A NOVEL NULL-STEERING BEAMFORMER BASED ON BAT ALGORITHM

BỘ ĐỊNH DẠNH BÚP SÓNG ĐIỀU KHIỂN NULL TRÊN GIẢN ĐỒ BỨC XẠ DỰA TRÊN THUẬT TOÁN ĐÀN DƠI

Tong Van Luyen[1,*]

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ABSTRACT

In this study, a null-steering beamformer for interference suppression of half-wave Dipole Uniformly Spaced Linear Array (DULA) has been proposed. The proposed beamformer has utilized BAT algorithm (BA) and the complex weight (both the amplitude and the phase) control of each array element. In order to verify the proposal, a number of scenarios of DULA pattern imposing the pre-set nulls have been carried out considering the mutual coupling among array elements. The proposed beamformer has demonstrated the capability to place with precision single, multiple, and broad nulls at arbitrary interference directions, suppress side lobes, and maintain a predefined beamwidth in the presence of mutual coupling.

Keywords: *Beamformer, interference suppression, mutual coupling.*

TÓM TẮT

Nghiên cứu này đề xuất một bộ định dạng búp sóng nhằm triệt nhiễu cho mảng anten tuyến tính cách đều (DULA) với các phần tử anten dipole nửa bước sóng. Bộ định búp sóng được đề xuất sử dụng thuật toán Dơi (BA) và kỹ thuật điều khiển trọng số phức (bao gồm cả biên độ và pha) của từng phần tử trong mảng anten. Để kiểm chứng đề xuất, một số kịch bản mô phỏng giản đồ bức xạ của DULA với các null được đặt trước đã được thực hiện, có xét đến sự ảnh hưởng của tương hỗ giữa các phần tử trong mảng. Bộ định dạng búp sóng này đã chứng minh khả năng đặt chính xác các null đơn, null đa, và dải null rộng tại các hướng nhiễu tùy ý, kiểm soát mức búp song phụ và duy trì độ rộng búp sóng xác định trước.

Từ khóa: *Bộ định dạng và điều khiển búp sóng, triệt nhiễu, tương hỗ.*

[1]School of Electrical and Electronic Engineering, Hanoi University of Industry, Vietnam

[*]Email: luyentv@haui.edu.vn

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1. INTRODUCTION

Adaptive beamformers are commonly employed in radar, sonar, and communication systems for performance improvement through boosting the efficiency of radio spectrum usage, interference suppression, and energy saving. Beamformers are capable of yielding proper weights for smart array antennas to achieve the required pattern [1]. These smart antennas with null-steering capabilities emerge as a promising solution for interference suppression.

A few nulling methods, namely, the position-only control, the amplitude-only control, the phase-only control, and the complex weight (including both the amplitude and the phase) control have been introduced in numerous studies and implementations [2 - 13]. Each of these methods, however, has its own advantages and shortcomings.

Among those, the complex weight method is the most complicated and expensive, because in this method, a controller, a phase shifter, and an attenuator are needed for each array element. Nevertheless, it has been considered the most flexible and efficient one [9 -13].

Recently, in order to overcome the limitations of the classical optimization techniques, including getting stuck in local minima in some conditions and inflexibility, various nature-inspired optimization algorithms based on ccomputational intelligence approaches have been developed. These algorithms such as genetic algorithm (GA), particle swarm optimization (PSO), firefly algorithm, bee algorithm, Cuckoo search, and BAT algorithm (BA) have been applied and proved to be better and more flexible than the classical techniques. Some of these algorithms have been proposed and implemented with their own benefits and limitations in the array pattern synthesis [2, 18].

