TRAJECTORY TRACKING CONTROL FOR A QUADCOPTER UAV BASED ON SLIDING MODE CONTROL AND PD COMBINED WITH FUZZY LOGIC

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ABSTRACT

Trajectory tracking control is one of the most critical issues that needs to be addressed in the field of unmanned aerial vehicles (UAVs), particularly for Quadcopters. This study proposes an advanced control approach that combines Fuzzy Logic, Proportional Derivative-Sliding Mode Control (PD-SMC), and Hierarchical Sliding Mode Control (HSMC) to develop an adaptive control system for a Quadcopter. In this approach, the fuzzy logic controller adaptively tunes the proportional and derivative gains of the PD controller based on the position error along the x-y axes and its derivative, thereby generating appropriate reference inclination angles according to the actual flight trajectory. Using these reference angles, the SMC computes control inputs to regulate the quadcopter's rotational angles (roll, pitch, and yaw), while the HSMC is employed to control its position. Simulation results performed in MATLAB & Simulink with a complex spiral trajectory demonstrate that the proposed Fuzzy-PD-SMC-HSMC method achieves higher trajectory tracking accuracy and significantly reduces tracking error amplitudes compared to the conventional PD-SMC-HSMC controller, owing to the adaptive flexibility of the proposed control system.

Keywords: *Fuzzy Logic, Proportional Derivative, Sliding Mode Control, Hierarchical Sliding Mode Control, Quadcopter*

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1. INTRODUCTION

Unmanned Aerial Vehicles (UAVs) are among the most advanced and influential technologies in the modern world, with significant applications in both civilian and military domains such as surveillance, delivery, rescue operations, and terrain mapping. Among various types of UAVs, the Quadcopter stands out due to its simple mechanical structure, high maneuverability, and ability to operate in confined spaces. However, achieving precise trajectory control of a Quadcopter remains a challenging task because of its strong nonlinearity, tight coupling between motors, and susceptibility to external disturbances. Therefore, the main objective of this study is to develop an advanced control method capable of adapting to varying conditions while maintaining high trajectory-tracking performance for a quadcopter.

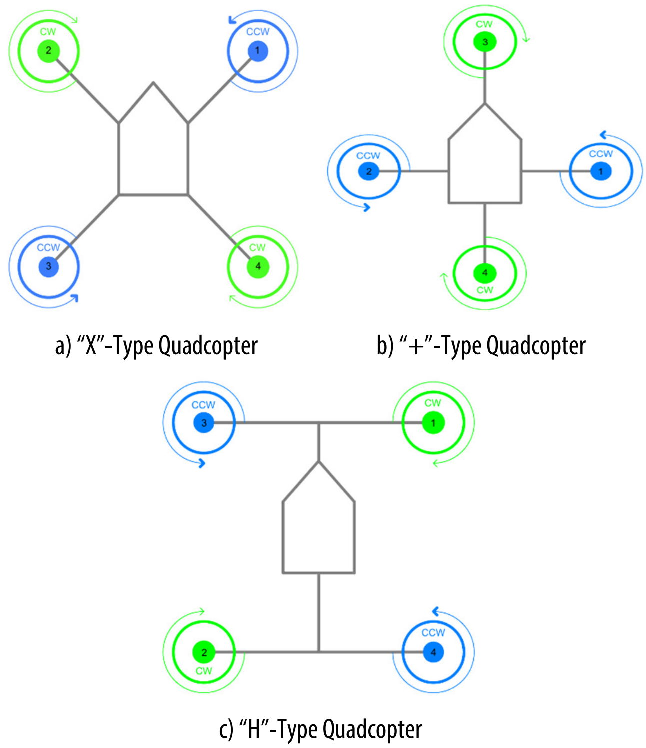
To address this issue, numerous efforts have been made by researchers through various control approaches. Several controllers have been implemented, including Proportional-Integral-Derivative (PID) control [1], Proportional-Derivative Sliding Mode Control (PD-SMC) and Hierarchical Sliding Mode Control (HSMC) [2, 3], Backstepping Sliding Mode Control (BSP-SMC) [4], Dynamic Surface Control combined with SMC (DSC-SMC) [5], Model Predictive Control (MPC) [6], and other intelligent control methods [7-9]. The PID controller [1] is simple and easy to implement, and it has been widely used in industrial systems. However, it struggles with highly nonlinear systems and requires careful tuning of the Proportional (P), Integral (I), and Derivative (D) gains to avoid oscillation and overshoot [10]. The PD-SMC-HSMC method [2, 3] combines the advantages of PD, SMC, and HSMC. Its cascaded structure provides good stability in both attitude and position control, enhances disturbance rejection, and improves trajectory-tracking accuracy. Moreover, HSMC helps mitigate the chattering phenomenon commonly observed in traditional SMC. This method has demonstrated its effectiveness in various nonlinear control and robotic applications [11-13]. However, the performance of PD-SMC-HSMC still depends heavily on manual tuning of PD gains and sliding-surface coefficients, making practical implementation under uncertain or varying load conditions challenging. The BSP-SMC method [4] integrates the recursive design approach of Backstepping based on Lyapunov theory with the robustness of SMC. Nonetheless, it suffers from the "explosion of terms" problem when applied to high-order systems. To overcome this issue, DSC-SMC [5] was introduced, using a low-pass filter to eliminate repeated differentiation in conventional Backstepping design, thereby reducing computational complexity and improving real-time feasibility. However, DSC-SMC still faces challenges in parameter tuning for optimal performance [14]. In [6], MPC was employed for UAV trajectory control, achieving high performance and system optimization. MPC predicts system behavior using dynamic models and generates optimal control inputs at each sampling instant. Nevertheless, this method requires intensive computation and relies strongly on model accuracy, which limits its real-time application on UAVs with constrained onboard resources. In addition, intelligent control approaches [7-9], such as Radial Basis Function Neural Networks (RBFNN) [7], Fuzzy Logic Control (FLC) [8], and Neuro-Fuzzy systems [9], have been developed to overcome the limitations of classical control methods. Among them, Fuzzy Logic shows outstanding potential due to its ability to handle uncertainty and nonlinearity without requiring an exact mathematical model. The fuzzy controller can flexibly adjust control parameters based on expert inference rules, improving stability, adaptability, and trajectory-tracking performance under varying conditions. However, pure fuzzy systems still face limitations related to subjective rule-base design and membership function tuning, which may affect performance [15].

