THE DESIGN OF TRAJECTORY TRACKING CONTROLLER FOR MOBILE ROBOTS USING FEEDFORWARD-FEEDBACK METHODS

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ABSTRACT

In recent years, the application of mobile robots in industrial production for material transportation has become increasingly prevalent, necessitating efficient, and flexible navigation capabilities. The ability to move along predefined trajectories plays a crucial role in the successful execution of autonomous missions. To address this challenge, this paper presents a novel combined controller that integrates both feedforward (FF) and feedback (FB) control techniques. The FF controller is utilized to predict the control signal output, while the FB controller ensures adaptability in the presence of uncertain disturbances and external factors that may affect the robot's trajectory-tracking performance. Through simulation, the proposed method demonstrates superior accuracy and stability compared to using FF and FB controllers in isolation.

Keywords: *Trajectory tracking, mobile robots, feedforward control, feedback control.*

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1. INTRODUCTION

Path tracking for mobile robots has been the subject of extensive research, with a significant emphasis on achieving highly accurate tracking of predetermined trajectories. These trajectories commonly involve fundamental segments, such as straight lines, circular arcs, or curves, which present a challenge in generating smooth recomputations of online and circular segments. The ultimate goal is to guide the robot seamlessly from its initial position to the desired goal while ensuring accurate trajectory tracking [1-3].

Traditional control methods in path tracking for mobile robots primarily rely on kinematic and dynamic governing equations. Kinematic control, which utilizes kinematic equations with pure velocity as control inputs and employs feedforward (FF) control, has been presented in previous studies [4-6]. This approach has certain limitations that lack the ability to automatically adapt to changing environments, handle complex disturbances, and ensure smooth transitions between trajectory segments. As a result, mobile robots may struggle to accurately follow the desired trajectory due to the absence of feedback (FB) control to adjust their trajectory. To address these limitations, dynamic control techniques have been developed, incorporating torque and requiring a dynamics model to design the controller [7-9]. However, constructing a perfect dynamics model in practical applications remains a challenging task. Fuzzy logic, proportional-integral-derivative (PID), and model predictive control (MPC) are instances of FB controller techniques that have come up with this problem. These methods aim to handle parameter uncertainties and improve the quality of motion control [10-13]. It is important to note that FB control may not provide predictive control action to compensate for known or measurable disturbances, which makes it less suitable for processes with large time constants or long-time delays. Significant and frequent disturbances can result in continuous operation in a transient state and failure to achieve the desired steady state.

This paper introduces a novel approach to enhance trajectory tracking for mobile robots by integrating FF and FB control. By effectively managing both linear and angular velocities while following a predefined trajectory, this method harnesses the advantages of both control techniques. FF control enables the robot to anticipate and generate control signals based on the desired trajectory, facilitating proactive trajectory tracking. Meanwhile, FB control continuously monitors the robot's position and velocity, making real-time adjustments to ensure accurate trajectory following, even in the presence of disturbances and uncertainties. The combination of FF and FB control offers improved performance and robustness in motion control for mobile robots, enabling them to navigate trajectories with greater accuracy and adaptability.

2. KINEMATICS OF DIFFERENTIAL DRIVE ROBOTS

The differential drive mobile robot is a mechanical system that uses two wheels on the same axis to move within the same axis to move within the XY plane. Its position and orientation are determined by the difference in rotation speeds of the wheels. The kinematic model describes the robot's motion based on its position (x, y), and angular position θ without considering forces or torques (Fig. 1).

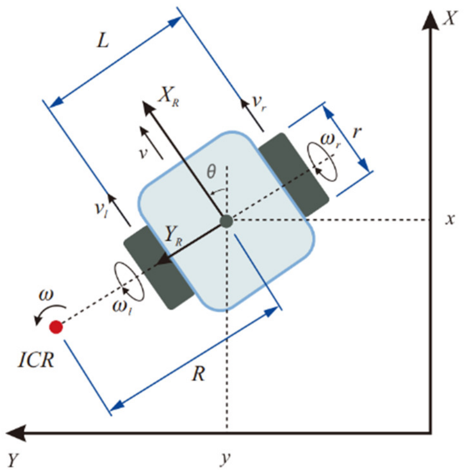


Fig. 1. The differential drive mobile robot

By considering the geometric factors of wheel radius r and axis length L, the kinematic model enables simulation of the robot's behaviour. When both wheels rotate at specified speeds, the robot moves with a linear velocity v and rotates around the Instantaneous Center of Rotation (ICR) with an angular velocity v and radius R. Thus, it can be seen that:

$$\begin{aligned} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= \omega \end{aligned} \tag{1}$$

Where $\dot{\theta}$ is the robot's rotating speed, $\dot{x}=dx/dt$ and $\dot{y}=dy/dt$ are the components of the velocity v at X and Y, respectively, at the current time t. The inputs for the robot can be represented by both variables v and ω in equation (1), as they can be defined in relation to the rotational speeds of the two wheels. The velocity can be determined by the equation $v=R\omega$, where R represents a constant factor.