BA is a novel evolutionary computation algorithm rooted in the typical behaviour of the bat which uses echolocation for finding prey, dodging obstacles, and identifying their roosts and crevices at night. This method has been applied successfully in order to solve a wide range of engineering problems [18, 19]. It has been proved that BA is better than PSO and GA with regards to convergence, robustness and precision [19]. The first time this algorithm was utilized for adaptive beamforming was in [20]. It was demonstrated in [20] that BA was a potential optimization means for adaptive beamforming regarding computation time. Still, this study was in initial stage, hence, it did not sufficiently analyse the utilization of BA in beamforming. Additionally, in the previous works [5, 8, 13 - 17], BA has been utilized to successfully develop beamformers for interference suppression of ULA, assuming that array elements were isotropic, and without mutual coupling. In fact, when array elements are close enough to each other, they transfer electromagnetic energy from one to another, causing mutual coupling effects. It has been shown in [21, 22] that mutual coupling between array elements plays an important role in an applied adaptive array, because it directly affected the efficiency and the performance of an adaptive array, such as side lobes and directions of nulls. This is very important to null-steering beamforming.

In this study, inspired by the proposal [13], a BA based null-steering beamformer will be developed for DULA to suppress interference in the presence of mutual coupling. The proposed beamformer has been verified in five scenarios including: operation speed, pattern nulling with single, multiple nulls, and broad null. The results show that the beamformer performs competently with regard to steering the nulls to interference directions, suppressing side lobes, and maintaining the main beam.

2. PROBLEM FORMULATION

2.1. Mutual coupling model

In the antenna technique, in order to gain a more exact radiation pattern of array antenna, the term named *active element pattern* of antenna element in the array has been introduced instead of a stand-alone pattern [23]. The authors of [24] and [25] show the effectiveness of applying the active element pattern in beamforming.

In a conventional way, the mutual coupling effect can be modeled by the coupling matrix. This coupling matrix can be defined by the term *active element pattern*, in which active element pattern is the product of a coupling matrix and the stand-alone [26]. So as to model the mutual coupling, an ULA array with *N* elements has been considered. The incident wave is assumed as plane wave. Then, the active element pattern matrix ($P_{sta}(\theta)$) is able to be expressed by stand-alone element pattern matrix ($P_{sta}(\theta)$) as:

$$P_{sta}(\theta) = MP_{sta}(\theta) \quad (1)$$

where: θ is elevation angle and *M* is mutual coupling matrix, respectively.

($P_{sta}(\theta)$) can be presented by:

$$P_{sta}(\theta) = [p_1(\theta)\ p_2(\theta)\ ...\ p_N(\theta)]^T \quad (2)$$

where: $p_i(\theta)$ is the stand-alone element pattern for i^{th} element in the array.

There are various approaches to define and calculate the mutual coupling matrix *M*. In this study, the open-circuit voltage method of [27] has been utilized to calculate mutual coupling and compensation in the context of pattern nulling as below.

The antenna array with N elements can be modeled as an N + 1 terminal network, in which one port is for an outside source and N ports are corresponding to N array elements. Each port of the N array element has been terminated in a load impedance Z_L. The Kirchhoff relations of the N + 1 terminal network are expressed as:

$$\begin{aligned} V_1 &= I_1Z_{11} + ... + I_iZ_{1i} + ... + I_NZ_{1N} + V_{1.oc} \\ &\vdots \\ V_i &= I_1Z_{i1} + ... + I_iZ_{ii} + ... + I_NZ_{iN} + V_{i.oc} \\ &\vdots \\ V_N &= I_1Z_{N1} + ... + I_iZ_{Ni} + ... + I_NZ_{NN} + V_{N.oc} \end{aligned} \quad (3)$$

where: $V_{i,oc}$ is the received voltage at i^{th} port if all of the array elements are in an open circuit condition, Z_{ij} is the mutual impedance between the ports (array elements) i^{th} and j^{th} is defined as $Z_{ij} = V_i / I_j |_{I_k=0, k\neq j}$, and $V_i = -Z_LI_i$ is the terminal voltage of i^{th} port.

