SEPARATION DYNAMICS IN A HALF-CAR MODEL

ĐỘNG LỰC HỌC QUÁ TRÌNH TÁCH BÁNH XE TRÊN MÔ HÌNH 1/2 NGANG XE

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ABSTRACT

The paper explores the vibration dynamics of a half-car model, focusing on whether the tires maintain contact with the road surface. A set of differential equations is developed to capture the two distinct states: with and without tire contact. A separation condition is applied to both tires, enabling a comparison of dynamic responses in these states. The vertical and roll dynamics are examined in relation to changes in vehicle structure and road inputs. The results demonstrate that the new model, incorporating the separation assumption, offers greater accuracy compared to previous vehicle vibration studies. Ultimately, this research provides novel insights that can inform future suspension design, as well as help develop effective control strategies to prevent rollovers and ensure lateral stability.

Keywords: *A half-car vibrations, vehicle vibrations, vibration dynamics, roll dynamics; tire-road separation, vehicle safety, ride comfort.*

TÓM TẮT

Bài báo nghiên cứu động lực học dao động của mô hình ½ ngang xe, tập trung vào nghiên cứu liệu bánh xe có tiếp xúc với bề mặt đường hay không. Một hệ phương trình vi phân được phát triển để mô tả dao động phương tiện cơ giới trong hai trạng thái: tiếp xúc và không tiếp xúc của lốp xe với mặt đường. Điều kiện tách bánh được áp dụng cho cả hai lốp xe bên trái và bên phải, cho phép so sánh các đặc tính động lực học trong hai trạng thái trên. Động lực học dao động theo phương thẳng đứng và góc nghiêng ngang được khảo sát khi thay đổi kết cấu xe và thay đổi kích thích mặt đường. Kết quả cho thấy mô hình mới, với giả định tách bánh xe, có độ chính xác cao hơn so với các nghiên cứu về dao động của hệ thống treo trước đây. Cuối cùng, nghiên cứu này cung cấp những kiến thức mới có thể được ứng dụng vào việc thiết kế hệ thống treo trong tương lai.

Từ khóa: *Dao động mô hình một phần hai; dao động phương tiện cơ giới; động lực học dao động, động lực học góc nghiêng ngang; tách bánh xe với mặt đường; an toàn phương tiện cơ giới; độ êm dịu chuyển động.*

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1. INTRODUCTION

Tire-road separation is a rare and exceptional event observed in the analysis of a vehicle's ride and handling [1] This phenomenon usually occurs as a brief, transient event that quickly stabilizes [2] Given the complexity of this dynamic system, two sets of equations are required to model the conditions when the tire is in contact with the road and when it is not [1, 3]. These equations need to be structured in a way that ensures continuity during the transition between the two states [4]. Tire-road separation can cause a sharp jolt or sudden shift in the vehicle's movement, temporarily reducing ride comfort [5]. The brief loss of contact, followed by re-establishing connection with the road, can transfer vibrations and shocks to the passengers [1, 6].

Recently, there are few researches have investigated the tire-road process in quarter-car and bicycle-car models [7, 8]. The researches focus on designing vehicles with suspension systems and tire properties that reduce the frequency and impact of tire-road separation under normal driving conditions [9-11]. However, no studies have been conducted on a half-car model that accounts for tire-road separation. Investigating the vibrations in this model is crucial, as it provides a deeper understanding of roll dynamics, which significantly affect the stability of moving vehicles. The safety of vehicle travel relies on good stability performance [12]. Therefore, this paper concentrates on examining how road excitations and vehicle structure influence both roll and vertical dynamics.

2. MATHEMATICAL MODELING

This section presents a 4-degree-of-freedom half-car model that incorporates both in-contact and no-contact states. The relevant systems of differential equations are outlined. Furthermore, the study introduces the concept of the separation condition to manage the transition between the two governing equation systems. This separation condition helps clarify how much the tire and road are apart in different driving situations.