Similarly, the tangential velocities of the wheels, represented as v_l and v_r, can be formulated in relation to the variables R, L, and ω as follows:

$$v_l = R_l\,\omega = \left(R - \frac{L}{2}\right)\omega \tag{2}$$

$$v_r = R_r\,\omega = \left(R + \frac{L}{2}\right)\omega \tag{3}$$

In which, R_l and R_r denote the rotation radius of each wheel relative to ICR. By solving the system of equations presented in equations (2) and (3) for variables ω and R, the following results are obtained:

$$\begin{aligned} \omega &= \frac{v_r - v_l}{L} \\ R &= \frac{L}{2}\left(\frac{v_r + v_l}{v_r - v_l}\right) \end{aligned} \tag{4}$$

Substituting ω and R in (2) gives:

$$v = \frac{v_r + v_l}{2} \tag{5}$$

Equations (4) and (5) indicate that the values of v and ω are influenced by the tangential velocities of the wheels, denoted as v_l and v_r. Conversely, the aforementioned velocities can be calculated by knowing the rotation speeds of the wheels, represented as ω_l and ω_r, using the formulas $v_l=(r/2)\omega_l$ and $v_r=(r/2)\omega_r$, where r signifies a specific factor related to the wheel.

The state of the differential mobile robot is represented by a vector that includes the minimum set of variables required to describe its behavior at any given time that can be defined by a set of generalized coordinates. The state vector for this robot can be expressed as follows:

$$z(t) = \left[x(t) \quad y(t) \quad \theta(t)\right]^T \tag{6}$$

Using the state vector described above, it is feasible to establish a state equation in the following manner:

$$\dot{z}(t) = f(z,u,t) \tag{7}$$

In the given context, z(t) represents the state vector, u(t) encompasses the system inputs (specifically v(t) and ω(t)), and t denotes the time. Consequently, the state equation can be derived as follows:

$$\dot{z}=\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix}=\begin{bmatrix}v\cos\theta\\ v\sin\theta\\ \omega\end{bmatrix}=\begin{bmatrix}\cos\theta & 0\\ \sin\theta & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix} \quad (8)$$

3. CONTROLLER DESIGN

In the path tracking, the robot will follow the point pose x, y in the trajectory. Therefore, by defining the state space of the robot with input $U=[u_1, u_2]^T=[\ddot{x},\ddot{y}]^T$, and the states $X=[x,\dot{x},y,\dot{y}]^T$, the state-space can be represented as follow:

$$\dot{X}=AX+BU$$

$$\dot{X}=\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{y}\\ \ddot{y}\end{bmatrix}=\begin{bmatrix}0&1&0&0\\0&0&0&0\\0&0&0&1\\0&0&0&0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ y\\ \dot{y}\end{bmatrix}+\begin{bmatrix}0&0\\1&0\\0&0\\0&1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix} \quad (9)$$

$$Y=CX$$

$$Y=\begin{bmatrix}1&0&0&0\\0&0&1&0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ y\\ \dot{y}\end{bmatrix} \quad (10)$$

As can be seen, the system has the same inputs and outputs with four states, and the relative degree obtain is $r=[2, 2]^T$. Hence, there are no internal dynamics, an u_{inv} can be found directly in a term of the reference trajectory.

$$u_{inv}=\begin{bmatrix}u_1\\ u_2\end{bmatrix}=\begin{bmatrix}\ddot{x}_d\\ \ddot{y}_d\end{bmatrix} \quad (11)$$

The controllability of the system outlined in equation (9) is established by examining its controllability matrix $Q_c=[B, AB]$, which possesses a complete rank. This indicates that a state controller can be implemented for any desired characteristic polynomial of the closed loop. The FB and FF controller will be combined as:

$$U=K(X_d-X)+u_{inv} \quad (12)$$

Where $X_d=[x_d,\dot{x}_d,y_d,\dot{y}_d]$ is the reference trajectory. The control gain matrix exhibits a specific structure in equation (9) due to the particular form of matrices A and B. In this form, the input u_1 exclusively affects states x and $\dot{x}$, while the input u_2 solely influences states y and $\dot{y}$. This characteristic results in a distinctive configuration of the control gain matrix.