Terminal voltages (3) can be rewritten as:

$$\begin{bmatrix} V_{1,oc} \\ V_{2,oc} \\ \vdots \\ V_{N,oc} \end{bmatrix} = \begin{bmatrix} 1+\frac{Z_{11}}{Z_L} & \frac{Z_{12}}{Z_L} & \frac{Z_{1N}}{Z_L} \\ \frac{Z_{21}}{Z_L} & 1+\frac{Z_{22}}{Z_L} & \frac{Z_{2N}}{Z_L} \\ \vdots & & \\ \frac{Z_{N1}}{Z_L} & \frac{Z_{N2}}{Z_L} & 1+\frac{Z_{NN}}{Z_L} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} \quad (4)$$

or

$$V_{oc} = Z_0 V \quad (5)$$

The authors of [26] have shown that: (i) if the received voltage vector V is obtained for different plane-wave incident angles and apart from an unimportant scaling factor, the resulting matrix obtained from stacking column-wise V is considered as the active pattern matrix P_{act} in (1); (ii) if the open-circuit voltage vector V_{oc} is assumed the same as the voltages received in stand-alone conditions, the resulting matrix obtained from stacking V_{oc} is regarded as the stand-alone pattern matrix P_{sta}. Therefore, the mutual coupling matrix can be defined from (1) and (5):

$$M = Z_0^{-1} \quad (6)$$

Based on the mutual coupling matrix in equation (6), mutual coupling compensation in array antennas could be performed efficiently (see [28] for more detail).

2.2. Array Factor of ULA

In our study, the ULA array of 2N half-wave dipole elements has been used and presented in Fig. 1. The array elements are positioned symmetrically around the center of the array, and the array factor can be defined as [29]:

$$AF(\theta) = \sum_{n=-N}^{N} w_n e^{jndk\sin(\theta)} \quad (7)$$

where: $w_n = w_n^{re} + jw_n^{im} = a_n e^{j\delta_n}$ is the complex excitation (weight) of n^{th} array element; $k = \frac{2\pi}{\lambda}$ is the wave number; λ is wave length; d is the distance between adjacent elements. Therefore, the array factor can be expressed by the real (Re{.}) and imaginary (Im{.}) parts as:

$$Re\{AF(\theta)\} = \sum_{n=-N}^{N} w_n^{re}\cos(ndk\sin(\theta)) - w_n^{im}\sin(ndk\sin(\theta)) \quad (8)$$

$$Im\{AF(\theta)\} = \sum_{n=-N}^{N} w_n^{im}\cos(ndk\sin(\theta)) - w_n^{re}\sin(ndk\sin(\theta)) \quad (9)$$

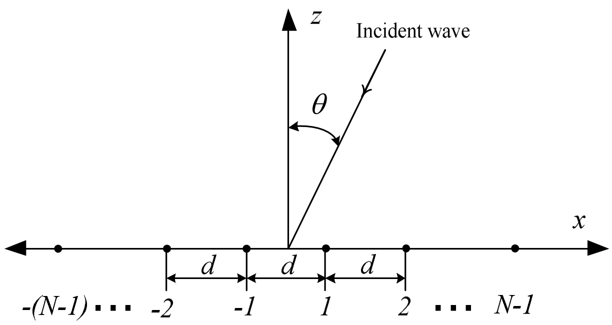

Fig. 1. Geometry of ULA array of 2N elements

2.3. Complex weight control

In our previous proposed null-steering beamformer based on controlling both the amplitude and the phase (complex weight) of each array element excitation [13], $w_{-n}^{im} = -w_n^{im}$ and $w_{-n}^{re} = w_n^{re}$. Therefore, from equations (7-9), the array factor can be rewritten as:

$$AF(\theta) = 2\sum_{n=-N}^{N} w_n^{re}\cos(ndk\sin(\theta)) - w_n^{im}\sin(ndk\sin(\theta)) \quad (10)$$

2.4. Fitness Function

The Fitness function *F* has been built from [8, 11]:

$$F = \begin{cases} N\sum_{i=1}^{I}[|AF_0(\theta_i)|^2], \text{ for } \theta=\theta_i \\ \sum_{\theta=-90^0}^{90^0}[|AF_0(\theta) - AF_d(\theta)|^2], \text{ elsewhere} \end{cases} \quad (11)$$

where: AF_0 and AF_d are the optimized array factor obtained by using an optimization algorithm, which will be BA in this paper, and the reference array factor in the range of theta angles except for nulls, respectively; θ_i are the angles of null points; and N is a parameter, which is experimentally defined during the investigation of the proposal. In this study, N has been chosen as 10000.

