LINEAR PARAMETER-VARYING MODEL PREDICTIVE CONTROL FOR QUADCOPTER ATTITUDE AND TRAJECTORY TRACKING

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ABSTRACT

This paper presents the design and validation of a Linear Parameter-Varying Model Predictive Control (LPV-MPC) scheme for quadcopter unmanned aerial vehicles. Due to the strongly nonlinear and coupled nature of quadcopter dynamics, conventional linear MPC approaches exhibit limited performance over a wide operating envelope, while nonlinear MPC often leads to excessive computational burden. To address this trade-off, the quadcopter attitude dynamics are modeled using an LPV framework, enabling the formulation of an efficient MPC controller with explicit constraint handling. Simulation results demonstrate accurate trajectory tracking and robust attitude regulation.

Keywords: *Quadcopter UAV, LPV systems, Model Predictive Control, Attitude control, Constraint handling.*

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1. INTRODUCTION

Unmanned aerial vehicles (UAVs), particularly quadcopter platforms, have attracted significant research and industrial interest over the past decade due to their high maneuverability, mechanical simplicity, and vertical take-off and landing capabilities.

These characteristics make quadcopters well suited for a wide range of applications, including aerial surveillance, infrastructure inspection, environmental monitoring, and search-and-rescue missions [1, 2]. Despite these advantages, the control of quad- copter UAVs remains a challenging task owing to their inherently nonlinear and under-actuated dynamics, strong coupling between translational and rotational motions, and the presence of physical constraints and external disturbances.

Model Predictive Control (MPC) has emerged as a powerful control paradigm for UAV systems, primarily due to its capability to explicitly handle state and input constraints while optimizing system performance over a finite prediction horizon [3, 4]. Linear MPC formulations based on locally linearized models are computationally efficient and have been successfully applied to quadcopter control in near-hover conditions. However, their performance degrades significantly during aggressive maneuvers or when the UAV operates far from the nominal operating point, where nonlinear effects become dominant [5].

Nonlinear Model Predictive Control (NMPC) provides a more accurate representation of quadcopter dynamics and has demonstrated superior tracking performance and robustness in simulation and experimental studies [6, 7]. Nevertheless, the resulting nonlinear optimization problems are generally non-convex and computationally demanding, which poses serious challenges for real-time implementation on embedded UAV hardware with limited computational resources. This trade-off between modeling accuracy and real-time feasibility has motivated the development of intermediate modeling and control frameworks.

Linear Parameter-Varying (LPV) modeling offers an attractive compromise by representing nonlinear dynamics as a family of linear models whose parameters vary as measurable scheduling variables. This approach preserves the linear structure required by efficient MPC solvers while capturing dominant nonlinear behaviors over a wide operating envelope [8,9]. Recent studies have demonstrated the effectiveness of LPV-based control and LPV–MPC strategies for aerial vehicles, showing improved performance compared to fixed linear controllers while maintaining real-time feasibility [10, 11].

Although LPV-MPC approaches for UAV systems have been investigated in recent works such as [9, 10, 11], the present study differs from these contributions in several important aspects. In particular, [9] mainly focuses on LPV modeling and experimental validation for quadrotor platforms, whereas this paper derives an LPV formulation specifically for the rotational (attitude) dynamics using measurable scheduling variables.

Furthermore, [10] provides a general LPV–MPC framework for constrained nonlinear systems, while this work develops a dedicated MPC structure for quadcopter attitude regulation, including control increment formulation and explicit moment/rate constraints. Compared with [11], which addresses combined attitude and position tracking using LPV-MPC, the proposed method emphasizes a hierarchical cascade control architecture, where LPV-MPC is implemented in the fast inner-loop for attitude stabilization and integrated with an outer-loop trajectory reference generator. Therefore, the main novelty of this paper lies in the specific LPV modeling of quadcopter attitude dynamics and its integration into a constrained hierarchical LPV-MPC control framework.

In this paper, an LPV–MPC controller is developed for quadcopter attitude control and integrated into a hierarchical cascade control architecture for trajectory tracking. The LPV-MPC operates in the inner loop to regulate the fast rotational dynamics, while an outer-loop controller generates attitude references based on position tracking objectives. This time-scale separation allows the proposed approach to achieve accurate tracking, constraint satisfaction, and improved robustness across a wide range of operating conditions.

