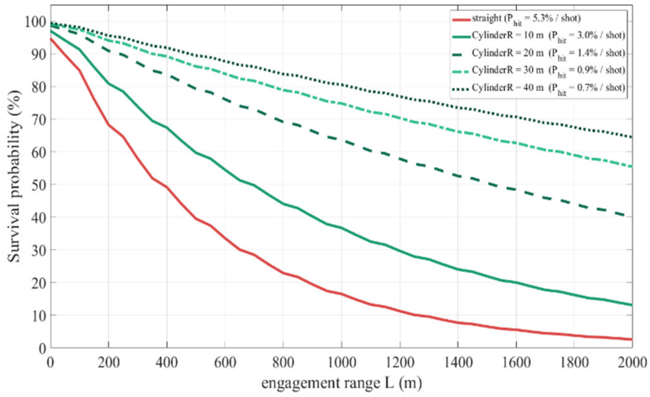

Figure 2. Survival probability as a function of engagement range (σ = 6m, d = 30m). Curves are shown for straight-line flight and CRT with R ∈ {10, 20, 30, 40} m

Figure 2 plots survival probability as a function of engagement range for straight-line flight and for the CRT with $R \in \{10, 20, 30, 40\}$ m. The survival probability of the straight-line UAV collapses rapidly: at 1 km, it retains only **2.9%** survival probability. By contrast, the CRT with $R = 20$m maintains **54.7%** survival probability at the same range, an improvement of nearly 19-fold. The exponential compounding of the per-shot advantage is clearly visible as engagement range increases, with larger R values providing progressively greater separation from the straight-line baseline.

3.3. Conditional hit probability vs. lateral offset

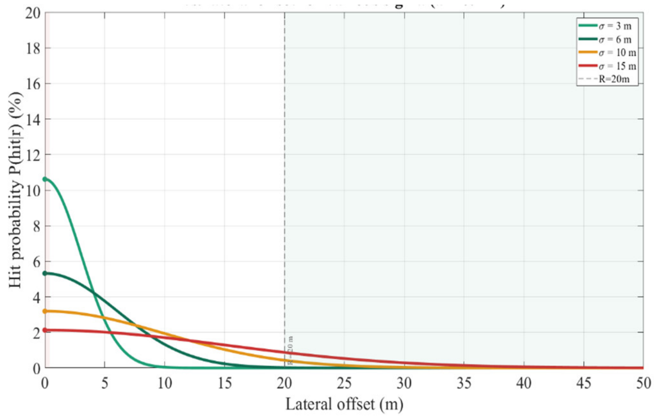

Figure 3. Conditional hit probability P(hit | r) as a function of lateral offset r for selected values of gunner dispersion σ

Figure 3 displays *P(hit|r)* from Eq. (4) as a function of lateral offset r for four values of gunner dispersion σ. As r increases, the hit probability decays monotonically toward zero, driven by the exponential tail of the Gaussian density. For the baseline case ($\sigma = 6$m, $R = 20$m), the mean lateral offset is approximately 12.7m, at which the conditional hit probability falls to roughly 1.5%. This behavior confirms that even moderate lateral excursions provide substantial protection, and that the benefit is more pronounced under lower gunner dispersion (smaller σ), where the Gaussian density is more concentrated around the aim point.

3.4. Parametric sensitivity analysis

Figure 4 presents single-factor sensitivity results evaluated at $L = 1$km. Three principal observations emerge:

- Increasing R monotonically improves CRT survival probability, although diminishing returns become apparent above $R \approx 30$m for $\sigma = 6$m, as the Gaussian tail is already effectively exploited at that offset.
- Both trajectory types benefit from larger gunner dispersion σ, but the CRT gains more steeply, widening the survival advantage over straight-line flight. This reflects the fact that higher dispersion depresses P_s and P_c proportionally, but the absolute reduction in P_c is larger.

- Shorter shot spacing *d* (denser fire) penalizes the straight-line UAV disproportionately, because its elevated per-shot hit probability compounds more rapidly with each additional firing opportunity.

