INFLUENCE OF MACHINING ERROR OF WORKPIECE POSITIONING SURFACES ON TOOTH PITCH ERRORS OF GLEASON SPIRAL BEVEL GEARS

NGHIÊN CỨU ẢNH HƯỞNG CỦA SAI SỐ CHẾ TẠO CÁC BỀ MẶT ĐỊNH VỊ CỦA PHÔI ĐẾN SAI SỐ BƯỚC RĂNG CỦA BÁNH RĂNG CÔN XOẮN HỆ GLEASON

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ABSTRACT

During the machining process of Gleason spiral bevel gears, errors of workpiece positioning surfaces cause positioning errors, which is one of the main reasons for tooth pitch errors. In this paper, the 3D models of Gleason spiral bevel gears was created according to the machining principle in Autodesk Inventor software and they were used to simulate the influence of positioning errors on the indicators for evaluating tooth pitch errors, including the maximum single pitch deviation, the maximum difference between adjacent pitches, and the total cumulative pitch deviation. The research results show that all these indicators depend linearly on the tilt angle and the distance between the workpiece axis and the z-axis of the machine-tool. Mathematical equations for determining the value of these indicators based on the tilt angle and distance between axes are also developed, and the manufacturing accuracy level of the workpiece positioning surfaces was determined so that the values of these indicators would not exceed 1/3 of the tolerance of DIN 6 accuracy level (according to Standard DIN 3965). The research results are significant for ensuring accuracy when machining Gleason spiral bevel gears.

Keywords: *Spiral bevel gears, Tooth pitch errors, Manufacturing accuracy level.*

TÓM TẮT

Trong quá trình gia công bánh răng côn xoắn hệ Gleason, sai số chế tạo các bề mặt định vị của phôi gây ra sai số gá đặt, đây là một trong các nguyên nhân chính tạo ra sai số bước răng. Trong bài báo này, một mô hình 3D của cặp bánh răng côn xoắn hệ Gleason được xây dựng theo nguyên lý gia công bằng phần mềm Autodesk Inventor và được sử dụng để mô phỏng ảnh hưởng của sai số gá đặt đến các chỉ tiêu đánh giá sai số bước răng bao gồm: sai số bước răng đơn lớn nhất, sai lệch hai bước răng liền kề lớn nhất và sai số bước răng tổng lớn nhất. Kết quả nghiên cứu cho thấy, tất cả các chỉ tiêu trên đều phụ thuộc tuyến tính vào góc nghiêng và khoảng cách giữa trục phôi và trục z của máy gia công. Từ đó đã tìm ra được các phương trình toán học để tính toán giá trị của các chỉ tiêu trên theo góc nghiêng và khoảng cách giữa trục phôi và trục z của máy gia công và đặt ra yêu cầu về cấp chính xác chế tạo của các bề mặt định vị của phôi sao cho giá trị của các chỉ tiêu trên không vượt quá 1/3 dung sai tương ứng với cấp chính xác DIN 6 theo tiêu chuẩn DIN 3965. Kết quả nghiên cứu có ý nghĩa quan trọng trong việc đảm bảo độ chính xác khi gia công bánh răng côn xoắn hệ Gleason.

Từ khóa: *Bánh răng côn xoắn, sai số bước răng, cấp chính xác chế tạo.*

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1. INTRODUCTION

When transmitting motion and moment between two non-parallel shafts, the spiral bevel gear transmission is generally preferred because of its advantages, including large power transmission capacity, stable ratio, significant applicable coefficient, and smooth transmission [1-3]. In industry, there are two main cutting methods of machining spiral bevel gears: face milling and face hobbing [4-7].

In terms of shape, face-milled bevel gears have circular and lengthwise tooth curves (also called Gleason spiral bevel gears), while face-hobbed gears have extended epicycloid lengthwise tooth curves. This difference is due to the cutting kinematics of the two methods. In the face milling method, only one slot is machined at a time until the total depth is cut completely. Subsequently, the workpiece is rotated to the position for machining the next slot, and the process will be repeated until all tooth spaces are finished (Figure 1). In the face hobbing method, all the tooth spaces are cut simultaneously. Therefore, the face milling method is called the single indexing method, and the face hobbing method is called the continuous indexing method.