The main contributions of this work can be summarized as follows:

• An LPV state-space model is derived for quadcopter attitude dynamics, capturing key nonlinear effects through parameter-dependent coefficients.

• An LPV-MPC controller with explicit input and rate constraints is formulated and implemented for inner-loop attitude regulation.

• The effectiveness of the proposed control framework is demonstrated through numerical simulation studies under representative flight scenarios.

• These contributions distinguish the proposed approach from recent LPV-MPC studies such as [9-11], where different modeling assumptions and control architectures were considered.

2. QUADCOPTER DYNAMIC MODEL

This section presents the dynamic model of the quadcopter UAV used for the design of the attitude controller. The model focuses on the rotational dynamics, which constitute the inner-loop subsystem in the hierarchical control architecture adopted in this work.

2.1. Coordinate Frames and State Variables

The orientation of the quadcopter is described with respect to an inertial (Earth- fixed) reference frame using the Euler angle parameterization, consisting of roll ϕ, pitch θ, and yaw ψ. The corresponding angular velocities expressed in the body-fixed frame are denoted by p, q, and r, respectively.

Based on this representation, the attitude-related state vector is defined as

$$x = [\phi \ \ \theta \ \ \psi \ \ p \ \ q \ \ r]^{\mathrm{T}}, \tag{1}$$

which combines the orientation variables and their associated body angular rates.

The control inputs to the rotational subsystem are the body moments generated by the quadcopter actuators. These inputs are collected in the control vector.

$$u = [U_2 \ U_3 \ U_4]^T \tag{2}$$

where U_2, U_3, and U_4 represent the roll, pitch, and yaw moments, respectively. These moments are produced through differential thrusts and reaction torques of the four rotors and serve as the direct control variables for attitude regulation.

2.2. Rotational Dynamics

The rotational motion of the quadcopter is governed by the rigid-body Euler equations expressed in the body-fixed frame. Assuming that the body axes are aligned with the principal axes of inertia, the angular rate dynamics are given by

$$\dot{p} = \frac{(I_{yy}-I_{zz})qr - J_{tp}q\Omega + U_2}{I_{xx}} \tag{3}$$

$$\dot{q} = \frac{(I_{zz}-I_{xx})pr - J_{tp}p\Omega + U_3}{I_{yy}} \tag{4}$$

$$\dot{r} = \frac{(I_{xx}-I_{yy})pq + U_4}{I_{zz}} \tag{5}$$

Where, I_{xx}, I_{yy}, and I_{zz} denote the moments of inertia about the body-frame axes, J_{tp} is the total rotor inertia, and Ω represents the aggregate rotor angular speed. The coupling terms involving products of angular rates account for Coriolis and gyroscopic effects, while the terms proportional to $J_{tp}\Omega$ capture the gyroscopic moments induced by the rotating propellers.

The kinematic relationships between the Euler angle derivatives and the body angular rates are expressed as

$$\begin{bmatrix}\dot\phi\\ \dot\theta\\ \dot\psi\end{bmatrix} = \begin{bmatrix}1 & 0 & -\sin\theta\\ 0 & \cos\phi & \sin\phi\cos\theta\\ 0 & -\sin\phi & \cos\phi\cos\theta\end{bmatrix}\begin{bmatrix}p\\ q\\ r\end{bmatrix} \quad (6)$$

This nonlinear kinematic transformation introduces trigonometric coupling among the attitude variables and leads to singularities when θ approaches $\pm\pi/2$. Despite this limitation, the Euler angle representation is adopted in this work due to its intuitive physical interpretation and its widespread use in hierarchical quadcopter control architectures.

3. LPV MODELING OF ATTITUDE DYNAMICS

This section presents the Linear Parameter-Varying (LPV) modeling framework adopted for the quadcopter attitude dynamics. The LPV representation serves as an intermediate modeling approach that bridges the gap between fixed linear models and fully nonlinear formulations, enabling accurate prediction over a wide operating envelope while preserving computational tractability.

3.1. LPV State-Space Representation

As shown in the previous section, the quadcopter rotational dynamics and Euler angle kinematics exhibit pronounced nonlinearities arising from trigonometric functions of the attitude angles and bilinear coupling terms involving the body angular rates. These nonlinear effects become significant during aggressive maneuvers and cannot be adequately captured by a single linear model obtained around a nominal operating point.