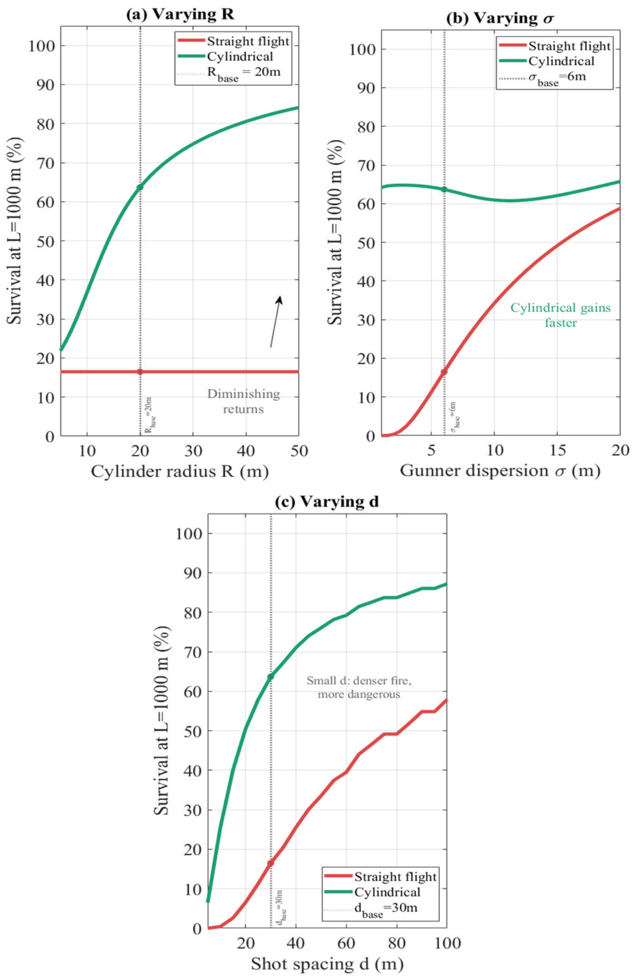

Figure 4. Sensitivity of survival probability at L = 1km to: (a) cylinder radius R; (b) gunner dispersion σ; (c) shot spacing d

Table 2 summarizes per-shot hit probability, gain ratio, and cumulative survival probability at 500m and 1,000m for a range of cylinder radii.

Table 2. Per-shot hit probability, survivability gain ratio, and cumulative survival probabilities at 500m and 1km for selected cylinder radii (σ = 6m, d = 30m, a = 0.4m)

R (m)	Pc / shot (%)	Gain G	S(500 m) (%)	S(1,000 m) (%)
Straight (R = 0)	5.30	1.0×	29.9	2.9
10	2.82	1.9×	51.1	26.1
20	0.92	5.8×	75.1	56.4
30	0.42	12.6×	87.2	76.0
40	0.23	23.0×	93.1	86.8

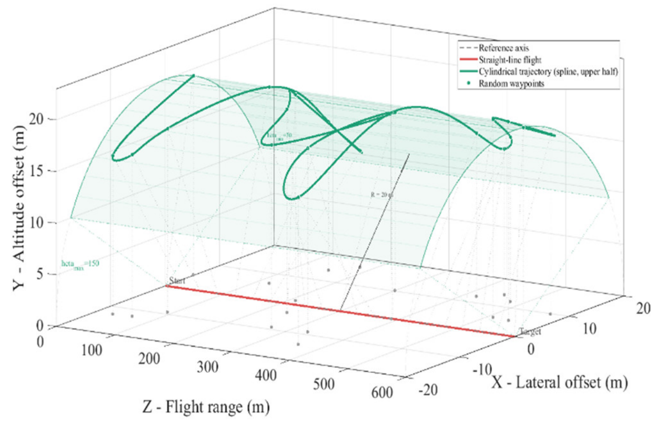

Figure 5a. Three-dimensional visualization of a representative CRT realization

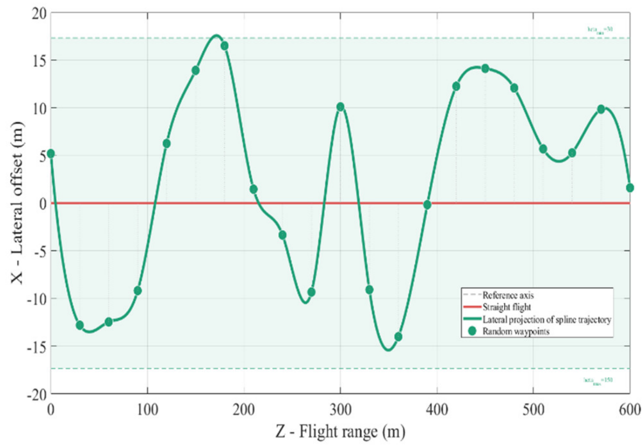

Figure 5b. Top-down (XZ-plane) projection of the same CRT realization

Figure 5 illustrates a representative CRT realization: Figure 5a shows the three-dimensional piecewise-linear path connecting the randomly sampled cylindrical-surface waypoints, and Figure 5b shows the corresponding top-down projection onto the XZ plane.

3.5. Physical interpretation

The survivability advantage of the CRT arises from two distinct but complementary mechanisms. First, the lateral offset *r* displaces the UAV from the vicinity of the gunner's aim point, exploiting the rapid exponential decay of the Gaussian probability density function in its tails. Second, the random and unpredictable angular component of each waypoint prevents adaptive aim-point correction: any observation of a previous waypoint provides no statistical information about the angular position of the next waypoint, rendering trajectory-learning strategies ineffective.