Based on these analyses, this study proposes an advanced control strategy combining Fuzzy Logic, PD-SMC, and HSMC to develop an adaptive control system for a quadcopter UAV. In this approach, the fuzzy logic controller automatically tunes the proportional (P) and derivative (D) gains of the PD controller according to the position error along the x-y axes and their derivatives, generating appropriate reference inclination angles corresponding to the actual flight trajectory. Using these reference angles, the SMC computes control inputs to regulate the quadcopter's attitude (roll, pitch, and yaw), while the HSMC is employed to control its position. Simulation results in MATLAB & Simulink using a complex spiral trajectory demonstrate that the proposed Fuzzy-PD-SMC-HSMC method achieves higher trajectory-tracking accuracy and significantly reduces tracking error amplitude compared to the conventional PD-SMC-HSMC controller [2, 3], owing to the adaptive flexibility of the proposed system.

The remainder of this paper is organized as follows: Section 2 presents the mathematical model of the "X"-type quadcopter; Section 3 describes the design of the Fuzzy-PD-SMC-HSMC controller; Section 4 discusses the simulation results and analysis; and Section 5 concludes the study.

2. MATHEMATICAL MODEL

2.1. Quadcopter's "X"-Type Model



a) "X"-Type Quadcopter b) "+"-Type Quadcopter

c) "H"-Type Quadcopter

Figure 1. Common Quadcopter frame configurations

These days, there are a few variants of the Quadcopter's frame, including the three most popular types: "X"-type, "+"-Type, and "H"-Type, each has its own characteristics, as illustrated in Fig. 1. The "+"-Type Quadcopter has 4 motors placed aligned directly with the cardinal direction, forming a "+" shape. The "H"-type Quadcopter is similar to the "+"-Type but has a rectangular body for extra mounting space, usually designed for heavy-duty missions. The "X"-Type Quadcopter, which is the most common type, has motors positioned at a 45 Degree offset to the body axes, creating an "X" shape. This design gives it the flexibility due to its balanced thrust distribution. Because of the wide applications of the "X"-Type, this paper will focus on this design.

To conduct this research, several assumptions have been made to simplify the modelling process: (i) the Quadcopter has a rigid and symmetrical frame, (ii) the center of gravity coincides with the midpoint of the airframe, and (iii) the effect of drag force is neglected considering the Quadcopter is operating in an ideal environment.

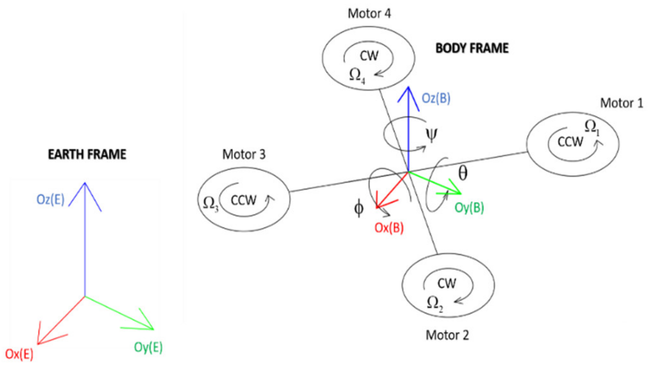


Figure 2. 'X'-Type Quadcopter model

The "X"-Type Quadcopter is described as a system with 12 states. These states describe the position, the orientation, and also the translational and rotational velocity along the axes in the Earth Coordinates:

$$\vec{x}=[x,y,z,\phi,\theta,\varphi,u,v,w,p,q,r]^T \tag{1}$$

In this model, x, y, z, u, v, w are translational states in a 3D world frame, with the first three being the positions and the others being velocity in x,y, and z directions, respectively; $\phi, \theta, \psi, p, q, r$ are rotational states which describe the quadcopter's orientation (attitude) and rotational velocities in the body frame. Further notice, as illustrated in Fig. 2, the world frame, or what can be called the Earth frame in some research, refers to the fixed coordinate system relative to the ground. The body frame, on the other hand, refers to the coordinate system fixed to the body of the quadcopter's physical structure, which moves along with the quadcopter. It is aligned with the quadcopter's structural axis and centred at its centre of mass. Here, CW is Clockwise and CCW is Counter-Clockwise. A quadcopter uses 4 motors to generate lift with 2 rotating Clockwise and 2 rotating counterclockwise in order to balance out the torque.

The system utilizes 4 control inputs $U = [U_1\ U_2\ U_3\ U_4]^T$ to control the quadcopter's attitude and position. The control input equations are inspired by the work [2, 8]:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} C_t & C_t & C_t & C_t \\ \frac{l}{\sqrt{2}}C_t & \frac{l}{\sqrt{2}}C_t & -\frac{l}{\sqrt{2}}C_t & -\frac{l}{\sqrt{2}}C_t \\ \frac{l}{\sqrt{2}}C_t & -\frac{l}{\sqrt{2}}C_t & -\frac{l}{\sqrt{2}}C_t & \frac{l}{\sqrt{2}}C_t \\ -C_d & C_d & -C_d & C_d \end{bmatrix} \begin{bmatrix} \Omega_1^2 \\ \Omega_2^2 \\ \Omega_3^2 \\ \Omega_4^2 \end{bmatrix} \tag{2}$$

With l is the length from the center of mass of the quadcopter to the center of mass of the motor axis, C_t, C_d are the thrust and drag aerodynamic coefficients of the propellers, $\Omega = [\Omega_1 \quad \Omega_2 \quad \Omega_3 \quad \Omega_4]^T$ are the propeller speeds of Motors 1, 2, 3, and 4, respectively.

2.2. Equation of Motion

The moment of inertia of the quadcopter's rigid body is described as:

$$J = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \tag{3}$$

Here, I_x, I_y, I_z are the moments of inertia with respect to Ox, Oy, and Oz coordinate, respectively. Since we assume our Quadcopter's body is symmetric, we can also assume $I_x = I_y$. The complete mathematical model of the Quadcopter can be written as [8]:

