RESEARCH ON THE MOTION STABILITY OF SELF-BALANCING TWO-WHEELERS

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ABSTRACT

A self-balancing two-wheeled vehicle is a dynamic system that is susceptible to imbalance from tilting and shaking because of variables like center of gravity coordinates and electric motor torque. The system is unable to concurrently satisfy needs like swing angle control and stability during movement due to its structural and weight features. However, depending on the goal function, the control system's design can maximize the vehicle's balance and stability. The paper presents theoretical research, examines controllability and system observation using LQR control theory, simulates the vehicle's stability, and assesses stability when the experimental vehicle's electric motor is stimulated by the supply unit's pulse.

Keywords: *Dynamics, Stability Optimization, Swing Angle, Modeling, Excitation Impact, LQR, PID, DSP.*

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1. INTRODUCTION

The stability of self-balancing two-wheelers, such as electric scooters, is influenced by complex dynamics that require robust control systems. These vehicles often function as under-actuated systems, facing stability challenges due to nonlinear behavior. Effective design and control strategies, particularly in response to real-time sensor feedback on posture and weight distribution, are crucial for maintaining balance [1, 2]. The Proportional-Integral-Derivative (PID) controller is commonly used, with recent improvements integrating particle swarm optimization to enhance stability and responsiveness [3]. Design of a Self-Balancing Vehicle with PID Control Based on Improved Particle Swarm Optimization Algorithm [4]. Adaptive control systems with neural networks and observers have also shown promise in refining the balancing mechanism [5, 6]. Additionally, control moment gyroscopes have been used to improve motion stability in confined spaces [7].

Safety remains a key concern, with accidents linked to instability during sudden braking or low-speed operation [8, 9]. Injuries from electric scooters further highlight the need for enhanced safety features [10, 11]. Technological advances, such as visual identification systems with sensor feedback, address control accuracy issues by reducing errors from gyroscopes and accelerometers [12]. Recent developments in alternative materials and structures also contribute to stability and safety [13, 14].

This study focuses on optimizing the stability of self-balancing two-wheelers, modeled as inverted pendulum systems. Challenges include complex movements, imbalances from rolling torque, and maintenance difficulties due to mechanical complexity. An optimization problem is formulated using the state-space model, with input voltage parameters for the electric motor calculated to achieve optimal stability under set motion conditions. The LQR optimization algorithm is applied, and Matlab-Simulink is used to design the controller, using the GEXTEK Hoverboard's parameters to assess dynamic stability. Therefore, the authors' group has studied the controllability and system observation using LQR control theory, simulated the vehicle's stability, and evaluated stability when the experimental vehicle's electric motor is stimulated by the supply unit's pulse.

2. RESEARCH THEORETICAL BASIS

2.1. Self-balancing two-wheeler dynamics model

Assuming that the car turns straight in the direction 0x, does not slip on the road surface ($\theta_{wL} = \theta_{wR} = \theta_w$), ignoring the force and rolling resistance torque applied to the wheels, ignoring the friction on the bearings of the electric motor.

The self-balancing two-wheeler model is the form of inverted pendulum move-ment, the modeling of the self-balancing two-wheeler (horizontally) is modeled as Fig. 1 [15].