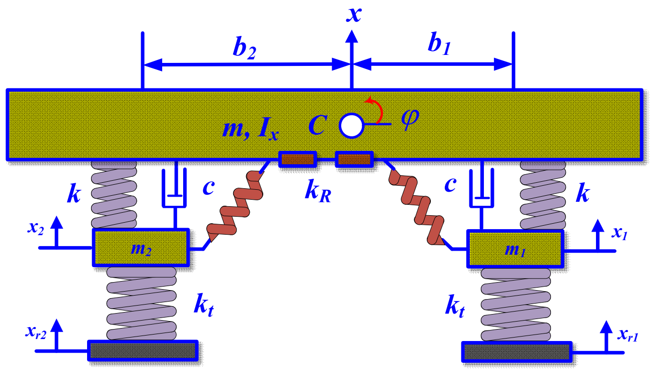

Figure 1. A half- car vibrating system of a vehicle

2.1. Governing Equation System

To investigate and optimize the roll vibration of a vehicle, a half-car vibrating model could be used [1], Figure 1 demonstrates the tire-road separation model for reference. This system involves the body displacement x, body roll φ, wheels hop x_1, x_2 as well as independent road inputs y_1 and y_2. The study will use a vehicle suspension system that includes both in-contact and no-contact states. This model will be employed to examine the transitions between the tire and road under periodic excitations, specifically represented as $x_{r1} = y_0\sin\omega t$; $x_{r2} = y_0\sin\omega t$, where x_{r1}, x_{r2} denote the displacements and ω represents the angular frequency of the road excitations.

In the contact state, the equations of motion for the model are written [1]:

$$m\ddot{x} + c(\dot{x} - \dot{x}_1 + b_1\dot{\varphi}) + c(\dot{x} - \dot{x}_2 - b_2\dot{\varphi}) + k(x - x_1 + b_1\varphi) + k(x - x_2 - b_2\varphi) = 0 \quad (1)$$

$$I_x\ddot{\varphi} + b_1c(\dot{x} - \dot{x}_1 + b_1\dot{\varphi}) - b_2c(\dot{x} - \dot{x}_2 - b_2\dot{\varphi}) + b_1k(x - x_1 + b_1\varphi) - b_2k(x - x_2 - b_2\varphi) + k_R\varphi = 0 \quad (2)$$

$$m_1\ddot{x}_1 - c(\dot{x} - \dot{x}_1 + b_1\dot{\varphi}) + k_t(x_1 - x_{r1}) - k(x - x_1 + b_1\varphi) = 0 \quad (3)$$

$$m_2\ddot{x}_2 - c(\dot{x} - \dot{x}_2 - b_2\dot{\varphi}) + k_t(x_2 - x_{r2}) - k(x - x_2 - b_2\varphi) = 0 \quad (4)$$

While the motion equations of the two wheels in the separation state are:

$$m_1\ddot{x}_1 - c(\dot{x} - \dot{x}_1 + b_1\dot{\varphi}) - k(x - x_1 + b_1\varphi) + (m_1 + \frac{b_2}{b}m)g = 0 \quad (5)$$

$$m_2\ddot{x}_2 - c(\dot{x} - \dot{x}_2 - b_2\dot{\varphi}) - k(x - x_2 - b_2\varphi) + (m_2 + m\frac{b_1}{b})g = 0 \quad (6)$$

2.2. Separation Condition

To solve the discontinuous system of differential equations above, it is necessary to have a separation condition for the wheels to transition from the in-contact state to the free-fall state and vice versa. This separation condition is applied for cases where either the left wheel or the right wheel is separated, or even when both wheels are separated simultaneously. To understand how the differential equations of motion work, we need to identify the separation condition. The tire consistently remains in contact with the road when its vertical displacement is less than its relaxed radial dimension. In contrast, the tire loses contact only when the vertical displacement exceeds the relaxed radius. Thus, the separation condition occurs when the relative displacement between the unsprung mass and road excitation becomes greater than the static compression of the tire.