3. PROPOSED BEAMFORMER

A BA based beamformer using complex weight control for interference suppression has been proposed from [13] and its flowchart is presented in Fig. 2. Operation of the beamformer is described as follows:

Initializing (I)

Setting the input data, for example: number of array elements (N), Direction of Arrival (DOA) of Interferences; number of iteration (i); maximum number of iterations (Max_I); and the termination criterion (Threshold).

Initializing bat population in which parameters of each bat are: location xi; velocity vi; pulse frequency fi; pulse rate ri; and loudness Ai. Each bat is in correspondence with a potential solution.

Finding the best solution (F)

The beamformer consecutively calculates and searches for the current best solution based on the BA. The operation is completed when the termination criterion or maximum number of iterations is satisfied. After that, the final best solution is obtained.

Building array element weights (B)

From the best solution, the beamformer calculates the corresponding complex weight excited at each element of ULA antenna. These weights will be used for ULA pattern nulling.

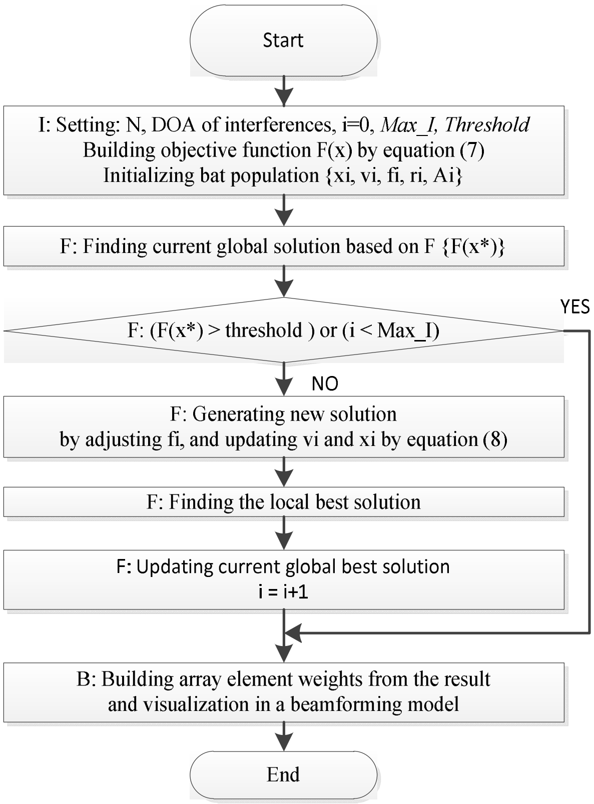

Fig. 2. Flowchart of the proposed beamformer

4. NUMERICAL RESULTS AND DISCUSSION

To evaluate the performance of the proposals for pattern nulling, four scenarios will be considered. It is well-known that the Dolph-Chebyshev array weights distribution produces the optimum pattern in terms of a trade-off between the side lobes level and the first-null beamwidth of main beam for equally spaced arrays [30]. Therefore, in this paper, array factor of Dolph-Chebyshev array, which side lobe level (SLL) is -30dB, inter-element spacing is λ/2, and 8 half-wave dipole elements, has been chosen as a reference one to control SLL and the beamwidth of the main beam. In order to design the 1×8 DULA, first of all, a half-wave dipole antenna working at 2.4GHz has been designed and verified. Its parameters have been shown in Fig. 3. Then, the 1 × 8 DULA has been built from this dipole with inter-element spacing of λ/2. Additionally, MATLAB R2014a and CST STUDIO SUITE 2016 have been utilized for designing the array and simulating in all scenarios.