To address this limitation, the attitude dynamics are reformulated within an LPV framework, in which the system is represented as a linear state-space model with matrices that depend on a vector of measurable scheduling variables. The continuous-time LPV model is expressed as

$$\dot{x}(t) = A(\boldsymbol{\rho}(t))\boldsymbol{x}(t) + B(\boldsymbol{p}(t))\boldsymbol{u}(t) \quad (7)$$

where $\boldsymbol{x}(t)$ and $\boldsymbol{u}(t)$ denote the state and control input vectors defined in the previous section, and $\boldsymbol{p}(t)$ is the scheduling parameter vector.

In this work, the scheduling variables are selected as measurable attitude angles and angular rates, namely

$$\boldsymbol{p}(t) = [\,\phi(t)\ \ \theta(t)\ \ p(t)\ \ q(t)\ \ r(t)\,] \quad (8)$$

This choice is motivated by the fact that the dominant nonlinearities in the rotational dynamics explicitly depend on these variables, while they can be directly measured or reliably estimated using onboard inertial sensors. Consequently, the LPV representation captures the essential nonlinear behavior of the quadcopter attitude dynamics without introducing additional states or unmeasurable parameters. For fixed values of the scheduling variables, the system reduces to a linear time-varying model.

Importantly, the state-space matrices remain affine functions of the state and input variables, which is a key requirement for the efficient solution of MPC optimization problems. Similar LPV formulations have been successfully applied to multirotor UAVs and other nonlinear aerospace systems [8, 9].

Accordingly, the parameter-dependent system matrix $A(\rho)$takes the form:

$$A(\rho) = \begin{bmatrix}0 & 0 & 0 & f_1(\phi,\theta) & f_2(\phi,\theta) & 0\\ 0 & 0 & 0 & f_3(\phi,\theta) & f_4(\phi,\theta) & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & a_{12}(r) & a_{13}(q)\\ 0 & 0 & 0 & a_{21}(r) & 0 & a_{23}(p)\\ 0 & 0 & 0 & a_{31}(q) & a_{32}(p) & 0\end{bmatrix}$$

where the coefficients $a_{ij}(\cdot)$represent the bilinear coupling terms:

$$a_{12}(r) = \frac{I_{yy}-I_{zz}}{I_{xx}}r, \quad a_{13}(q) = \frac{I_{yy}-I_{zz}}{I_{xx}}q,$$
$$a_{21}(r) = \frac{I_{zz}-I_{xx}}{I_{yy}}r, \quad a_{23}(p) = \frac{I_{zz}-I_{xx}}{I_{yy}}p,$$
$$a_{31}(q) = \frac{I_{xx}-I_{yy}}{I_{zz}}q, \quad a_{32}(p) = \frac{I_{xx}-I_{yy}}{I_{zz}}p.$$

The input matrix $B(\rho)$remains constant:

$$B = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 1/I_{xx} & 0 & 0\\ 0 & 1/I_{yy} & 0\\ 0 & 0 & 1/I_{zz}\end{bmatrix}.$$

3.2. Discrete-Time LPV Model

For digital implementation within the model predictive control framework, the continuous-time LPV model is discretized using a zero-order hold assumption on the control inputs. Given a sampling period T_s, the discrete-time LPV model can be written as

$$\boldsymbol{x}_{k+1} = A_d(\boldsymbol{\rho}_k)\boldsymbol{x}_k + B_d(\boldsymbol{\rho}_k)\boldsymbol{u}_k \quad (9)$$

Where $\boldsymbol{\rho}_k = \boldsymbol{\rho}(kT_s)$ denotes the scheduling vector evaluated at the current sampling instant.

The discrete-time system matrices $A_d(\boldsymbol{\rho}_k)$ and $B_d(\boldsymbol{\rho}_k)$ are obtained by discretiz- ing the corresponding continuous-time matrices at each time step. In practice, this discretization can be performed using first-order or higher-order numerical inte- gration methods, depending on the desired accuracy and computational budget. Since the attitude controller operates in the inner loop at a high sampling rate, the zero-order hold discretization provides a suitable compromise between accuracy and efficiency.