Contact condition for right wheel:

$$x_1 - x_{r1} < x_{T1} \quad (7)$$

Free-fall condition for right wheel:

$$x_1 - x_{r1} \geq x_{T1} \quad (8)$$

Contact condition for left wheel:

$$x_2 - x_{r2} < x_{T2} \quad (9)$$

Free-fall condition for left wheel:

$$x_2 - x_{r2} \geq x_{T2} \quad (10)$$

where, $x_{T1} = \dfrac{(m_1 + m\frac{b_2}{b})g}{k_t}$ and $x_{T2} = \dfrac{(m_2 + m\frac{b_1}{b})g}{k_t}$;

They are static compression.

Numerical integration is utilized to examine the vibration dynamics of vehicles moving over a road bump, aiming to identify possible occurrences of tire-road separation. In this study, MATLAB's ode45 solver is applied to handle the numerical integrations corresponding to the separation condition.

3. RESULTS AND DISCUSSION

The half-car model can vary between the front and rear halves due to differences in suspension systems and mass distribution. Additionally, the front and rear halves may use different anti-roll bars with varying torsional stiffness k_R. In this study, the front half of a half-car model is analyzed using the parametric data for a practical half-car model provided in Table 1 [1].

Table 1. Dimensional parameters of a half-car model

Parameter	Value [Unit]	Parameter	Value [Unit]
m	420kg	I_x	820kgm^2
m_1	53kg	m_2	53kg
b_1	0.7m	b_2	0.75m
k	10000N/m	k_t	200000N/m
c	1000Ns/m	k_r	25000N/m

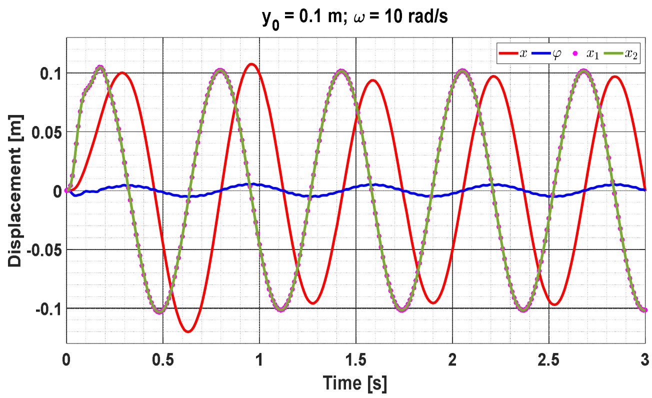

Figure 2. Time response of a half-car model in the contact state

The time response is useful for analyzing the steady-state behavior of a vehicle traveling over a series of sine wave-shaped roads. Figure 2 illustrates the time domain in the contact state. Although the road amplitude is high at $y_0 = 0.1\text{m}$, both wheels remain in constant contact with the road surface due to the low input frequency $\omega = 10\text{rad/s}$. In this case, the body bounce x and the wheel hops x_1, x_2 resemble harmonic functions. Moreover, the displacements of the wheels are identical, as the dimensions b_1 and b_2 are nearly equal. As a result, the roll vibration φ is negligible.

To detect the loss of tire-road contact, two indicator functions, I_1, I_2, are utilized for the corresponding wheels. Let us define the indicator functions to identify tire-road separation during the vehicle's motion:

$$I_1 = \text{Heaviside}(x_{11} - x_{r1} - x_{T1}) = \begin{cases} 0.05 & x_1 - x_{r1} \geq x_{T1} \\ 0 & x_1 - x_{r1} < x_{T1} \end{cases} \quad (11)$$

$$I_2 = \text{Heaviside}(x_2 - x_{r2} - x_{T2}) = \begin{cases} 0.05 & x_2 - x_{r2} \geq x_{T2} \\ 0 & x_2 - x_{r2} < x_{T2} \end{cases} \quad (12)$$

To observe how the indicators work, this research runs the model over the road at a frequency where tire-road separation occurs. When the road frequency increases to a high value, $\omega = 20\text{rad/s}$, the separation phenomenon becomes more apparent. In this scenario, the model will no longer maintain constant contact with the road profile for a certain period during each cycle, a condition referred to as steady-state separation [6]. Notably, the left wheel experiences more separation than the right. This can be explained by the difference in the distance from the center of mass to the two wheels. In this study, while the excitations for both tires are the same, the distance from the center of mass to the left wheel is greater than to the right wheel, as shown in Table 1. Furthermore, the displacements of the two tires differ slightly, causing body roll to fluctuate significantly compared to the previous case, as shown in Figure 3.