Initial parameters for all investigation scenarios as: step size of random walk is 0.01; boundary frequency values: $f_{min} = 0$ and $f_{max} = 1$; variable phase of the weights is limited in the range of - 0.1 to 0.1 radian and variable amplitude of weight is in the range of 0 to 1; the population size (pop) is 500; and the number of iterations is 100 except for the first scenario in section 4.1.

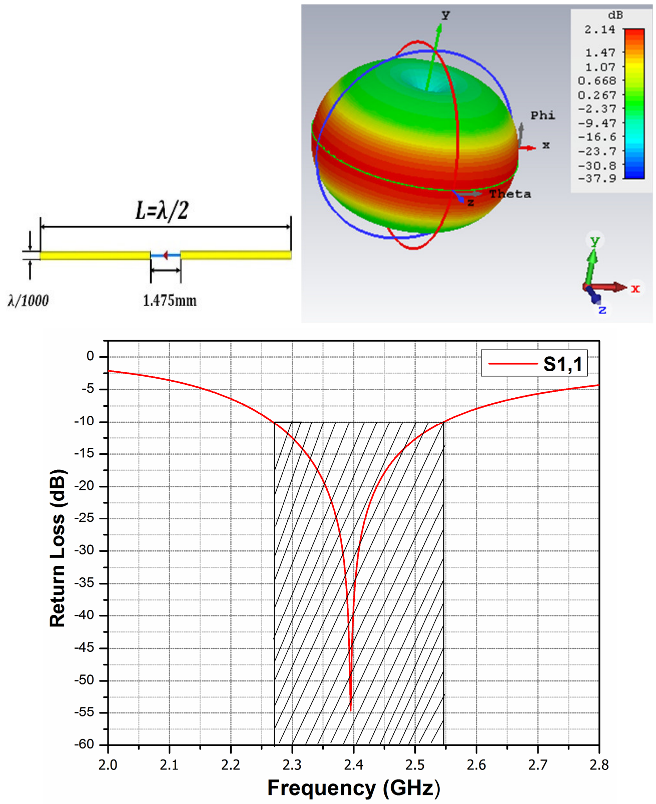

Fig. 3. Single antenna element as half-wave dipole

In order to demonstrate the capability of our proposed beamformer for interference suppression, five scenarios have been constructed. The first scenario named Convergence Characteristics is the initial stage to evaluate operation of the proposal beamformer. This has been achieved by investigating the convergence rate of the fitness function on BA (section 4.1). The second scenario is about compensation of mutual coupling in the case of Dolph-Chebyshev array pattern. The third to fifth scenarios, which are Pattern with Single Null, Pattern with Multiple Nulls, and Pattern with Broad Null, are used for the purpose of investigating the ability of null-steering of the proposal (Section 4.2 - 4.4) in the presence of mutual coupling.

4.1. Convergence Characteristics

In the first scenario, first of all, the convergence ability of our proposed beamformers has been investigated in the case of obtaining the desired optimization pattern as Dolph-Chebyshev array pattern with -30dB SLL. In order to do that, their convergence rates with population of 500 and 200 iterations have been evaluated for side lobe suppression and illustrated in Fig. 4. It is clear that the beamformer converges at high speed with fitness function values less than 0.005 after 100 iterations.