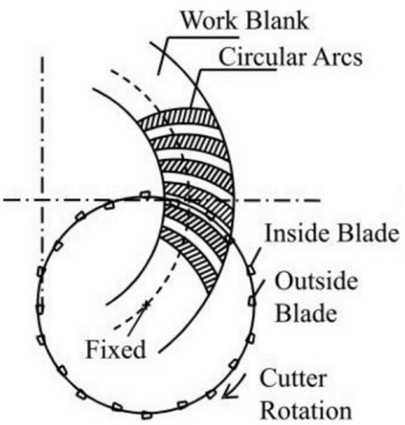

Figure 1. Kinematic of face milling method [8].

Nowadays, scientists are very interested in reducing the noise and vibration of spiral bevel gear transmission. One of the essential reasons for noise and vibration of spiral bevel gear transmission is tooth pitch errors. To reduce tooth pitch error, it is first necessary to find out the reasons of such errors. Machining errors are decomposed into constant systematic, variable systematic, and random errors. The reasons for constant systematic errors can be listed as follows: errors of manufacturing machine, geometrical errors of cutting tools, errors of cutting method, and elastic deformation of the process system (machine, tools, fixtures, workpiece). The variable system errors appear in large-series production due to wear of the cutting tool and thermal deformation during the cutting process. Random errors are caused by several reasons, such as uneven workpiece hardness, uneven machining allowance, cutting tool removal, installation, positioning error.

In this paper, errors due to machines, cutting tools, and cutting methods are not studied, because they cannot be changed. The variable systematic errors are also not analyzed because this paper does not study large-series production. This paper also does not analyze errors due to uneven workpiece hardness and uneven machining allowance because of their difficulty in control. Thus, there are two reasons of machining errors that need attention to reduce: elastic deformation of the process system and positioning errors. The elastic deformation of the process system depends on the system stiffness and the cutting force. The positioning errors are affected by the geometrical error of the fixture and the manufacturing error of the workpiece positioning surfaces. If the fixture is manufactured with high accuracy, its geometrical errors will be so small that they can be ignored. In that case, the positioning errors mainly depend on the manufacturing error of the workpiece positioning surfaces. This paper studies the influence of the machining error of workpiece positioning surfaces on tooth pitch errors of face-milled spiral bevel gears by measuring and calculating the tooth pitch errors on a 3D-CAD model, that was created with positioning error simulation.

In the tooth cutting process, plane B and cylindrical surface C are used to position the workpiece for machining face-milled spiral bevel gear (Figure 2). The plane B restricts three degrees of freedom (translating motion along the z-axis rotary motion along the x and y axes), and the cylindrical surface C restricts two more degrees of freedom (translating motion along the x and y axes). In the ideal case, plane B coincides with plane Oxy, and the axis of cylindrical surface C coincides with the z-axis of the machine. In reality, plane B is not entirely flat, cylindrical surface C is not uniformly round, and the axis of cylindrical surface C is not entirely perpendicular to plane B. Therefore, positioning error can appear. Due to the positioning error, the axis of the cylindrical surface C will not coincide with the z-axis of the machine, as shown in Figure 3. In this case, the general relationship of the axis of the cylindrical surface C is described by equation (1):

$$y = z.tg\theta + \Delta \quad (1)$$

where Δ is the distance between the workpiece axis and the z-axis of the machine-tool, and θ is the tilt angle.