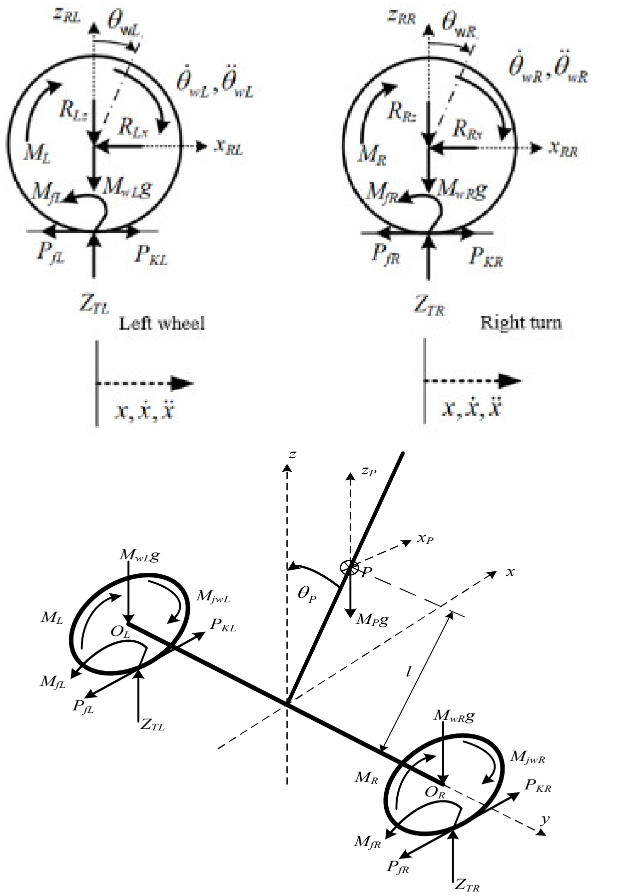

Fig. 1. Force and torque applied to the car

2.2. Research Vehicle Model Basic Parameters

Table 1. Basic parameters of the research model

Ampersand	Meaning	Unit
x	Wheel center displacement in the direction of 0x	m
$\dot{x}$	Wheel center velocity	m/s
θ	Body rotation angle in the vertical side of the vehicle	rad
$\dot{\theta}$	Body rotation angle speed	rad/s
l	Distance from the center of the wheel to the center of gravity of the vehicle	m
M_p	Equivalent weight of the car body	kg
r	Wheel radius	m
M_w	Equivalent weight of the wheel	kg
R_L, R_R	Jet from the wheels to the left and right body	N
$M_{L,R}$	Electric motor torque on wheels	Nm
$P_{fL,R}$	Rolling resistance applied to the left and right wheels	N
k_f	Friction coefficient on the shaft of an electric motorcoefficient on the shaft of an electric motor	
I_R	Moment of inertia of electric motor armature	
i	Gravity current strength	A
R	Armature resistance	Ω
L	Electric motor self-induction coefficient	H
ω	Rotational Velocity of Electric Motor	rad/s
V_a	Electric motor input voltage	V
V_{emf}	Electromotive Capacitor	V

2.3. Motion Mathematical Models

The mathematical model is established through rotational movements around the axes 0x, 0y, and 0z and translational movements in the 0x direction with the condi-tions that the vehicle does not slip on the road surface (θwL = θwR = θw), ignoring the force and moment of resistance to rolling on the wheels, ignoring the friction of the motor bearing, The wheels are always in contact with the road surface; we have

Wheel dynamics equation

$$x = \theta_w.r \quad , \dot{x} = \dot{\theta}_w.r \qquad , \ddot{x} = \ddot{\theta}_w.r \tag{1}$$

The equation for balancing the force of the car body in the x-square is

$$\begin{gathered} \sum F_x = M_p\ddot{x} \\ (R_{Lx} + R_{Rx}) - M_p l\ddot{\theta}\cos\theta_p + M_p l\theta_p^2 \sin\theta_p = M_p\ddot{x} \end{gathered} \tag{2}$$

Wheel equilibrium equation and state space equation of wheel motion, Newton II. Suppose $\theta_p = \pi + \phi$ with ϕ describe a small angle from the vertical upward direction:

$$R_{Lx} + R_{Rx} = 2(M_w + \frac{I_w}{r^2})\ddot{x} + \frac{2k_m k_e}{Rr^2}\dot{x} - \frac{2k_m}{Rr}V_a \tag{3}$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{2k_e k_m(M_p lr - I_p - M_p l^2)}{Rr^2\alpha} & \frac{M_p^2 g l^2}{\alpha} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2k_e k_m(r\beta - M_p l)}{Rr^2\alpha} & \frac{M_p g l\beta}{\alpha} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2k_m(M_p l^2 + I_p - M_p lr)}{Rr\alpha} \\ 0 \\ \frac{2k_m(M_p l - r\beta)}{Rr\alpha} \end{bmatrix} V_a$$

or

$$\begin{cases} \dot{X} = AX + Bu \\ Y = CX + Du \end{cases} \quad (4)$$

The principle of stability control for a self-balancing two-wheeler is illustrated in Fig. 2. The car is in a balanced state when the body is upright, with the center of gravity coinciding with the unstable balance point. However, because this equilibrium point is unstable, even a small impact force causes the car to leave its position and it cannot return on its own without control. Therefore, it is necessary to impact the A2 control signal in the same direction of movement to bring the car to a balanced state. In addition, it is also important to control the car to follow the set value of the dis-placement and rotation angle.