The LPV model is discretized using a zero-order hold (ZOH) method, assuming constant control inputs over each sampling interval. The discrete-time matrices are computed as:

$$A_d(\rho_k) = \exp(A(\rho_k)T_s)$$

$$B_d(\rho_k) = \int_0^{T_s} \exp(A(\rho_k)\tau)B\, d\tau$$

In the simulations, the sampling period is selected as: $T_s = 0.01s$

which is consistent with typical inner-loop attitude control frequencies (100Hz) for quadcopter UAVs.

The resulting discrete-time LPV model forms the prediction model employed by the LPV-MPC controller in the subsequent section. By updating the system matrices online according to the current scheduling variables, the proposed approach achieves improved prediction accuracy compared to fixed linear MPC.

4. LPV-MPC FORMULATION

Based on the discrete-time LPV model derived in the previous section, a Linear Parameter-Varying Model Predictive Control (LPV-MPC) scheme is developed for quadcopter attitude regulation. The objective of the controller is to track attitude reference trajectories while explicitly accounting for actuator constraints and ensuring smooth control actions suitable for practical implementation.

4.1. Prediction Model

In quadcopter attitude control, abrupt variations in control moments may lead to actuator saturation, excessive motor wear, and degraded tracking performance. To address this issue, the control problem is formulated in terms of control increments rather than absolute control inputs. Specifically, the control increment is defined as

$$\Delta\boldsymbol{u}_k = \boldsymbol{u}_k - \boldsymbol{u}_{k-1} \quad (10)$$

An augmented state vector is introduced to incorporate the previous control input into the prediction model, which allows the control rate to be directly penalized in the MPC cost function. Let $\tilde{\boldsymbol{x}}_{\boldsymbol{k}}$ denote the augmented state at time step *k*. Over

a prediction horizon of length *N*, the stacked prediction model can be compactly written as

$$\tilde{\boldsymbol{X}} = G\Delta\boldsymbol{U} + \hat{A}\tilde{\boldsymbol{x}}_0, \quad (11)$$

where $\Delta\boldsymbol{U}$ is the stacked vector of future control increments, and $\tilde{\boldsymbol{X}}$ collects the predicted augmented states over the horizon. The matrices G and $\hat{A}$ are constructed from the discrete-time LPV system matrices and depend implicitly on the current scheduling variables.

This augmented formulation is widely adopted in MPC-based UAV control, as it improves numerical conditioning and enables direct handling of actuator rate constraints without increasing the complexity of the optimization problem [3, 4].

4.2. Cost Function

The LPV-MPC optimization problem aims to minimize a quadratic cost function that balances attitude tracking accuracy and control smoothness. The cost function is defined as

$$J = \frac{1}{2}\boldsymbol{e}_N^{\top}Q_f\boldsymbol{e}_N + \frac{1}{2}\sum_{k=0}^{N-1}\left(\boldsymbol{e}_k^{\top}Q\boldsymbol{e}_k + \Delta\boldsymbol{u}_k^{\top}R\Delta\boldsymbol{u}_k\right) \quad (12)$$

where $\boldsymbol{e}_{\boldsymbol{k}}$ denotes the attitude tracking error at step *k*, and *Q*, Q_f, and *R* are positive (semi-)definite weighting matrices.

The stage cost penalizes deviations from the reference attitude as well as large control increments, while the terminal cost enhances closed-loop performance and improves stability properties. By tuning the weighting matrices, the designer can explicitly trade off tracking performance against actuator effort and smoothness. This quadratic structure preserves convexity of the optimization problem and is well suited for real-time implementation [3, 10].

4.3. Constraints

A key advantage of MPC lies in its ability to explicitly incorporate physical constraints into the control design. In the proposed LPV–MPC framework, the following constraints are enforced at each prediction step:

$$\boldsymbol{u}_k \& \in \mathcal{U} \quad (13)$$

$$\Delta\boldsymbol{u}_k \in \Delta\mathcal{U} \quad (14)$$

$$\boldsymbol{x}_k \in \mathcal{X} \quad (15)$$

where $\mathcal{U}$ denotes the admissible set of control moments determined by actuator limitations, $\Delta\mathcal{U}$ represents bounds on control rate variations, and $\mathcal{X}$ defines allowable bounds on attitude angles and angular rates.

These constraints ensure that the computed control sequence remains feasible with respect to physical actuator limits and safety requirements throughout the prediction horizon. Under the LPV formulation, the resulting optimization problem can be expressed as a convex quadratic program (QP) and efficiently solved at each sampling instant using standard QP solvers. This property is crucial for real-time attitude control in quadcopter UAVs operating at high control frequencies [10, 11].