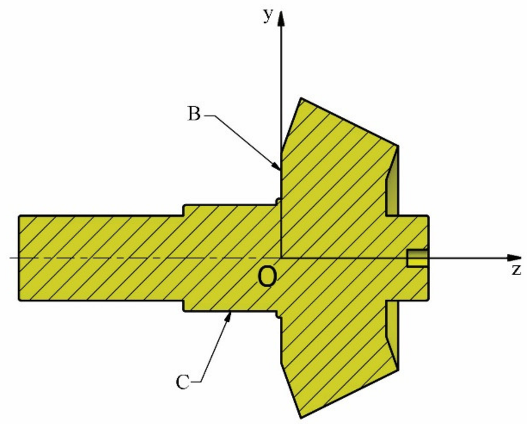

Figure 2. Positioning the workpiece

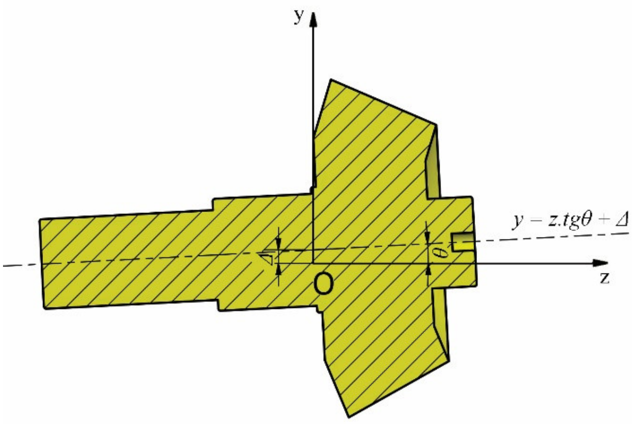

Figure 3. Positioning error

In this paper, the influence of the parameters θ and Δ on the indicators for evaluating tooth pitch errors, including f_p max - maximum single pitch deviation, f_u max - maximum difference between adjacent pitches, F_p - total cumulative pitch deviation (Figure 4), was studied to determine the maximum values of the parameters θ and Δ so that the values of the indicators do not exceed 1/3 tolerance of DIN 6 accuracy level according to Standard DIN 3965. From there, the machining accuracy level of the workpiece positioning surfaces was requested.

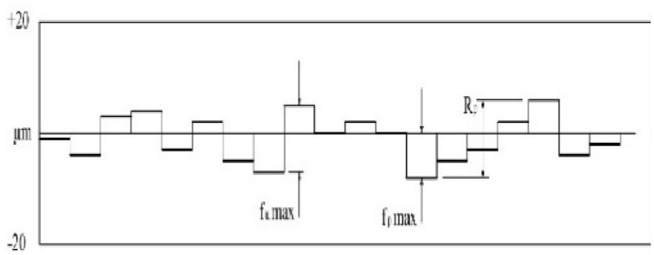

Figure 4. Indicators for evaluating the tooth pitch errors

2. RESEARCH OBJECT AND METHOD

2.1. Research object

The research object in this paper is a pair of face-milled spiral bevel gears designed according to standard ISO 23509:2006 with parameters as shown in Table 1.

Table 1. Main geometrical parameters

Parameters	Symbol	Unit	Value	
			Pinion	Gear
Number of teeth	Z		14	39
Mean normal module	m_{mn}	mm	3.213	
Angle of shaft axes	Σ	degree	90	
Hypoid offset	a	mm	0	
Face width	b	mm	25.4	
Outer pitch cone distance	R_e	mm	93.973	
Mean normal pressure angle	α	degree	20	20
Mean spiral angle	β_m	degree	35	35
Reference cone angle	δ	degree	19.747	70.253
Mean pitch diameter	d_m	mm	54.918	152.987
Outer pitch diameter	d_e	mm	63.500	176.893
Outside tip circle diameter	d_{ae}	mm	75.324	178.288
Spiral direction			Right	Left

With the mean regular module in the range of 2 - 3.55mm and the outer pitch diameter of the pinion in the range of 50 - 125mm, it is determined that the tolerance of DIN 6 accuracy level for the indicators for evaluating tooth pitch errors of the pinion is as shown in Table 2.