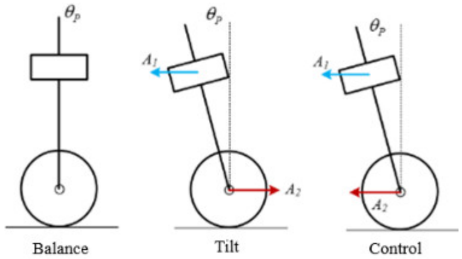

Fig. 2. The driving principle of a self-balancing two-wheeler

The body is considered balanced when in the vertical position, coinciding with the positive axis 0z. If there is an angular deviation from the 0z axis, the car will be out of balance with the tilt angle θP. The two main kinematic factors to consider in the mathematical model are controlling the angular deviation of the body and the for-ward motion of the entire system. When the car is moving, the jamming forces act on the body, causing the car to rotate at an angle to the 0z axis, which is directly propor-tional to the intensity and direction of the noise force. The control is achieved by minimizing the error between the actual angle θm and the desired angle of inclination θd = 0. Two control signals from the feedback loops will be used to adjust the engine voltage, helping the car return to the desired equilibrium position.

2.4. Optimal control of the system according to Lyapunov

The simulation of the stability of a self-balancing two-wheeler begins by investi-gating the controllability and observability of the system through LQR control theory. The control system can be designed to optimize stable quality (body balancing) based on a defined target function. The control problem aims to achieve the desired output responses when the system is subjected to input stimuli, including interference. The Lyapunov stabilization method allows the system to return itself to the nearest equilibrium point after being impacted without needing a control signal.

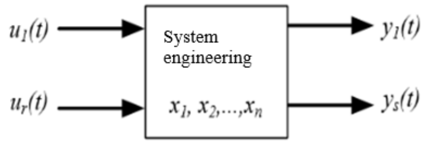

Fig. 3. System Diagram

The input signal is typically random, as when a two-wheeled vehicle moves on the road, the center of gravity coordinates and the engine torque generate the tilting and swaying of the body. With random signals, it is not feasible to apply a specific time function; instead, the occurrence can only be described probabilistically. In control systems, the stationary stochastic process is commonly analyzed, where parameters like mx and rx(τ) are defined based on the x(t) element, resulting in an ergodic stochastic process.

$$m_x = \lim_{T\to\infty}\frac{1}{T}\int_0^T x(t)dt \text{ and } r_x(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_0^T x(t)x(t+\tau)dt \quad (5)$$

Use SISO, MIMO, ODE, or Runge Kutta models in Matlab to determine the results of the above problems.

2.5. System Stimulation

The forced process is the system's response when it has an output state of 0 and re-ceives the stimulus signal U(t). The free process is the system's response when it is not stimulated but has an initial state other than 0. Thus, when the system is stimulated by a signal to U(t) due to the linearity of the system, the state model has 2 compo-nents: Y(t) = Yc(t) + Yt(t). Yc(t) is the solution of the state equation corresponding to the given U(t) and the initial state X(0) = 0, which is the equation describing the forced process.

$$Y_c(t) = C\int_0^t e^{A(t-\tau)}BU(\tau)d\tau + DU \quad (6)$$

On the basis of predetermined parameters, such as dimensions and volume, it is nec-essary to calculate the input voltage parameters for the electric motor so that the swing angle of the vehicle achieves the best stability indicators under the set motion conditions.

3. GEXTEX HOVERBOARD VEHICLE DYNAMICS OPTIMIZATION REVIEW

3.1. GexTex Hoverboard Test Vehicle Parameters

Table 2. Properties of GEXTEK Hoverboard

Describe	Ampersand	Value	Unit
The weight of the wheel	Mw	0,51	kg

Moment of inertia of the wheel	Iw	$5,1.10^{-4}$	$kg.m^2$
Wheel radius	r	0,062	m
Body Weight	Mp	59	kg
Moment of inertia of the car body	Ip	0,228	$kg.m^2$
Distance	l	1,3	m
Field Acceleration	g	9,81	m/s^2
Armature resistance	R	1	Ω
Moment constant	Km	1	Nm/A
Reaction electrodynamic constant	Ke	1	Vs
Engine power	P	350	W
Working Current	I	9,7	A

Fig. 4. GEXTEK Hoverboard Test Vehicle

3.2. Build an m-file

m-file including: Formulate state equations and calculate the coefficients of LQR controllers.