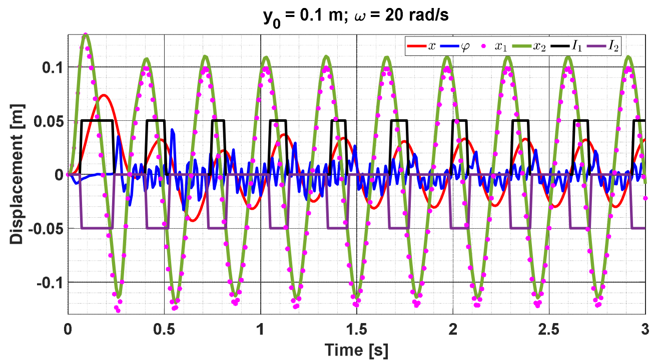

Figure 3. Time response of a half-car model in the no-contact state

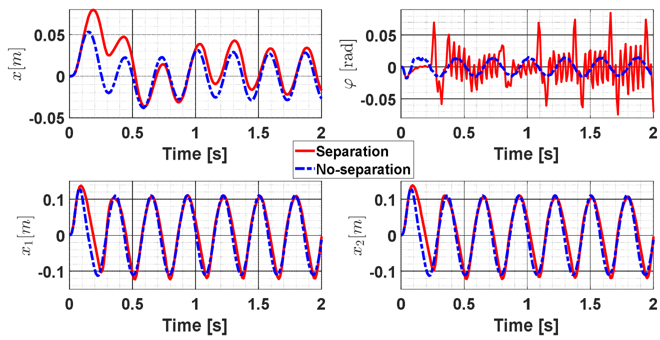

Figure 4. Comparison of time responses in the two states

A comparsion of the time responses for tire-road separation and tire-road in contact assumption will reveal important insights. This comparison highlights how the time responses differ when considering separation. The difference addresses the question of how much error is incurred in calculating vibration dynamics without the separation assumption.

At an input frequency of $\omega = 22\text{rad/s}$, Figure 4 illustrates the numerical calculation of the time response, with dashed lines representing the assumption that the tire will never separate from the road, and solid lines indicating the time response when tires can separate from the road. The time responses diverge after an initial period, with the actual response differing significantly from the one that does not consider tire-road separation. In more detail, the body roll shows a difference of over 10 percent, even 50 percent at certain points.

The roll angle of a vehicle typically ranges from 2 to 5 degrees when the vehicle is travelling normally on a smooth roadway. However, when the vehicle turns or encounters unstable conditions, the roll angle might rise but generally does not exceed 7 to 10 degrees for most vehicles [13]. If the body roll exceeds the limit, the vehicle becomes unstable and may experience a rollover. Therefore, investigating the roll response while considering tire-road separation is necessary. As seen in Figure 5, the roll angle of the half-car depends on the roadway input frequency. The roll angle over a sufficiently long simulation interval is high at a high input frequency, reaching a maximum of approximately 0.1 rad, equivalent to 6 degrees. Meaning that a rollover will occur if the input frequency is increased.

Without considering tire-road separation, the vibration response may be reduced remarkably and may not accurately reflect real-world situation. Thereby, Figure 6 compares a series of body bounces corresponding to distinct road amplitudes under two assumptions including separation and no separation. The body displacements follow a sine function at a low road excitation amplitude, $y_0 = 0.04\text{m}$, and the differences between the two responses are negligible. Nevertheless, the vertical displacement with separation consideration goes notably up at high road amplitudes, it reaches approximately to 0.14m after the transient period at a road amplitude of $y_0 = 0.1\text{m}$.