Table 2. Tolerance for the indicators for evaluating tooth pitch errors

Indicators	f_p max	f_u max	F_p
Tolerance (μm)	11	14	38
1/3 Tolerance (μm)	3.67	4.67	12.67

2.2. Research method

A 3D spiral bevel gear pair model was created in Autodesk Inventor software, including three steps exactly like the face-milled method (Figure 5). The drawing of the final pinion and gear is shown in Figure 6.

To create positioning errors, when constructing the sketch in step 1, the position of the rotation axis was changed to not coincide with the coordinate axis (with the tilt angle θ and the distance Δ). By measuring the length of chords on the mean pitch circle, as shown in Figure 7, the tooth pitches were calculated, and then the indicators f_p max, f_u max, and F_p were determined. Changing the values of parameters θ and Δ will clarify the influence of positioning error on tooth pitch errors.

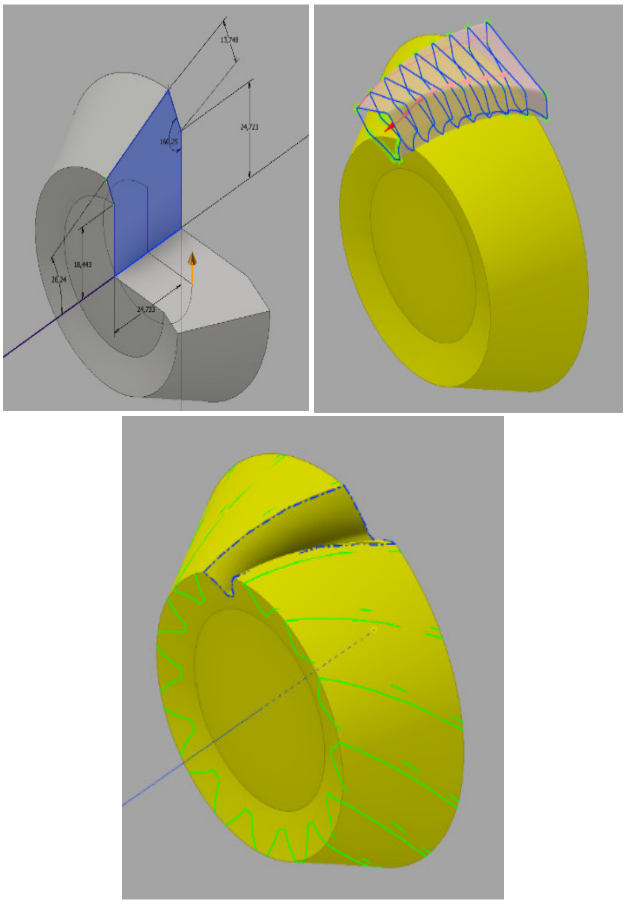

Figure 5. Steps to create a 3D model

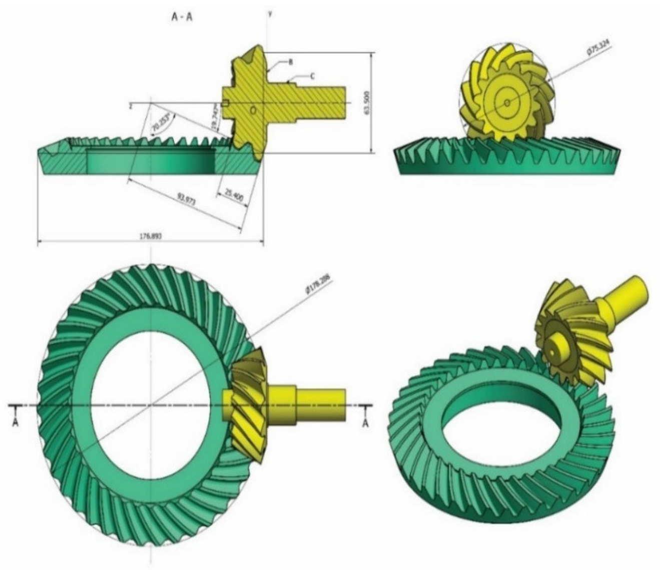

Figure 6. Drawing of the final pinion and gear

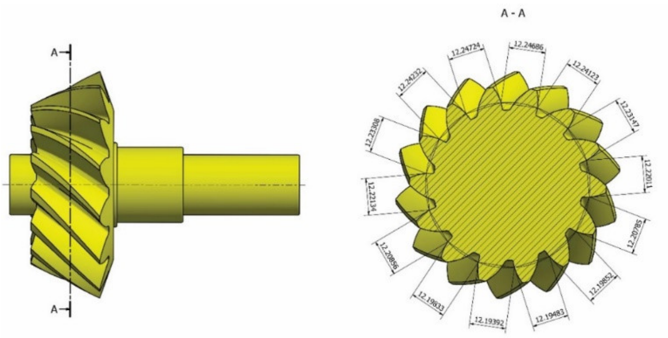

Figure 7. Measuring the length of chords on the mean pitch circle

3. RESULTS AND DISCUSSION

3.1. Influence of tilt angle on tooth pitch error

The main reason for the tilt angle θ is the non-perpendicularity between plane B and the axis of cylindrical surface C (Figure 2). Look up the perpendicularity tolerance table to determine the perpendicularity tolerance for each manufacturing accuracy level, then calculate the tilt angle θ as shown in Table 3.

Table 3. Tilt angle θ according to manufacturing accuracy level

Manufacturing accuracy level	**IT4**	**IT5**	**IT6**	**IT7**