$$\begin{cases} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} c\theta c\psi & s\mathbf{f}s\theta c\psi - c\mathbf{f}s\psi & c\mathbf{f}s\theta c\psi + s\mathbf{f}s\psi \\ c\theta s\psi & s\mathbf{f}s\theta s\psi + c\mathbf{f}s\psi & c\mathbf{f}s\theta s\psi - s\mathbf{f}c\psi \\ -s\theta & s\mathbf{f}c\theta & c\mathbf{f}c\theta \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \\ \ddot{x} = \frac{1}{m}\left(\left(c\mathbf{f}s\theta c\psi + s\mathbf{f}s\psi\right)U_1\right); \\ \ddot{y} = \frac{1}{m}\left(\left(s\mathbf{f}s\theta s\psi - s\mathbf{f}c\psi\right)U_1\right); \ddot{z} = \frac{1}{m}\left(c\mathbf{f}c\theta U_1\right) - g \\ \begin{bmatrix} \dot{\mathbf{f}} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & s\mathbf{f}t\theta & c\mathbf{f}t\theta \\ 0 & c\mathbf{f} & -s\mathbf{f} \\ 0 & s\mathbf{f}\sec\theta & c\mathbf{f}\sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \\ \ddot{\mathbf{f}} = \dot{\theta}\dot{\psi}\frac{I_y - I_z}{I_x} + \frac{U_2}{I_x}; \ddot{\theta} = \dot{\mathbf{f}}\dot{\psi}\frac{I_z - I_x}{I_y} + \frac{U_3}{I_y}; \\ \ddot{\psi} = \dot{\mathbf{f}}\dot{\theta}\frac{I_x - I_y}{I_z} + \frac{U_4}{I_z} \end{cases} \tag{4}$$

Here, g is the acceleration due to gravity, m is the mass of the Quadcopter, c, s and t stands for cos, sin and tan respectively, and $sec\theta = \frac{1}{cos\theta}$.

3. CONTROLLER DESIGN

In this research, the main objective is to control the quadcopter to follow a predefined desired path. To achieve this, a closed-loop control system based on the

Fuzzy-PD-SMC-HSMC approach is proposed. A Fuzzy Logic Controller is implemented to adaptively tune the proportional and derivative gains of the PD controller, which generate the reference angles ϕ_{ref} and θ_{ref}. From these reference angles, the SMC computes control inputs U_2, U_3, U_4 to regulate the quadcopter's rotational angles (roll, pitch, and yaw). Meanwhile, the HSMC is employed to control its position via control input U_1. The control inputs will then be fed to the mathematical model of the Quadcopter to get the new set of states. The loop will continue to run until the time exceeds. The overall structure of the proposed Fuzzy-PD-SMC-HSMC control system is illustrated in Fig. 3.

the Quadcopter moves in that direction. The controller is designed based on [2, 3]. Firstly, a PD controller is designed to minimize the horizontal position error:

$$\begin{cases} u_x = -\hat{K}_{p_x}(x_{ref} - x) + \hat{K}_{d_x}(\dot{x}_{ref} - \dot{x}) \\ u_y = \hat{K}_{p_y}(y_{ref} - y) - \hat{K}_{d_y}(\dot{y}_{ref} - \dot{y}) \end{cases} \quad (5)$$

Here, $\hat{K}_{p_x}, \hat{K}_{d_x}, \hat{K}_{p_y}, \hat{K}_{d_y}$ are PD gains for x, and y errors, respectively. The negative sign between the u_x and u_y is because the Quadcopter needs to tilt forward to move forward, thus it needs a negative roll angle. Furthermore, the Quadcopter is a heading-dependent device, which means that its x-body and y-body rotate alongside yaw.

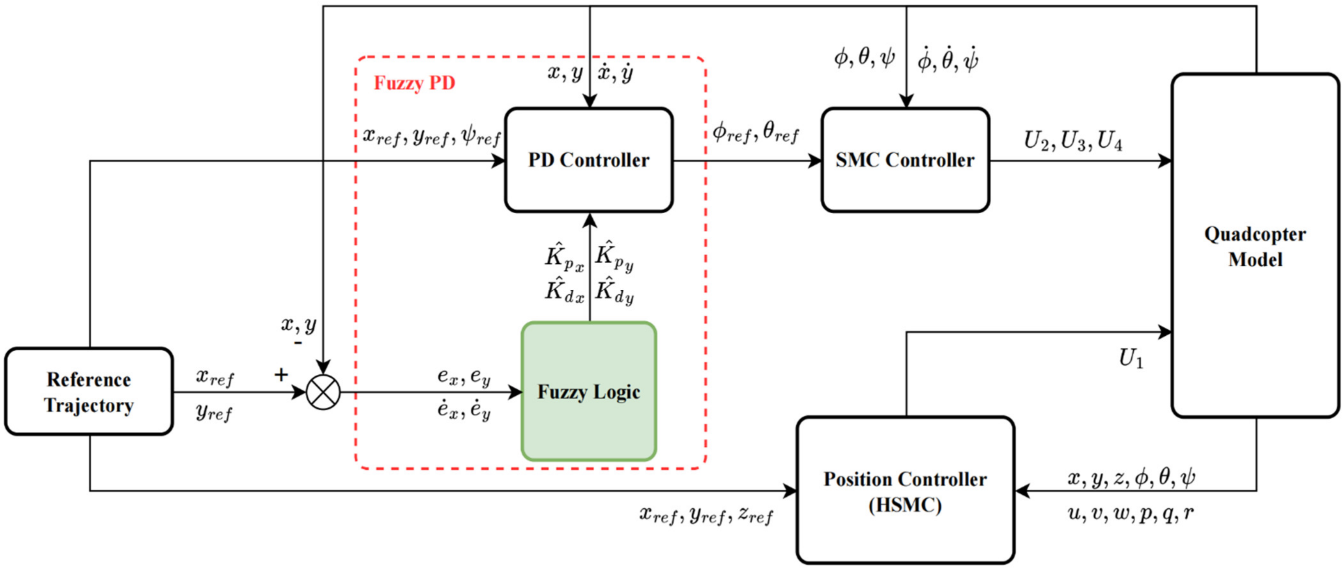


Figure 3. The overall control schematic of the system

3.1. Fuzzy-PD-SMC Controller

The Fuzzy-PD-SMC Controller is very important in the structure of the system because the output of this block needs to be as precise as possible. It uses a Fuzzy Logic Controller to tune the gains of the PD Controller, which generates the reference angles ϕ_{ref} and θ_{ref}. The better the reference angles are, the more accurate the Quadcopter track. For the reference angle generator, a PD Controller is designed because of its simplicity, but it still maintains robustness with a low steady state error in dynamic systems. Then, an SMC controller is implemented to compute the control inputs responsible for the roll, pitch, and yaw angles control from the reference angle and the previous state of the system.