The upper-semicircle constraint ($y_i \geq 0$) is operationally significant, as it ensures the UAV maintains a non-negative altitude offset relative to the reference axis throughout the mission, eliminating the risk of terrain contact due to downward lateral excursions. Crucially, this safety constraint incurs no survivability penalty: by symmetry, the distribution of $r = |R \cdot sin\theta|$ is identical for $\theta \in [0, \pi]$ (upper semicircle) and $\theta \in [-\pi, \pi]$ (full circle).

3.6. Optimal cylinder radius

The gain ratio *G* is a monotonically increasing function of *R* for all σ > 0, implying that a larger cylinder radius is always preferable from a pure survivability standpoint. However, practical constraints impose an upper bound $R \le R_{max}$, where R_{max} is determined by terrain clearance requirements, obstacle avoidance envelopes, and mission-specific geometric constraints. Within this bound, *R* should be maximized.

A secondary consideration is path-length growth. The Euclidean inter-waypoint distance scales as approximately $\sqrt{(d^2 + 4R^2)}$ in the worst case. For the baseline parameters (*R* = 20 m, *d* = 30 m), this yields ≈36 m - a 20% path-length increase which must be accounted for in energy and endurance budgets of limited-range platforms.

3.7. Model Limitations

The present model rests on four principal simplifying assumptions:

• A single threat source with fixed dispersion σ is assumed. Real engagements may involve multiple simultaneous shooters at diverse azimuthal angles, which would further degrade aim-point predictability and likely amplify the CRT advantage.

• Shot outcomes are modeled as independent Bernoulli trials. Burst fire and gun-laying dynamics may introduce inter-shot correlation not captured here.

• The UAV is represented by a rectangular cross-section with a fixed effective half-width a, neglecting aspect-angle dependence of the presented area.

• The gunner is assumed non-adaptive. An adversary capable of inferring R from trajectory observation could partially offset the survivability gain; this scenario is deferred to future work.

Additionally, the model does not account for angular-slew-rate limitations of gun systems. For large *R* and small *d*, the required lateral tracking rate may exceed the weapon's mechanical slew limit, conferring additional CRT advantage beyond the present model's predictions.

4. CONCLUSION

This paper has presented a closed-form probabilistic framework for quantifying UAV survivability under a cylindrical random trajectory strategy against direct-fire threats. The three principal contributions are: (1) analytical expressions for the per-shot hit probability for both straight-line and CRT flight, derived from a Gaussian projectile dispersion model via the error function; (2) cumulative survival probability curves as a function of engagement range, obtained by compounding independent per-shot survival factors; and (3) a parametric sensitivity analysis characterizing the individual effects of cylinder radius *R*, gunner dispersion σ, and shot spacing *d* on survivability.

Numerical evaluation with representative parameters demonstrates that the CRT reduces the per-shot hit probability by a factor of approximately six relative to straight-line flight, translating to a survival probability of 54.7% versus 2.9% at a 1km engagement range. This advantage grows exponentially with range, making the CRT strategy increasingly effective in long-range engagements.

The model provides a practical design tool for trajectory planners: given an estimate of the threat dispersion σ and an operational bound on the lateral excursion radius *R*, Eqs. (4)-(9) yield the expected survival probability at any engagement range. Future work will extend the framework to adaptive gunners, burst-fire threat models, multi-threat geometries, and real-time onboard path-planning integration subject to terrain constraints.

REFERENCES

[1]. Ball R. E., *The Fundamentals of Aircraft Combat Survivability Analysis and Design* (2nd ed.). AIAA Education Series, 2003.

[2]. Przemieniecki J. S., *Mathematical Methods in Defense Analyses* (2nd ed.). AIAA, 1994.

[3]. Boyko N., Sheridan T. B., "Trajectory randomization for UAV survivability in hostile environments," in *Proceedings of the 9th AIAA Aviation Technology, Integration and Operations Conference*, 2009.

[4]. Foo J. L., Knutzon J., Kalivarapu V., Oliver J., Winer E., "Path planning of unmanned aerial vehicles in a constrained environment," in *AIAA Infotech@Aerospace Conference*, 2006.

[5]. Jun M., D'Andrea R., "Path planning for unmanned aerial vehicles in uncertain and adversarial environments," *Cooperative Control: Models, Applications and Algorithms*, Springer, 95-110, 2002.

[6]. McCoy R. L., *Modern Exterior Ballistics*. Schiffer Publishing, 1999.

[7]. Driels M., *Weaponeering: Conventional Weapon System Effectiveness*. AIAA Education Series, 2013.