Perpendicularity tolerance (µm)	2.5	4	6	10
Tilt angle *θ* (degree)	0.006	0.009	0.014	0.023

When fixing Δ = 0 and changing θ according to the values in Table 3, the results are determined as shown in Table 4. Based on the results in Table 4, the influence graphs of the tilt angle of workpiece axis to the z-axis of the machine on the indicators for evaluating tooth pitch errors were built, as shown in Figure 8.

Table 4. Result of determination of the influence of the tilt angle on the indicators for evaluating tooth pitch errors

θ (degree)	0	0.006	0.009	0.014	0.023
f_p max (µm)	0	0.880	1.320	2.052	3.369
f_u max (µm)	0	0.392	0.587	0.914	1.501
F_p (µm)	0	3.954	5.928	9.219	15.135

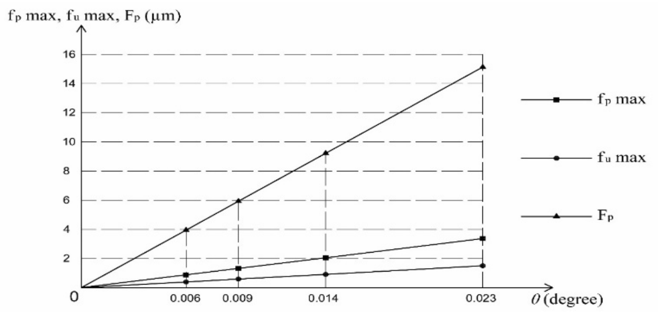

Figure 8. The influence graphs of the tilt angle on the indicators for evaluating tooth pitch errors

As shown in Figure 8, it can be seen that the influence graphs of the tilt angle of workpiece axis to the z-axis of the machine on the indicators for evaluating tooth pitch errors are all straight lines. It proves that all indicators for evaluating tooth pitch error depend linearly on the tilt angle. The mathematical equations of this dependence are determined as follows:

f_p max = 146.6*θ;

f_u max = 65.28*θ;

F_p = 658.47*θ.

In which the indicators f_p max, f_u max, and F_p are measured in micrometers, *θ* is measured in degrees.

Using these equations, the value of θ can be determined so that the values of indicators do not exceed 1/3 of the tolerance of accuracy level DIN 6, as shown in Table 2. For f_p max ≤ 3.67μm, θ ≤ 0.025 is needed. For f_u max ≤ 4.67μm, θ ≤ 0.071 is needed. For F_p ≤ 12.67μm, θ ≤ 0.019 is required.