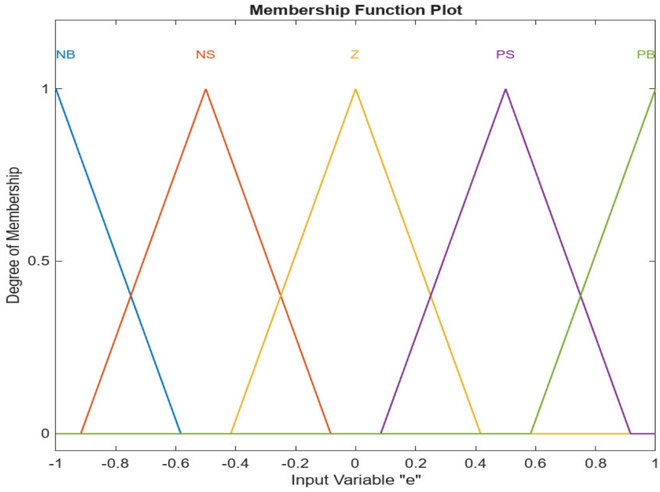
In the case of a Quadcopter, in order to move along the Ox and Oy axes, the Quadcopter needs to alter its roll and pitch angles. The higher the angle value, the faster

Ultimately, we can compute the desired roll and pitch angle:

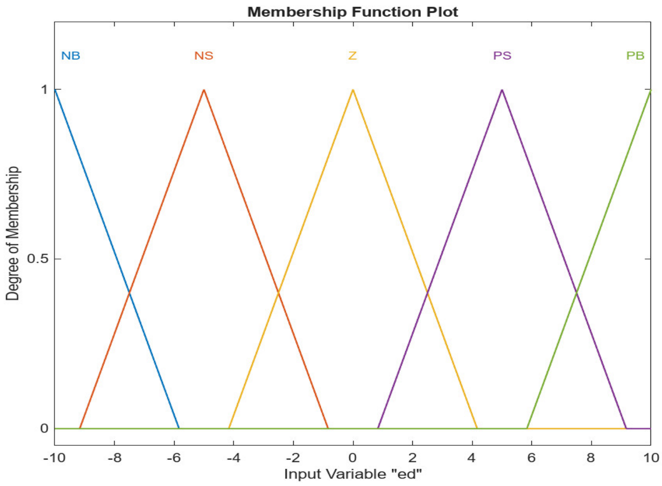
$$\begin{cases} \phi_{ref} = u_y \cos(\psi) + u_x \sin(\psi) \\ \theta_{ref} = -u_y \sin(\psi) + u_x \cos(\psi) \end{cases} \quad (6)$$

However, to further improve the transient response and stability of the quadcopter, Fuzzy logic is used to further tune the gain values of the Quadcopter. The Fuzzy rules for K_p, K_d are declared in tables 1 and 2, respectively. Here, NB: Negative Big, NS: Negative Small, Z: Zero, PS: Positive Small, PB: Positive Big; VL: Very Large, L: Large, M: Medium, S: Small, VS: Very Small. Each Fuzzy Logic has 2 input values, including the position error and the error rate, and produces 1 output value, which is the PD gain. The input Error range is [-1 1] while the input Error rate range is [-10 10] for both Fuzzy Logic controllers. The value range of K_p is [0.8 1.3] while for the K_d is [0.3 0.8]. The complete set of Fuzzy Logic is designed through a trial-and-error analysis process. The efficiency of this set of values will be proven in the simulation process.

Fig. 4 and Fig. 5 illustrate the Fuzzy Logic membership function and control surface for computing the proportional gain for the PD controller. The Fig. 6 and Fig. 7 similarly illustrate the fuzzy logic design for computing the derivative gain of the PD controller.



a) Input error membership function



b) Input error rate membership function

Figure 4. K_p Fuzzy Membership Function

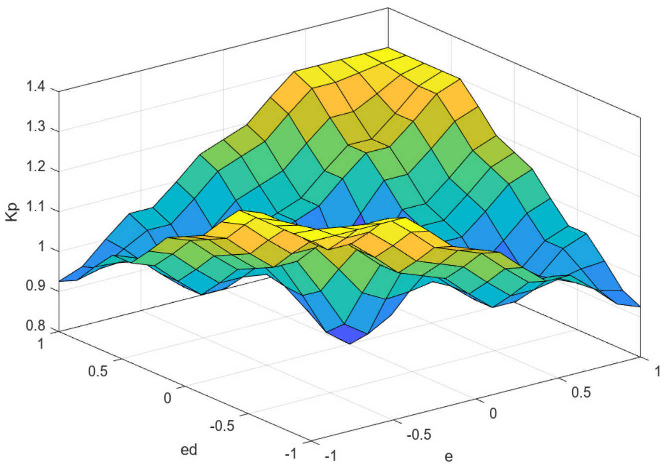
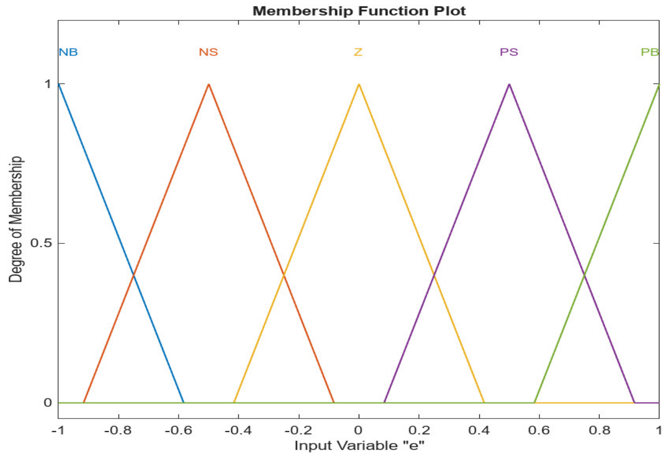


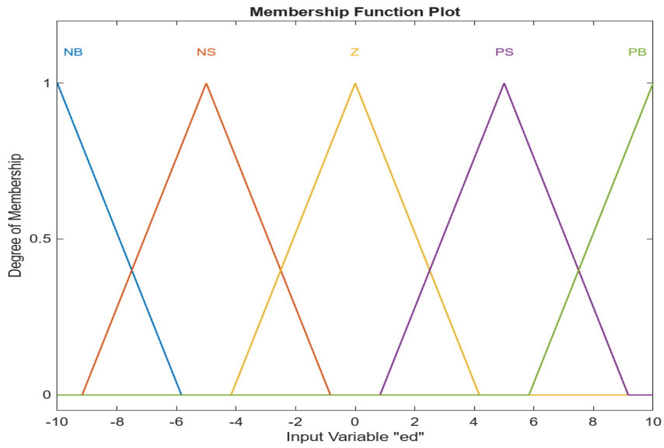
Figure 5. K_p Fuzzy Logic Control Surface