5. SIMULATION RESULTS

Simulation studies are conducted using a high-fidelity quadcopter model with realis- tic physical parameters. The proposed LPV-MPC controller achieves accurate attitude tracking while respecting actuator constraints. Control inputs remain smooth and bounded throughout the simulations.

Table 1. Quadcopter physical parameters used in simulation

Parameter	Symbol	Value
Mass	(m)	1.2kg
Moment of inertia (x-axis)	(I_{xx})	0.015kg·m^2
Moment of inertia (y-axis)	(I_{yy})	0.015kg·m^2
Moment of inertia (z-axis)	(I_{zz})	0.030kg·m^2
Maximum roll moment	($U_{2,max}$)	±0.8N·m
Maximum pitch moment	($U_{3,max}$)	±0.8N·m
Maximum yaw moment	($U_{4,max}$)	±0.5N·m
Control rate limit	(ΔU_{max})	±0.2N·m/step
Sampling time	(T_s)	0.01s
Prediction horizon	(N)	20

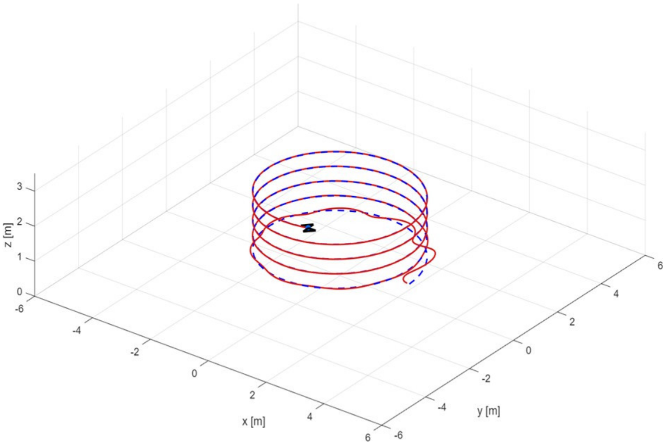

Figure 1. Three-dimensional trajectory tracking using the proposed LPV-MPC controller

Figure 1 shows the three-dimensional trajectory tracking performance of the quad- copter. The actual trajectory closely follows the reference path throughout the maneuver, demonstrating accurate spatial tracking and coordinated motion. The smooth evolution of the trajectory indicates stable closed-loop behavior and effective compensation of nonlinear coupling effects.

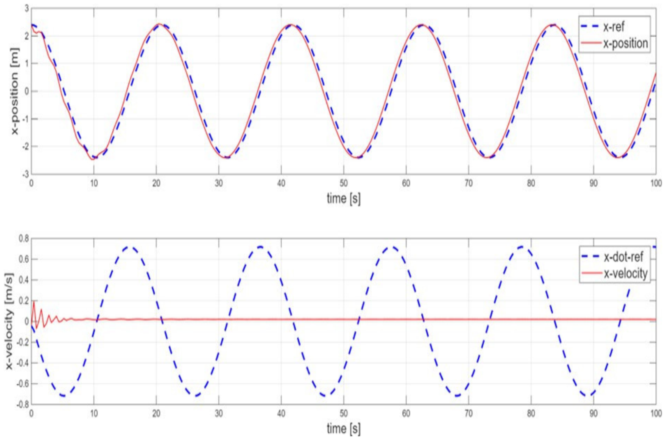

Figure 2. Tracking performance along the x-axis using the Feedback Linearization- LPV-MPC controller

As illustrated in Figure 2, the quadcopter position along the *x*-axis closely follows the sinusoidal reference trajectory. A small transient tracking error is observed during the initial phase; however, the response quickly converges and the steady- state error remains negligible, indicating effective horizontal position regulation.