Thus, for all values of the indicators for evaluating tooth pitch errors not to exceed 1/3 of the tolerance of DIN 6 accuracy level according to Standard DIN 3965, the tilt angle of the workpiece axis to the z-axis of the machine must not exceed 0.019 (equivalent to accuracy level IT7).

3.2. Influence of distance on tooth pitch error

The distance between the workpiece axis and the z-axis of the machine depends mainly on the roundness of the cylindrical surface C. Given that the distance between the axes is half the roundness tolerance of the cylindrical surface C, determine the value of the distance between the axes according to the manufacturing accuracy level, as shown in Table 5.

Table 5. Distance between the axes Δ according to manufacturing accuracy level

Manufacturing accuracy level	IT4	IT5	IT6	IT7
Roundness tolerance (μm)	2.5	4	6	10
Distance between the axes *Δ* (μm)	1.25	2	3	5

Table 6. Result of determination of the influence of the distance between the axes on the indicators for evaluating tooth pitch errors

Δ (μm)	0	1.25	2	3	5
f_p max (μm)	0	0.817	1.308	1.962	3.270
f_u max (μm)	0	0.364	0.583	0.873	1.456
F_p (μm)	0	3.674	5.878	8.817	14.695

When fixing θ = 0 and changing Δ according to the values in Table 5, the results are determined as shown in Table 6. Based on the results in Table 6, the influence graphs of the distance between the workpiece axis and the z-axis of the machine on the indicators for evaluating tooth pitch errors were built, as shown in Figure 9.

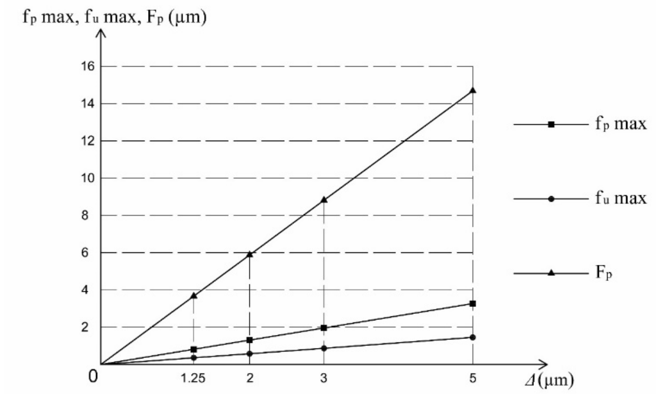

Figure 9. The influence graphs of the distance between the axes on the indicators for evaluating tooth pitch errors

Similar to the dependence of the indicators for evaluating tooth pitch error on tilt angle, the indicators also depend linearly on the distance between axes. The mathematical equations of this dependence are determined as follows:

f_p max = 0.654*Δ;

f_u max = 0.291*Δ;

F_p = 2.939*Δ.

Using these equations, the value of b can be determined so that the values of indicators do not exceed 1/3 of the tolerance of DIN 6 accuracy level, as shown in Table 2. For f_p max ≤ 3.67μm, Δ ≤ 5.61μm is needed. For f_u max ≤ 4.67μm, Δ ≤ 16.05μm is necessary. For F_p ≤ 12.67μm, Δ ≤ 4.31μm is required.

Thus, for all values of the indicators for evaluating tooth pitch errors not to exceed 1/3 of the tolerance of DIN 6 accuracy level according to Standard DIN 3965, the distance between the workpiece axis and the z-axis of the machine must not exceed 4.31μm (equivalent to accuracy level IT7).

3.3. The combined influence of tilt angle and distance between axes on tooth pitch error

The results of determining the combined influence of the tilt angle and the distance between axes on the indicators for evaluating tooth pitch error are shown in Table 7. Results of comparing the values of indicators for evaluating tooth pitch errors due to the combined influence of the tilt angle and the distance between the axes with the corresponding values of these indicators due to each individual influence as shown in Tables 8, 9, 10.