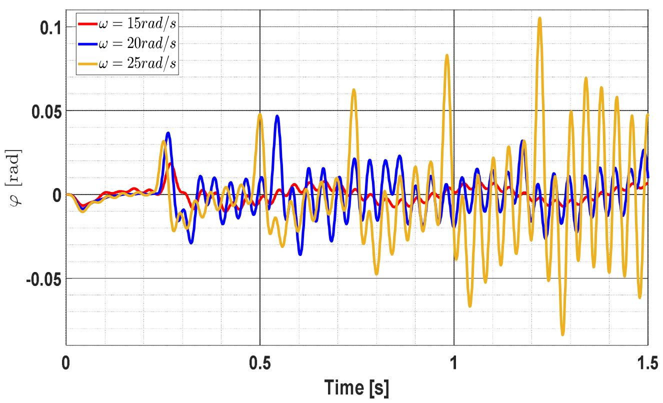

Figure 5. Roll angles with different roads

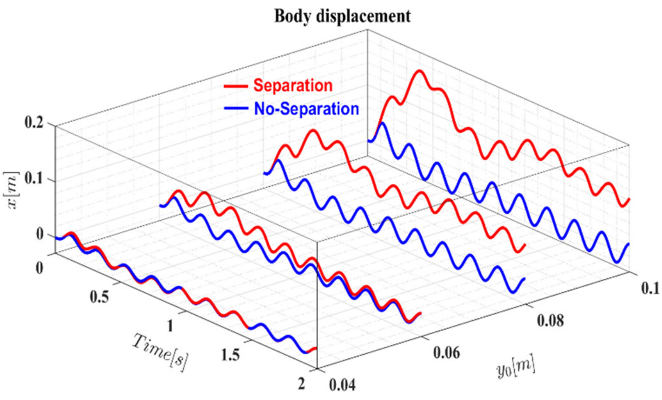

Figure 6. Body displacements with different roads

Safety and stability depend not only on roadway conditions but also on vehicle structure. Therefore, the analysis of vibration dynamics needs to be carried out by changing the position of the vehicle's center. The position of the vehicle's center is represented by the distances b_1, b_2, and b, which can be expressed as a position ratio $p = b_1/b$. Figure 7 depicts the body roll angle across a wide range of center positions. When the vehicle's center is positioned in the middle, $p = 0.5$, the body roll does not appear during the entire investigation interval. In contrast, the roll response increases significantly when the centre moves farther from the midle. In summary, the value of roll angle is proportional to the value of centre postion ratio. Based on this study, consequently, the vehicle should be designed with the mass center positioned near the middle to avoid rollover as well as enhance stability.

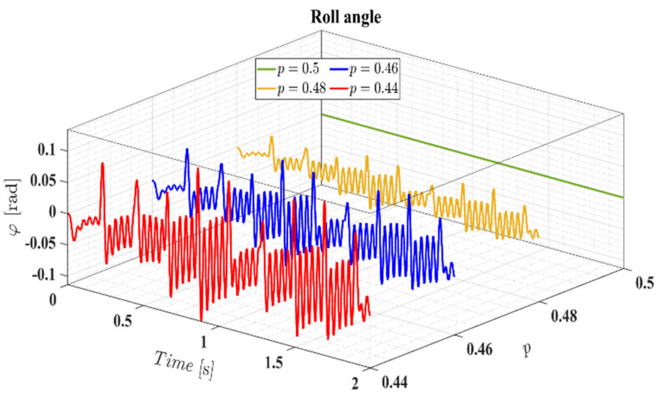

Figure 7. Roll angles with different length ratios

Based on the results in Figures 5 and 7, the vibrating half-car model studied in the paper closely resembles a real-world vehicle because it includes roll motions. Indeed, it can be concluded that the proposed model is superior to a quarter-car model that has only vertical dynamics and does not account for roll dynamics.

4. CONCLUSION

The tire-road separation dynamics were analyzed using a practical half-car model. Initially, the time response was simulated for the in-contact state, and then the results were compared to the no-contact scenario, revealing a difference of more than 10 percent at higher road excitation frequencies. Vertical displacements of the vehicle body were examined across a wide range of road inputs, showing significant fluctuations at high road amplitudes. The roll response was also studied in relation to road excitations and the position of the mass center. The findings indicate that the roll angle increases notably when the mass center is positioned farther from the middle.

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THÔNG TIN TÁC GIẢ

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