$$K=\begin{bmatrix}k_1&k_2&0&0\\0&0&k_3&k_4\end{bmatrix} \quad (13)$$

Because the control output of the controller is $\ddot{x},\ddot{y}$ while the input to the kinematic model of the differential mobile robot is v, ω it requires a transform between the output control and the system model. By derivatives the kinematic model in equation (8) with states x, y to find the convert form:

$$\ddot{x}=\dot{v}\cos\theta-v\dot{\theta}\sin\theta \quad (14)$$

$$\ddot{y}=\dot{v}\cos\theta+v\dot{\theta}\sin\theta \quad (15)$$

It is clear that both velocities v, ω are present in the second derivatives. The system of equations is reformulated to express the individual inputs $(\dot{v},\omega)$ of the kinematic model in terms of control outputs. This rewriting allows us to describe $\dot{v}$ and ω as functions dependent on the control outputs:

$$\begin{bmatrix}\dot{v}\\ \omega\end{bmatrix}=F^{-1}\begin{bmatrix}\ddot{x}\\ \ddot{y}\end{bmatrix}=\begin{bmatrix}\cos\theta & \sin\theta\\ -\frac{\sin\theta}{v} & \frac{\cos\theta}{v}\end{bmatrix}\begin{bmatrix}\ddot{x}\\ \ddot{y}\end{bmatrix} \quad (16)$$

The obtained solution ω from equation (16) serves as the direct input for the robot, while the solution $\dot{v}$ should be integrated before it can be used as an input for the kinematic model. The combined overall control structure is shown in Fig. 2.

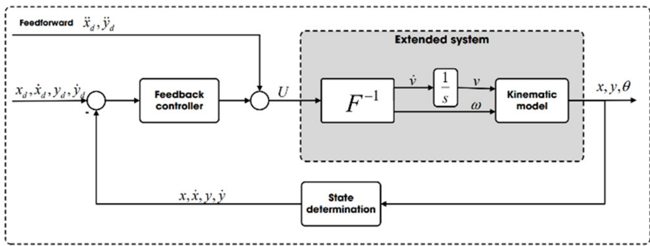
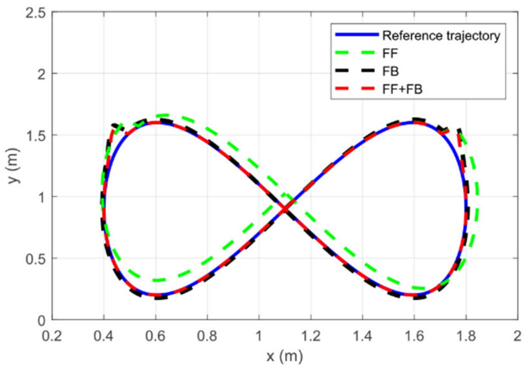


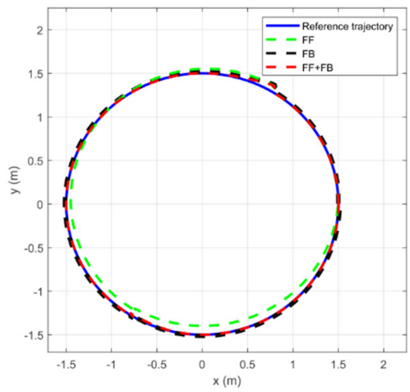
Fig. 2. FF and FB control structure of path tracking for the mobile robot

4. RESULT AND DISCUSSION

To evaluate the path-tracking performance of the proposed method for mobile robots, three controllers FF, FB, and the proposed method by combined FF and FB (FF+FB) were analyzed with two reference trajectories were chosen for simulation: the infinity trajectory and the circular trajectory. The trajectories tracking of the mobile robot are illustrated in Fig. 3 which included the infinity trajectory and the circular trajectory. Moreover, two disturbances and the uncertainty model were introduced during the robot's movement to verify the ability of the proposed controller when affected by the environment. These problems occurred at time intervals of 5 seconds and 20 seconds in separate cases. The errors between the controllers along the x-axis and y-axis are illustrated in Figs. 4 and 7 for the infinity trajectory and circular trajectory, respectively. The velocities of the robot created by three controllers for two trajectories are demonstrated in Fig. 5 and Fig. 7.