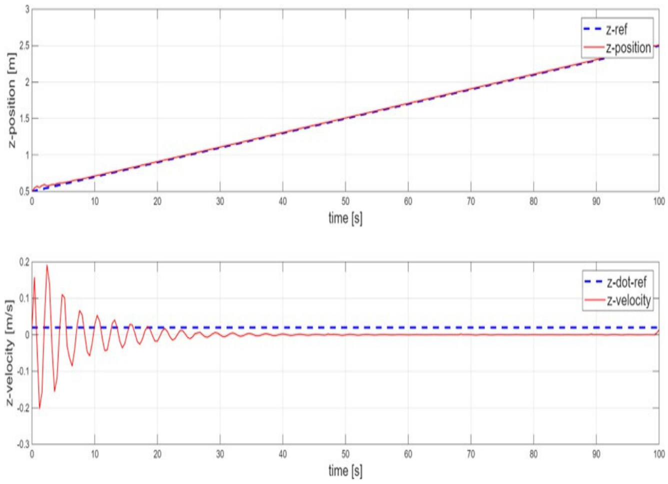

Figure 3. Tracking performance along the z-axis using the Feedback Linearization- LPV-MPC controller

Figure 3 presents the tracking results along the *z*-axis. Similar to the *z*-direction, the controller achieves accurate tracking with minimal steady-state error. The smooth transient response confirms the ability of the proposed control scheme to handle coupled translational dynamics.

The Euler angle responses shown in Figure 4 indicate stable and accurate attitude regulation. The roll and pitch angles exhibit small oscillations at the beginning of the maneuver, which are rapidly attenuated. The yaw angle tracks a time-varying reference without inducing instability, highlighting the effectiveness of the LPV- MPC controller in handling rotational dynamics.

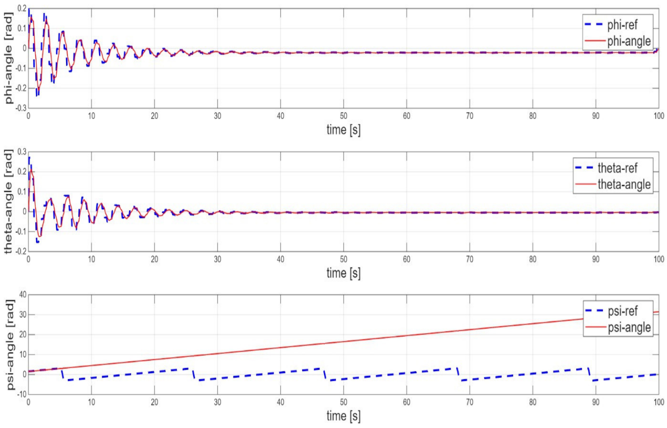

Figure 4. Euler angle tracking performance using the Feedback Linearization-LPV-MPC controller

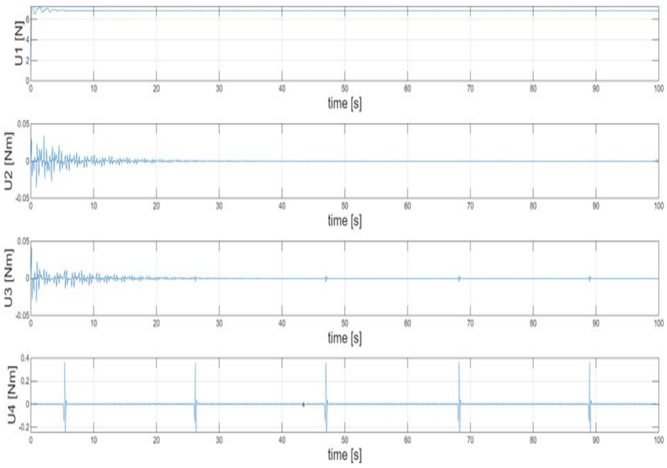

Figure 5. Control inputs generated by the LPV-MPC controller

Figure 5 illustrates the control inputs produced by the LPV-MPC controller. The thrust and control moments remain within their admissible bounds throughout the simulation. Short-duration peaks occur during rapid reference changes, particularly in the yaw channel, but no sustained actuator saturation is observed, confirming effective constraint handling and smooth control action.

6. CONCLUSION

This paper presented a Linear Parameter-Varying Model Predictive Control (LPV- MPC) framework for quadcopter attitude control within a hierarchical control archi- tecture. By exploiting an LPV representation of the attitude dynamics, the proposed controller captures dominant nonlinear effects while retaining the computational ef- ficiency of quadratic programming-based MPC. Simulation results demonstrated accurate trajectory tracking, stable attitude regulation, and effective constraint handling under representative flight scenarios. Future work will focus on formal robustness analysis and experimental validation on real quadcopter platforms.

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