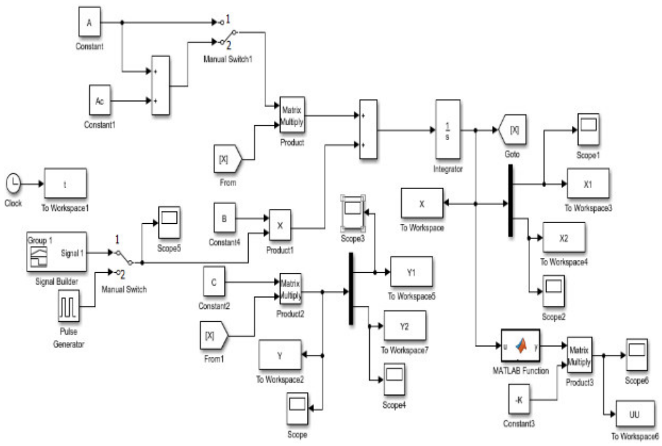

Fig. 5. Simulink Diagram Simulating the System

4. RESULTS AND DISCUSSION

4.1. Results with excitation source with unit pulse and no control

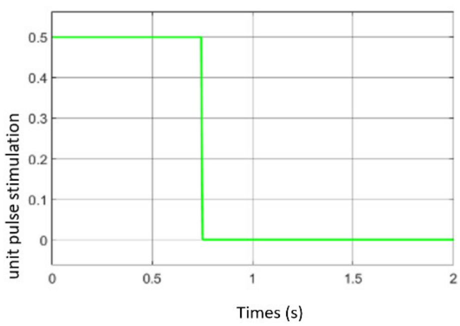

Fig. 6. Excitation pulse with unit pulse

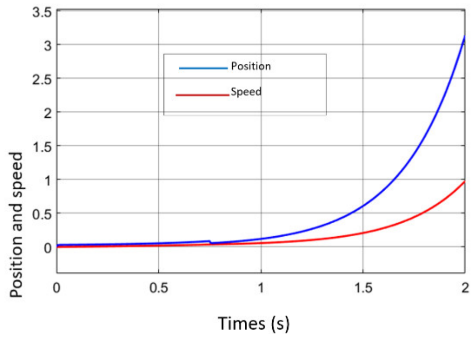

Fig. 7. When the unit pulse excites the vehicle body, its position and speed remain uncontrolled

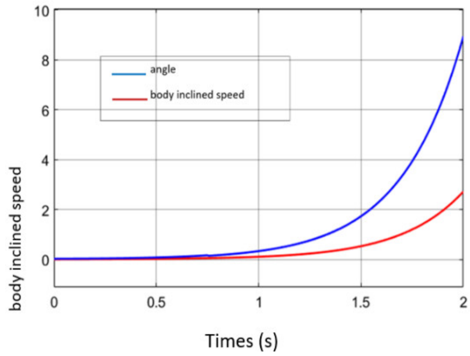

Fig. 8. Tilt angle and tilt speed when not controlled

Two-wheeled vehicles are stimulated with unit pulses to produce results that corre-spond to the system's response $(x,\dot{x},\phi,\dot{\phi})$.

With the results in Figs. 7 and 8, we can see that when the increase time starts from about 0.5 (s) onwards, the coordinate position of the center of gravity as well as the tilt angle of the body also gradually increase and do not return to the original position, which means that the movement system of the two-wheeled vehicle is unbalanced.

4.2. Results with LQR controlled stimulation source

When the system is adjusted by the LQR controller, the position and tilt angle of the car body are as shown in Fig. 9.

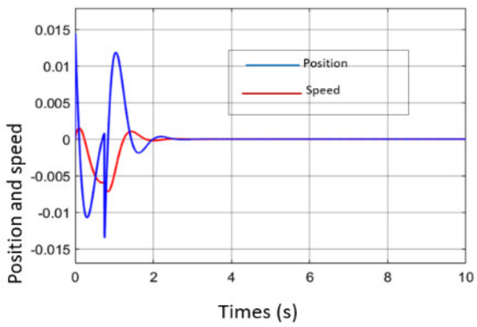

Fig. 9. Position and speed of the vehicle body when excited with the unit pulse when using LQR