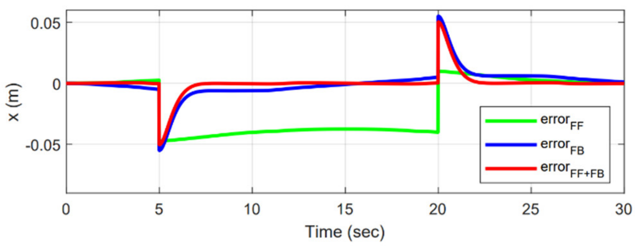
(a)



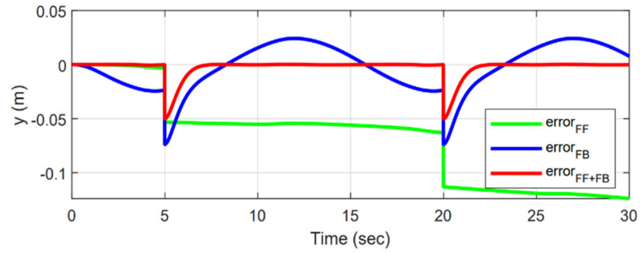
(b)

Fig. 3. Trajectory tracking of the mobile robot: (a) Infinity trajectory and (b) Circular trajectory

It is evident from the evaluation that, in the absence of disturbances, all three controllers (FF, FB, and FF+FB) demonstrate stable operation. Notably, the FF controller exhibits the smallest control errors on the x and y axes for both trajectories, particularly within the initial 0 to 5-second timeframe. In contrast, the FB controller, relying on position errors for control signal adjustments, leads to comparatively larger errors at the current time due to this feedback-based mechanism.

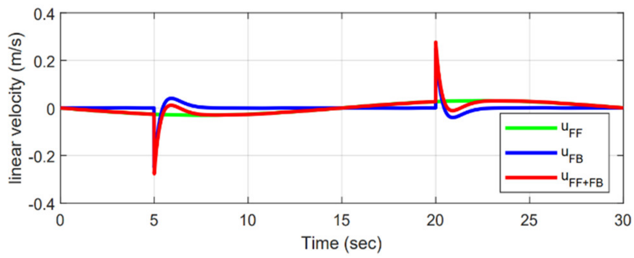


(a)

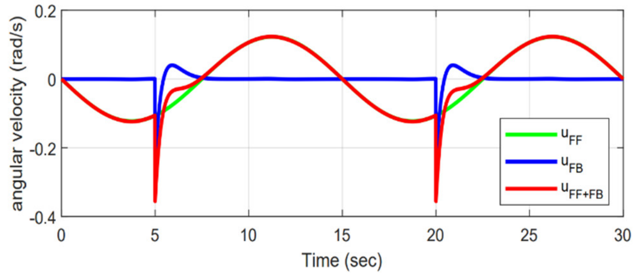


(b)

Fig. 4. The error of the mobile robot with the infinity trajectory: (a) The error in the x-axis and (b) The error in the y-axis



(a)



(b)

Fig. 5. The velocities control with the infinity trajectory: (a) Linear velocity and (b) Angular velocity

However, at the 5-second mark, when a disturbance and the uncertainty model affect the robot, the performance of the FF controller is noticeably impacted, causing it to deviate from the desired path. In contrast, FF+FB controllers demonstrate improved performance due to the incorporation of current position FB. By combining the FF and FB controllers, the error in the robot's position movement is minimized for both trajectories. It is worth mentioning that the control signals for linear and angular velocity from the FF+FB controller closely resemble those of the FF controller (Figs. 4 and 7) they have differences when affected by the disturbance and the uncertainty model. Due to the integration of the FB controller, these signals will change by the FB controller and can adapt and adjust in the presence of disturbance and the uncertainty model to steer the robot back onto the desired trajectory. This behavior remains consistent at the 20-second mark.

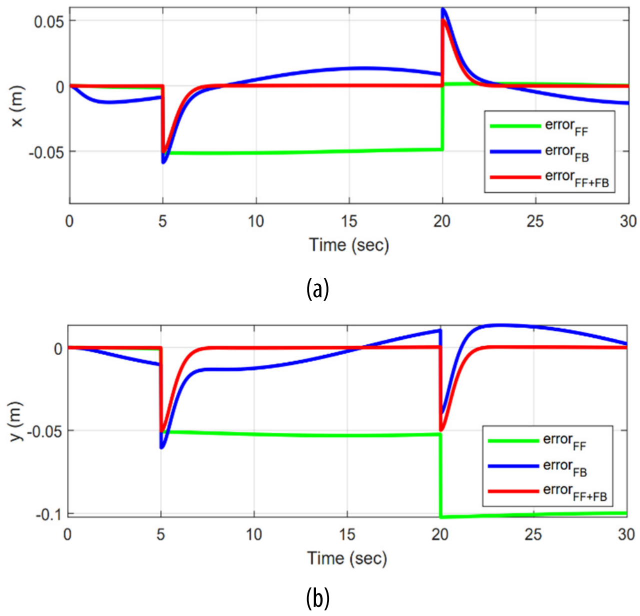


Fig. 6. The error of the mobile robot with the circular trajectory: (a) The error in the x-axis and (b) The error in the y-axis