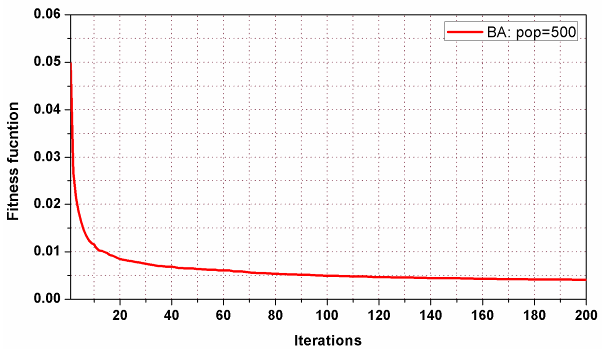

Fig. 4. Fitness function

4.2. Dolph-Chebyshev Array Pattern with Mutual Coupling

In the second scenario, the 1×8 DULA array patterns weighted by Dolph-Chebyshev method have been investigated in three cases: without mutual coupling (*Cheb_ideal*); with mutual coupling (*Cheb_MC*); with compensation of mutual coupling by open-circuit voltage method (*Cheb_OC*). The simulation results have been presented in Fig. 5. It can be seen that mutual coupling makes the Dolph-Chebyshev array pattern distortion in the side lobes regions. Particularly, the peak of the first side lobe has increased to -29.3dB, the peak of the third lobe has decreased to -31.2dB, and the locations of nulls have been slightly changed. Additionally, the results show that the mutual coupling can be compensated well by the open-circuit voltage method.

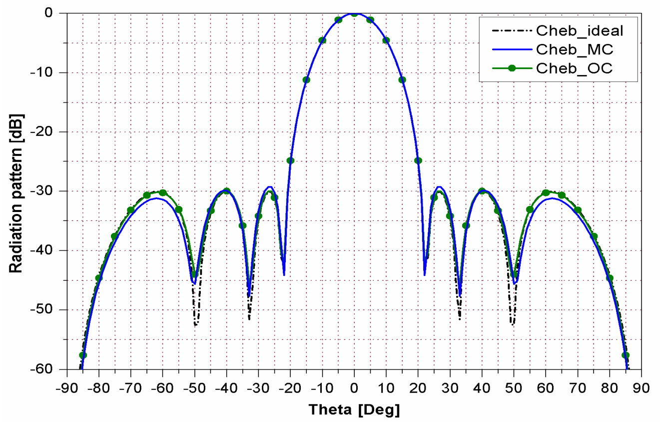

Fig. 5. Dolph-Chebyshev array pattern

4.3. Pattern with Single Null

The third scenario demonstrates the optimized pattern with single null. The setting of this null can be arbitrary at any angle, which is selected at the peak of the second side lobe (40^0) in this situation. The initialization of the population has been done as weights of Dolph-Chebyshev array with -30 dB SLL. Fig. 6 illustrates the optimized patterns with single null achieved by our proposal based on BA in three cases: no mutual coupling (*Single null_ideal*); with mutual coupling (*Single null_MC*); with compensation of mutual coupling by open-circuit voltage method (*Single null_OC*). Although the NDL is not as good as in the ideal case, it is important to highlight that a null at predefined location of 40^0 has been placed successfully in the presence of mutual coupling impact. The NDL has been 6dB lower than that before. Besides, the optimized pattern retains characteristics of the Dolph-Chebyshev pattern like main beam and SLL of approximately -30dB except for the peak of first side lobe of -28.4dB.