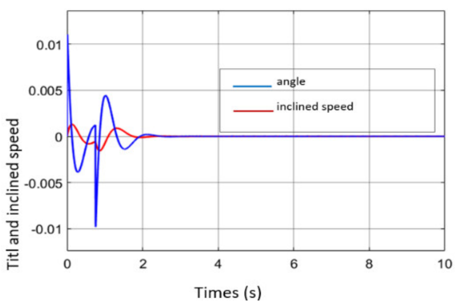

Fig. 10. Tilt angle and tilt speed when excited with single pulse when using LQR

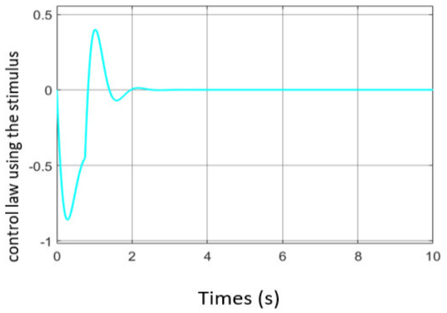

Fig. 11. Control law when excitation adapts to unit pulse

The data presented in Figs. 9 and 10 demonstrate that when subjected to a unit pulse input, the two-wheeled vehicle controlled by the Linear Quadratic Regulator (LQR) controller exhibits forward and backward oscillations. The maximum deviation of the center of gravity is recorded at 14cm, with stabilization occurring within approximately 2 seconds as per the control law shown in Fig. 11. This behavior reflects the vehicle's dynamic response, where the control system continuously works to counteract instability. The LQR controller, aimed at optimizing performance by minimizing positional and tilt angle errors, effectively mitigates the initial oscillations induced by the pulse input and promptly stabilizes the system. The vehicle is powered by torque from two electric motors, operating at a maximum supply voltage of 12V (Fig. 10), which control its motion. Figs. 11 and 12 display the vehicle's positional and tilt angle changes over time, showcasing the control system's capacity to maintain stability during operation.

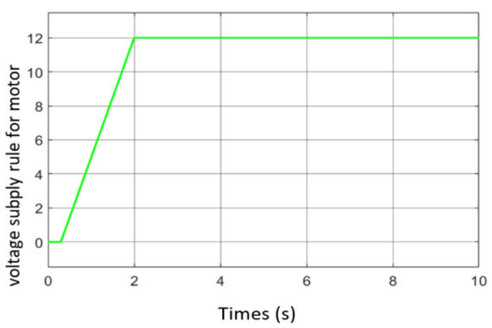

Fig. 12. Voltage supply rules for electric motors

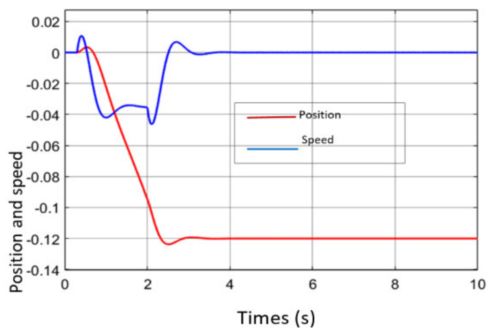

Fig. 13. Position and speed of the vehicle body when applying voltage to the electric motor and controlled by the LQR controller

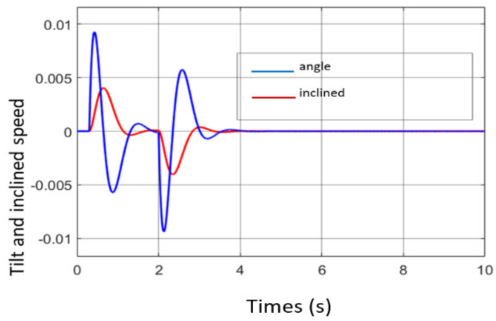

Fig. 14. Tilt angle and body tilt speed when applying voltage to electric motors and controlled by LQR controllers

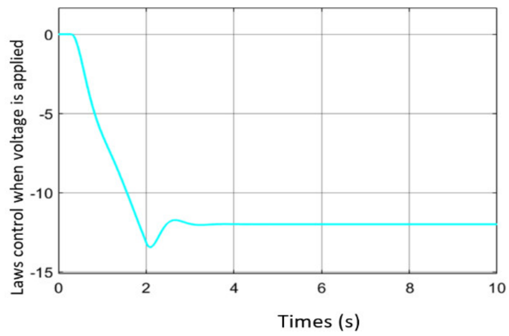

Fig. 15. Laws for stable control of two-wheel vehicle when voltage is applied to electric motors and controlled by LQR

The graphs in Figs. 14 and 15 illustrate the simulation results for the position and tilt angle changes of the vehicle's body $(x,\dot{x},\phi,\dot{\phi})$. These results correspond to a 12V power supply and LQR controller operation. Specifically, the coordinate position of the center of gravity is around 12 cm, and after approximately 3 seconds, the system starts stabilizing according to the control law shown in Fig. 15.

5. CONCLUSION

The objective is to explore the theoretical foundation of optimal control for the propulsion system and apply the LQR control algorithm to the linear propulsion system, aiming to improve the design parameters of the self-balancing two-wheeled vehicle.

The study uses a specific set of self-balancing two-wheelers to simulate and analyze the vehicle's dynamic characteristics under two conditions: with and without LQR controllers.

The simulation results indicate that, with the LQR controller, the vehicle successfully maintains self-balance, returning to its stable position in 2 seconds with a deviation of only 20cm, which remains within the system's stability limits.

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