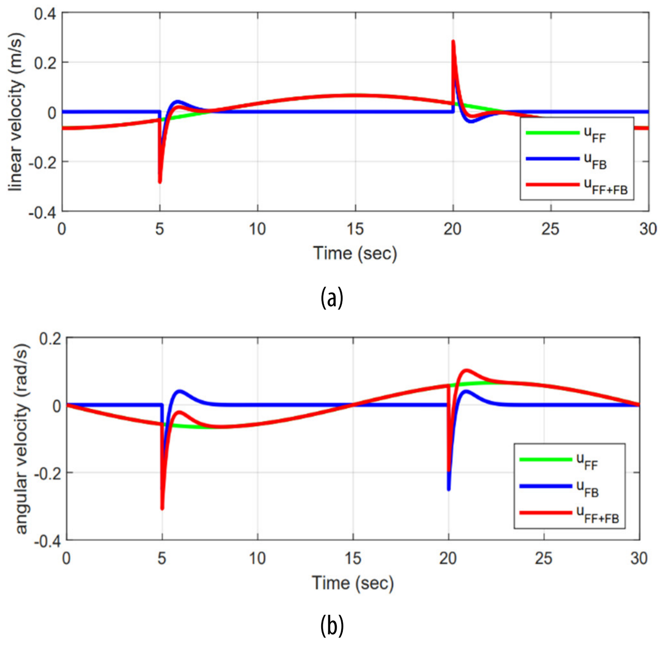


Fig. 7. The velocities control with the circular trajectory: (a) Linear velocity and (b) Angular velocity

To evaluate the performance of the proposed method, the standard deviation of Root Mean Square Error (RMSE) is applied, in which p_i is the actual position and p_{ref} is the reference position. As shown in Table 1, the FF+FB method produces smallest position errors in x-axis and y-axis in circular and infinity trajectories, which are 0.0088 and 0.0090m, respectively.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(p_i - p_{ref})^2} \quad (17)$$

Table 1. The comparison of position errors in different methods

Trajectory type	Controller	x-axis (m)	y-axis (m)
Circular trajectory	FF	0.0350	0.0691
	FB	0.0138	0.0132
	FF+FB	0.0088	0.0096
Infinity trajectory	FF	0.0281	0.0789
	FB	0.0117	0.0215
	FF+FB	0.0097	0.0090

Thus, while the FF controller may exhibit good performance in the absence of disturbances and uncertainties, real-world scenarios often involve environmental disturbances and uncertainties that can affect the controller's performance. This is a situation that makes the FB controller becomes crucial. The FB controller takes into account the current position errors and allows for adjustments in response to disturbances, helping the robot maintain better trajectory tracking performance in such challenging conditions. Furthermore, the combination of two controllers FF and FB, offers significant advantages. By leveraging the strengths of both controllers, the combined controller can compensate for the limitations of each individual controller. The FF controller provides accurate trajectory tracking during normal conditions, while the FB controller introduces corrective measures to counteract the effects of disturbances and uncertainties. The synergy between the two controllers enhances the robot's ability to follow the desired trajectory more effectively and adapt to varying environmental conditions, providing improved overall performance and robustness.

5. CONCLUSION

The paper addressed the challenges of disturbances and uncertainty in tracking trajectories for mobile robots. It proposed a controller design that combines FF and FB control to mitigate these issues. The goal is to enable a two-wheeled differential mobile robot to accurately follow a pre-set trajectory, even in the presence of environmental disturbances. To assess the effectiveness of the suggested controller, simulation experiments were carried out on two distinct paths. These experiments aimed to evaluate the performance of the controller under varying conditions and scenarios. The results indicate that combining FF and FB control leads to significantly improved tracking ability and reduced moving errors compared to using either control method individually. Future work will include designing a predictive controller combined with FF controller that enables the robot to follow a trajectory and avoid obstacles simultaneously.

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