Table 1. Fuzzy Rule for K_p

De/e	NB	NS	Z	PS	PB
NB	VL	VL	L	M	S
NS	VL	L	M	S	M
Z	L	M	VS	M	L
PS	M	S	M	L	VL
PB	S	M	L	VL	VL



a) Input error membership function



b) Input error rate membership function

Figure 6. K_d Fuzzy Membership Function

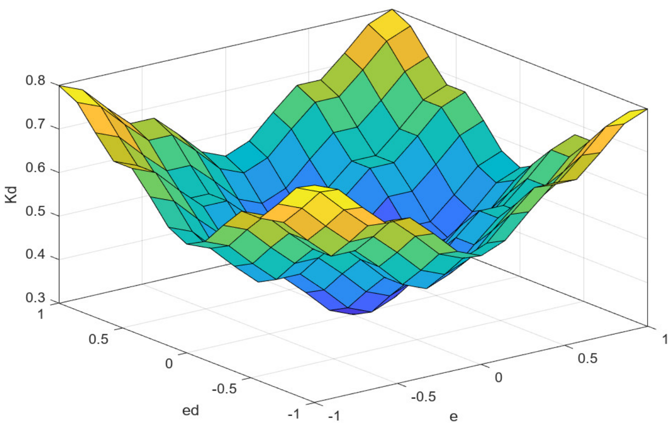


Figure 7. K_d Fuzzy Logic Control Surface

Table 2. Fuzzy Rule for K_d

De/e	NB	NS	Z	PS	PB
NB	VL	L	M	L	VL
NS	L	M	S	M	L
Z	M	S	VS	S	M
PS	L	M	S	M	L
PB	VL	L	M	L	VL

In terms of computing the control inputs for roll, pitch, and yaw angle, we design an SMC Controller inherited from [2, 3]. We can rewrite the quadcopter's model as:

$$\begin{cases} \dot{x}_1 = x_2 ; \dot{x}_2 = f_x + b_x U_1 \\ \dot{x}_3 = x_4 ; \dot{x}_4 = f_y + b_y U_1 \\ \dot{x}_5 = x_6 ; \dot{x}_6 = f_z + b_z U_1 \\ \dot{x}_7 = x_8 ; \dot{x}_8 = f_\phi + b_\phi U_2 \\ \dot{x}_9 = x_{10} ; \dot{x}_{10} = f_\theta + b_\theta U_3 \\ \dot{x}_{11} = x_{12} ; \dot{x}_{12} = f_\psi + b_\psi U_4 \end{cases} \quad (7)$$

With f_i, b_i $(i = x, y, z, \phi, \theta, \psi)$, and $x_1 = x, x_2 = \dot{x}$, $x_3 = y, x_4 = \dot{y}, x_5 = z, x_6 = \dot{z}, x_7 = \phi, x_8 = \dot{\phi}, x_9 = \theta$, $x_{10} = \dot{\theta}, x_7 = \psi, x_8 = \dot{\psi}$ can be derived from (4).

The Sliding Surface are chosen as:

$$s_\phi = c_\phi(\phi - \phi_{ref}) + (\dot{\phi} - \dot{\phi}_{ref}) \quad (8)$$

$$s_\theta = c_\theta(\theta - \theta_{ref}) + (\dot{\theta} - \dot{\theta}_{ref}) \quad (9)$$

$$s_\psi = c_\psi(\psi - \psi_{ref}) + (\dot{\psi} - \dot{\psi}_{ref}) \quad (10)$$

Here, c_ϕ, c_θ, c_ψ are positive numbers.

The control inputs are defined as follows [3]:

$$U_2 = \frac{1}{b_\phi}\begin{bmatrix} -c_\phi \phi - (c_\phi + 1)\dot{\phi} + c_\phi \phi_{ref} \\ +(c_\phi + 1)\dot{\phi}_{ref} - f_\phi - K_\phi sat(s_\theta) + \ddot{\phi}_{ref} \end{bmatrix} \quad (11)$$

$$U_3 = \frac{1}{b_\theta}\begin{bmatrix} -c_\theta \theta - (c_\theta + 1)\dot{\theta} + c_\theta \theta_{ref} + (c_\theta + 1)\dot{\theta}_{ref} \\ -f_\theta - K_\theta sat(s_\theta) + \ddot{\theta}_{ref} \end{bmatrix} \quad (12)$$

$$U_4 = \frac{1}{b_\psi}\begin{bmatrix} -c_\psi \psi - (c_\psi + 1)\dot{\psi} + c_\psi \psi_{ref} + (c_\psi + 1)\dot{\psi}_{ref} \\ -f_\psi - K_\psi sat(s_\psi) + \ddot{\psi}_{ref} \end{bmatrix} \quad (13)$$

Considering the Lyapunov stability, we have the Lyapunov candidate from (8) to (10):

$$V = V_\phi + V_\theta + V_\psi = \frac{1}{2}s_\phi^2 + \frac{1}{2}s_\theta^2 + \frac{1}{2}s_\psi^2 \quad (14)$$

Deriving (14), together with equations (8) to (10), we can obtain:

$$\begin{aligned} \dot{V}_\phi &= -s_\phi^2 - s_\phi K_\phi sat(s_\phi) - s_\theta^2 - s_\theta K_\theta sat(s_\theta) \\ &\quad - s_\psi^2 - s_\psi K_\psi sat(s_\psi) \le 0 \end{aligned} \quad (15)$$

The $sat()$ function is defined as:

$$sat(x) = \begin{cases} \text{-1 if } x \le 1 \\ x \text{ if } -1 < x < 1 \\ \text{1 if } x \ge 1 \end{cases} \quad (16)$$

From (15), we can prove that the system is stable by Lyapunov stability.

3.2. HSMC Controller

The control input U_1, which controls the heights of the Quadcopter, is calculated using a Hierarchical Sliding Mode Control (HSMC) inherited from [2, 3]. Basically, Hierarchical Sliding Mode Control is a robust control technology that extends the classical Sliding Mode Control by dividing the system states into many layers to converge the errors to zero in a finite time. The lower layers of the HSMC focus on individual subsystems, while the higher layers integrate outputs from lower layers to solve the control problem. The specific method used in this research is Aggregated HSMC. This method aggregates lower layers into the higher ones, thus enhancing the finite-time convergence and smooth control inputs.