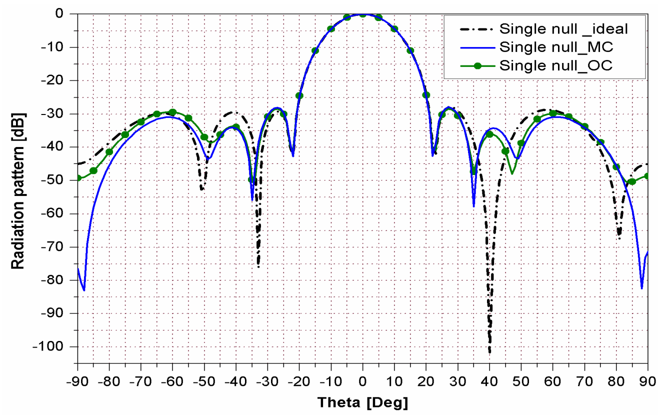

Fig. 6. Optimized patterns with single null at 40^0

4.4. Pattern with Multiple Null

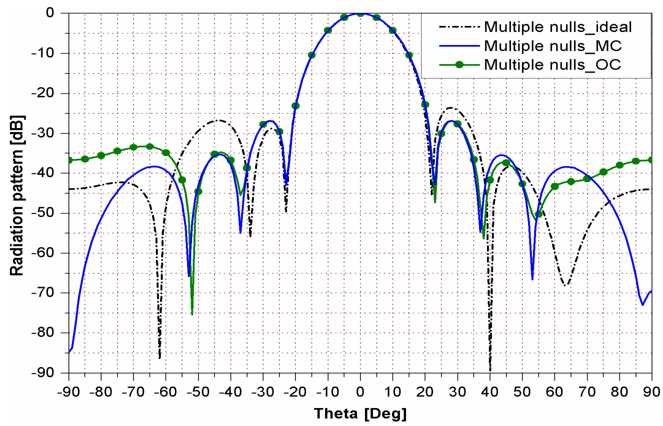

Fig. 7. Optimized pattern with two nulls at 40^0 and 62^0

In the fourth scenario, the proposed beamformer will be used to separately set multiple nulls at 40^0 and 62^0, which corresponds to the peaks of the second and the third side lobes next to the main beam of Dolph-Chebyshev array pattern. As shown in Fig. 7, the patterns with multiple nulls at the predefined locations have been exactly obtained. For pattern with compensation of mutual coupling by open-circuit (*Multiple nulls_OC*), all the NDLs are deeper than -41dB while all the SLLs are lower than -26.8dB and the main beam is approximately equal to that of the Dolph-Chebyshev pattern.

4.5. Pattern with Broad Null

In the application of interference suppression, if the DOA of interferences show minor changes in accordance with time or are not identified with precision, or a null is constantly steered for gaining a proper signal-to-noise ratio, a broad null is required. In the fifth scenario, the patterns with a broad null placed at the target sectors of [40^0, 45^0] has been obtained and illustrated in Fig. 8 as a demonstration of the capability of broad interference suppression. It can be observed that a broad null (minimum NDL of -37dB) on the optimized patterns (*Broad null_OC*) at that target sector has been obtained when the mutual coupling has been compensated by the open-circuit voltage method. The beamwidth stays the same and maximum SLL of -28.7dB.

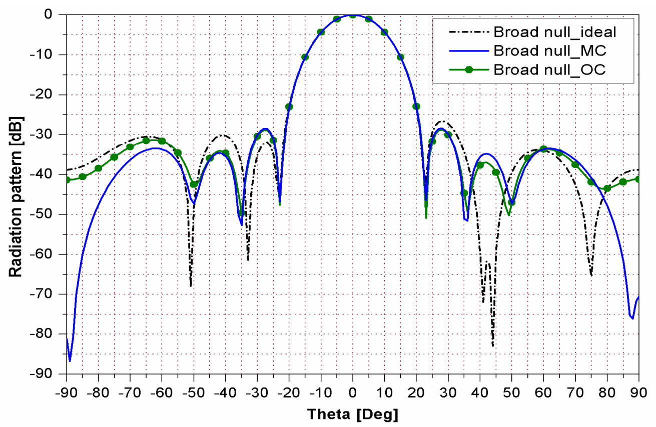

Fig. 8. Optimized pattern with a broad null from 40^0 to 45^0

5. CONCLUSION

In this paper, a BA based null-steeing beamformer for DULA antennas pattern nulling, which has utilized complex weight control method, has been proposed and implemented successfully. The pattern nulling capability of the proposal has been verified by five scenarios consisting of operation speed, Dolph-Chebyshev array pattern with mutual coupling, pattern nulling with single, multiple and broad nulls in ideal cases or in the presence of mutual coupling. Furthermore, to deal with mutual coupling effect, the open-circuit voltage method has been applied for compensation. The simulation results show that the above mentioned nulls can be placed accurately to arbitrary interference directions by making use of our proposed beamformer, while the patterns have maintained the main beam and low SLL in both conditions. Toward realistic electromagnetic effects in relation with antennas and propagation, more effective methods for compensating mutual coupling and real antenna arrays should be investigated in further works.

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THÔNG TIN TÁC GIẢ

Tống Văn Luyên

Trường Điện - Điện tử, Trường Đại học Công nghiệp Hà Nội