The lower layer of the HSMC is defined as:

$$s_1 = c_x(x - x_{ref}) + (\dot{x} - \dot{x}_{ref}) \quad (17)$$

$$s_2 = c_y(y - y_{ref}) + (\dot{y} - \dot{y}_{ref}) \quad (18)$$

$$s_3 = c_z(z - z_{ref}) + (\dot{z} - \dot{z}_{ref}) \quad (19)$$

Where c_x, c_y, c_z are positive numbers.

The higher layers are denoted as:

$$S_1 = s_1 \quad (20)$$

$$S_2 = \lambda_1 S_1 + s_2 \quad (21)$$

$$S_3 = \lambda_2 S_2 + s_3 \quad (22)$$

With λ_1, λ_2 are control parameters.

From [3], we have a set of control laws:

$$u_{eqx} = \frac{-c_x(\dot{x} - \dot{x}_{ref}) + \ddot{x}_{ref} - f_x}{b_x} \quad (23)$$

$$u_{eqy} = \frac{-c_y(\dot{y} - \dot{y}_{ref}) + \ddot{y}_{ref} - f_y}{b_y} \quad (24)$$

$$u_{eqz} = \frac{-c_z(\dot{z} - \dot{z}_{ref}) + \ddot{z}_{ref} - f_z}{b_z} \quad (25)$$

$$\begin{aligned} u_{sw} &= -\frac{\lambda_1\lambda_2 b_x(u_{eqy} + u_{eqz}) + \lambda_2 b_y(u_{eqx} + u_{eqz})}{\lambda_1\lambda_2 b_x + \lambda_2 b_y + b_z} \\ &\quad - \frac{b_z(u_{eqx} + u_{eqy}) + K_a S_3 + \eta sat(S_3)}{\lambda_1\lambda_2 b_x + \lambda_2 b_y + b_z} \end{aligned} \quad (26)$$

$$U_1 = u_{eqx} + u_{eqy} + u_{eqz} + u_{sw} \quad (27)$$

With K_a, η are positive numbers, $b_x, b_y, b_z, f_x, f_y, f_z$ can be derived from (7).

The stability of the system is proven in [2, 3]. From the sliding surface, we declare our Lyapunov candidate as follows:

$$V = \frac{1}{2} S_3^2 \quad (28)$$

Differentiating (28) with respect to time, we have:

$$\dot{V} = -K_a S_3^2 - \eta S_3 sat(S_3) \le 0 \quad (29)$$

Thus, the system is stable by Lyapunov stability.

4. SIMULATION AND RESULT

To effectively recreate the dynamics and environment in which the Quadcopter operates and test the proposed method, we use MATLAB/Simulink 2023a, guarantee a precise simulation. Also, for our simulated plant, we have chosen a set of parameters for the UAV. The detailed system parameters are chosen as in Table 3.

Table 3. Detailed System Parameters

Parameters	Values	Parameters	Values
mm	$1.5kg$	$c_\phi = c_\theta$	3.5
g	$9.81\frac{m}{s^2}$	c_ψ	0.5
$I_x = I_y$	$0.0015kg/m^2$	$K_\phi = K_\theta$	0.4
I_z	$0.005kg/m^2$	K_{psi}	0.2
$c_x = c_y$	0.05	c_z	1
λ_1	0.05	K	0.34
η	0.25	λ_2	0.05

4.1. The UAV system operates under ideal conditions

The proposed method will be put directly into comparison with the method presented in [3], with a sample time of $T = 30s$. The comparison is done under the same ideal conditions, with the same tracking problem - a spiral figure. The results are shown in the figures below. Here, the green line stands for the PD-SMC-HSMC method proposed in research [3], the red line is the proposed method in this paper, which is Fuzzy-PD-SMC-HSMC, and the blue line is the reference, or the desired value. As can be observed in Fig. 8 to Fig. 11, the UAV quadcopters react well to the given trajectory since both systems can follow the spiral line. In Fig. 8, it can be observed that the Fuzzy-PD-SMC-HSMC method helps the system exhibit fewer oscillations, particularly in the lower and middle regions of the spiral. The proposed method (red line) follows the reference trajectory (blue line) more closely than the other method (green line). This indicates that the desired positions along the x, y, and z axes can be maintained under dynamic conditions with higher accuracy and stability when applying the Fuzzy-PD-SMC-HSMC approach.

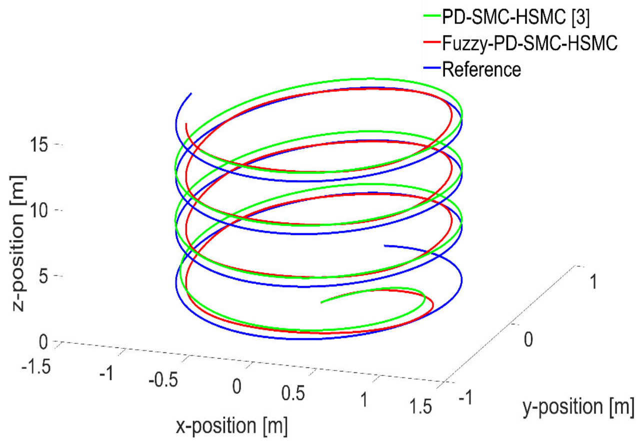


Figure 8. Trajectory tracking result under ideal conditions

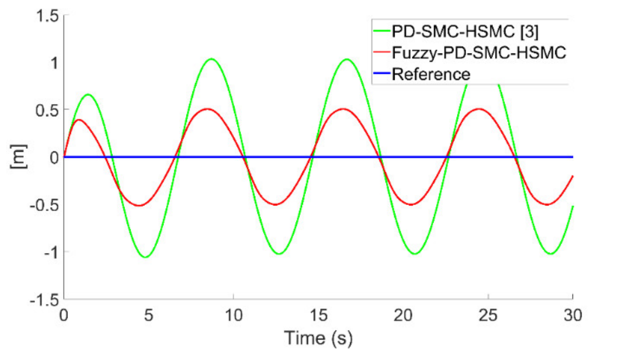


Figure 9. X-position error under ideal conditions

In terms of error, a significant reduction in position error along all three axes (x, y, and z) can be observed when applying the Fuzzy-PD-SMC-HSMC method, demonstrating a considerable improvement in trajectory tracking accuracy as well as superior control performance compared to the conventional PD-SMC-HSMC approach. The Fuzzy Logic Controller added helps the system perform better in terms of tracking error. The tracking accuracy of the proposed controller is increased by approximately 50%, as shown in Table 4, where the Root Mean Square Error is calculated and compared. In terms of X-Position error, the proposed method has a 50.66% better tracking ability (0.3527m RMSE) than the method PD-SMC-HSMC in [3] (0.7147m RMSE). For the Y-Position error, the value is 46.3% with 0.3724m compared to 0.6936m. And for Z-Position error, the value is 9.49% (0.0410m RMSE to 0.0453m RMSE). Overall, the Fuzzy-PD-SMC-HSMC technique offers increased tracking performance and control efficiency, making it a more effective solution for applications requiring precise path following, such as quadrotor UAV maneuvers.

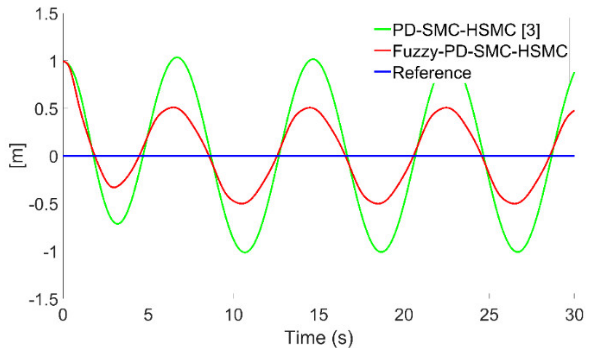


Figure 10. Y-position error under ideal conditions

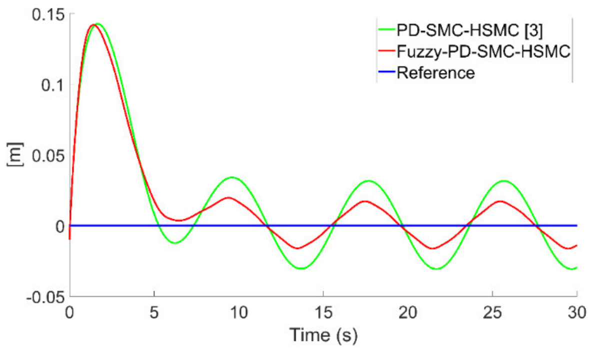


Figure 11. Z-position error under ideal conditions

Table 4. RMSE under ideal conditions (m)

Methods	X-Position	Y-Position	Z-Position
PD-SMC-HSMC [3]	0.7147	0.6936	0.0453
Fuzzy-PD-SMC-HSMC	**0.3527**	**0.3724**	**0.0410**

4.2. The UAV system operates under the influence of noise

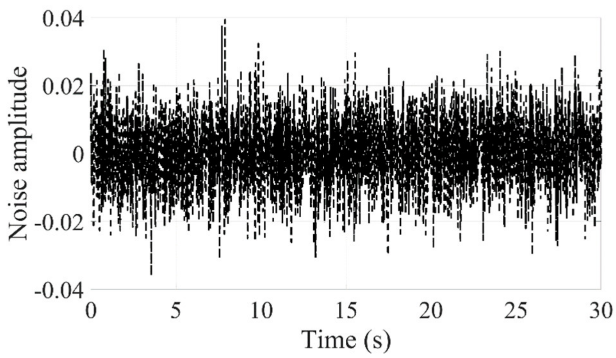


Figure 12. Unknown external disturbances affecting the UAV

In this scenario, the UAV system operates under conditions where disturbances directly affect the control signals, with the disturbance amplitude illustrated in Fig. 12. Based on the trajectory tracking results presented in Fig. 13, it can be observed that the proposed Fuzzy-PD-SMC-HSMC method exhibits significantly more stable and accurate tracking performance compared to the PD-SMC-HSMC controller. Specifically, owing to the fuzzy inference mechanism integrated into the PD control structure, the system is capable of automatically adjusting the control gains in response to both the magnitude of the error and its rate of change. This enables improved adaptability under the influence of external disturbances or modeling uncertainties. In contrast, the conventional PD-SMC-HSMC controller employs fixed control parameters, resulting in less flexible responses and larger trajectory deviations when subjected to noise.

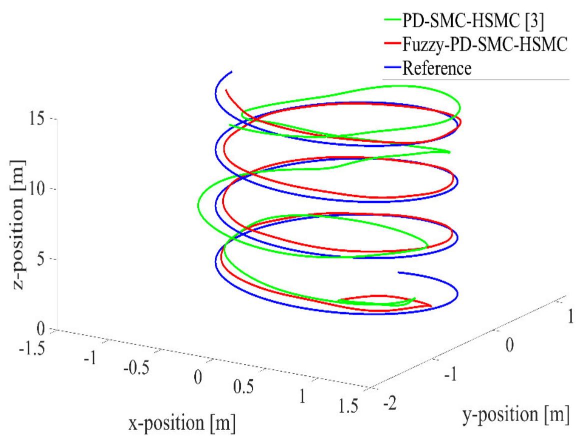


Figure 13. Trajectory tracking result under the influence of noise

The trajectory tracking errors along the three axes under the influence of disturbances are illustrated in Figs. 14 ÷ 16, while the corresponding RMSE values are detailed in Table 5. The results indicate that the proposed Fuzzy-PD-SMC-HSMC method achieves significantly smaller root mean square errors across all three axes compared with the PD-SMC-HSMC method [3], particularly along the X and Y axes. Specifically, the RMSE values decrease from 0.6722 to 0.3557 for the X-axis and from 0.8239 to 0.3656 for the Y-axis, corresponding to improvements of approximately 47% and 55%, respectively, while the Z-axis error is also slightly reduced (from 0.0460 to 0.0410). These results demonstrate that the adaptive adjustment mechanism based on fuzzy logic enables the system to maintain stable trajectory tracking performance and effectively suppress the influence of random disturbances on UAV position accuracy. From a practical perspective, this finding is particularly significant since the substantial reduction in trajectory tracking errors allows the UAV to perform tasks with higher precision. This makes the proposed approach well-suited for high-accuracy missions such as target tracking, infrastructure inspection, or smart agriculture operations.

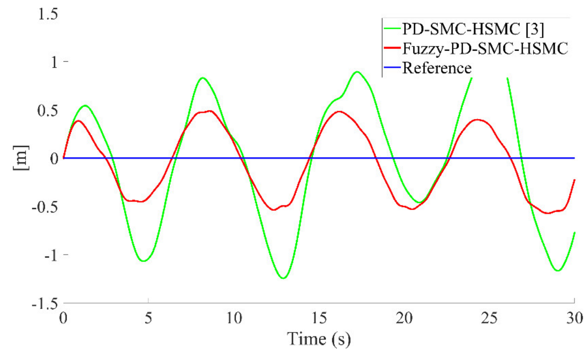


Figure 14. X-position error under the influence of noise

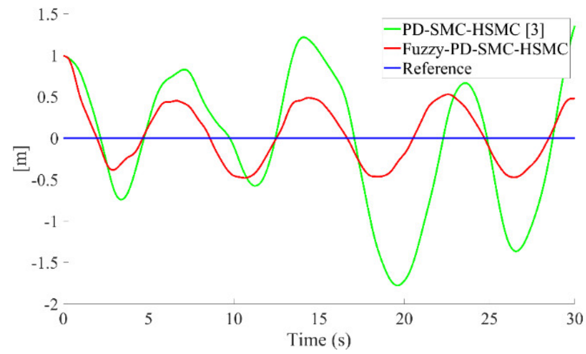


Figure 15. Y-position error under the influence of noise

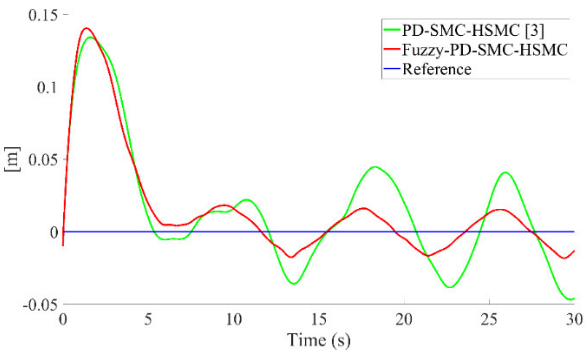


Figure 16. Z-position error under the influence of noise

Table 5. RMSE under the influence of noise (m)

Methods	X-Position	Y-Position	Z-Position
PD-SMC-HSMC [3]	0.6722	0.8239	0.0460
Fuzzy-PD-SMC-HSMC	**0.3557**	**0.3656**	**0.0410**

5. CONCLUSION

This paper proposes an adaptive control method utilizing a Fuzzy-PD-SMC-HSMC controller to accurately track the trajectory of a quadcopter. The simulation results provide a comparison between the Fuzzy-PD-SMC-HSMC and PD-SMC-HSMC methods, highlighting the superior efficiency of the proposed approach. The MATLAB & Simulink simulations show that, in terms of tracking accuracy and error minimization along all three coordinate axes, the proposed Fuzzy-PD-SMC-HSMC approach with a dynamic gain-tuning mechanism outperforms the PD-SMC-HSMC method. However, the Fuzzy-PD-SMC-HSMC method still faces limitations, as the fuzzy rules must be carefully tuned to achieve optimal performance. Future research may explore additional scenarios related to this problem, such as implementing the proposed method on a real-world system, since this study was conducted entirely on a simulation platform.

REFERENCES

[1]. Sy MTN., Dinh XM., Do QQ., Ta NT., Pham VD., Ta ND., Bui XM., "Development of autonomous landing system for reconnaissance UAVs," in *Proc. Int. Conf. on Applied Mathematics and Computer Science,* Cham: Springer Nature Switzerland, 475-487, 2024.

[2]. Nguyen AD., Walunj P., Le C., La HM., "SM-NMPC: Sliding mode-based nonlinear model predictive control for UAVs under degraded motor on microcontrollers," *IEEE Robotics and Automation Letters*, 2025.

[3]. Nguyen VC., La HM., "A class of hierarchical sliding mode control based on extended Kalman filter for quadrotor UAVs," *arXiv preprint arXiv:2504.02851*, 2025.

[4]. Huang S., Huang J., Cai Z., Cui H., "Adaptive backstepping sliding mode control for quadrotor UAV," *Scientific Programming*, 2021(1), 3997648, 2021.

[5]. Lin X., Yu Y., Sun CY., "A decoupling control for quadrotor UAV using dynamic surface control and sliding mode disturbance observer," *Nonlinear Dynamics*, 97(1), 781-795, 2019.

[6]. Tuan Anh TN., Dinh XM., Do QQ., Ta NT., Le XH., "Adaptive-PD and model predictive control for UAV quadcopter trajectory tracking," in *Proc. 4th Int. Conf. on Advances in Information and Communication Technology* (ICTA 2025), Cham: Springer Nature Switzerland, 2025.

[7]. Zhang D., Meng L., Hao Y., Xia R., "Adaptive sliding mode fault-tolerant control of UAV systems based on radial basis function neural networks," *Scientific Reports*, 15(1), 27504, 2025.

[8]. Madebo NW., Abdissa CM., Lemma LN., "Enhanced trajectory control of quadrotor UAV using fuzzy PID-based recurrent neural network controller," *IEEE Access*, 2024.

[9]. Choi HD., Kim KS., Shi P., Ahn CK., "Adaptive neuro-fuzzy sliding mode tracking for quadrotor UAVs," *IEEE Transactions on Automation Science and Engineering*, 2025.

[10]. Dang S.T., Dinh X.M., Than Ngoc T.A., Nguyen V.T., Kim D.T., Le X.H., "Adaptive Sliding Mode Control for Robust Trajectory Tracking of Car-Like

Mobile Robots in Autonomous Navigation Systems," *Journal of Applied and Computational Mechanics*, 2025.

[11]. Dang ST., Dinh XM., Kim TD., Xuan HL., Ha MH., "Adaptive backstepping hierarchical sliding mode control for 3-wheeled mobile robots based on RBF neural networks," *Electronics,* 12(11), 2345, 2023.

[12]. Kim TD., Nguyen LN., Dinh XM., Le HX., "Adaptive hierarchical sliding mode control design for 3D ship-mounted container crane with saturating actuators," *Journal of Control, Automation and Electrical Systems*, 33(6), 1643–1658, 2022.

[13]. Hoang TTU., Kim DT., Pham VA., Dinh XM., Le HX., "Designing an adaptive controller for two-wheeled self-balancing mobile robot using hierarchical sliding control strategy and radial basis function neural network," *HaUI Journal of Science and Technology*, 57, 39-48, 2021.

[14]. Dinh XM., Dang XD., Le HX., Nguyen NL., "Precise trajectory tracking control of 3-DOF delta robots using dynamic surface control combined with sliding mode control," *Journal of Mechanical Science and Technology*, 1-16, 2025.

[15]. Dinh XM., Luong TG., Le XH., Nguyen VT., Kim DT., "Adaptive fuzzy dynamic surface control with sliding mode control for enhanced trajectory tracking of delta robots," *Journal of Control, Automation and Electrical Systems*, 1-25, 2025.