Table 7. Result of determination of the combined influence of tilt angle and distance between axes on indicators for evaluating tooth pitch errors

Manufacturing accuracy level	Tilt angle θ (degree)	Distance between the axes Δ (μm)	f_p max (μm)	f_u max (μm)	F_p (μm)
IT4	0.006	1.25	1.575	0.701	7.075
IT5	0.009	2	2.435	1.083	10.937
IT6	0.014	3	3.722	1.655	16.715
IT7	0.023	5	6.156	2.737	27.640

Table 8. Result of comparing the values of the maximum single pitch deviation

Manufacturing accuracy level	f_p max (μm)				Percentage of the sum
	Individual influences			Combined influence	
	Tilt angle	Distance between the axes	Sum		
IT4	0.880	0.817	1.697	1.575	92.8%
IT5	1.320	1.308	2.628	2.435	92.7%
IT6	2.052	1.962	4.014	3.722	92.7%
IT7	3.369	3.270	6.639	6.156	92.7%

Table 9. Result of comparing the values of the maximum difference between adjacent pitches

Manufacturing accuracy level	f_u max (μm)				Percentage of the sum
	Individual influences			Combined influence	
	Tilt angle	Distance between the axes	Sum		
IT4	0.392	0.364	0.756	0.701	92.7%
IT5	0.587	0.583	1.170	1.083	92.6%
IT6	0.914	0.873	1.787	1.655	92.6%
IT7	1.501	1.456	2.957	2.737	92.6%

Table 10. Result of comparing the values of the total cumulative pitch deviation

Manufacturing accuracy level	F_p (μm)				Percentage of the sum
	Individual influences			Combined influence	
	Tilt angle	Distance between the axes	Sum		
IT4	3.954	3.674	7.628	7.075	92.8%
IT5	5.928	5.878	11.806	10.937	92.6%
IT6	9.219	8.817	18.036	16.715	92.7%
IT7	15.135	14.695	29.830	27.640	92.7%

From the above results, it can be seen that the value of the indicators for evaluating tooth pitch errors due to the combined influence of the tilt angle and the distance between the axes is about 92.7% of the sum of the corresponding values of these indicators due to each influence.

When the manufacturing accuracy is IT6, the maximum single pitch deviation (f_p max) and total cumulative pitch deviation (F_p) exceed 1/3 tolerance of the DIN 6 accuracy level.

Thus, for all values of the indicators for evaluating tooth pitch errors not to exceed 1/3 of the tolerance of DIN 6 accuracy level according to Standard DIN 3965, the manufacturing accuracy level of perpendicularity of the positioning plane to the axis of the positioning cylindrical surface and roundness of the positioning cylindrical surface is at least IT5.

4. CONCLUSION

This paper focuses on investigating the influence of machining error of workpiece positioning surfaces on tooth pitch errors of gleason spiral bevel gears. The 3D spiral bevel gear pair model was created to simulate different scenarios. The research results of the paper show that:

1. When machining face-milled spiral bevel gears, the machining error of workpiece positioning surfaces (including a plane and a cylindrical surface) causes the tilt angle and distance between the workpiece axis and the z-axis of the cutting machine. Both the tilt angle and the distance between the axes affect the indicators for evaluating tooth pitch errors according to linear functions. The value of the indicators due to the combined influence is about 92.7% of the sum of the corresponding values of these indicators due to each individual influence.

2. For the requirement that the error caused by positioning is 1/3 of the manufacturing error and the indicators for evaluating tooth pitch errors reach DIN 6 accuracy level (according to Standard DIN 3965), the manufacturing accuracy levels of the workpiece positioning surfaces, including accuracy level of perpendicularity of the positioning plane to the axis of the positioning cylindrical surface and accuracy level of roundness of the positioning cylindrical surface are at least IT5.

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THÔNG TIN TÁC GIẢ

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Trung tâm Công nghệ, Trường Đại học kỹ thuật Lê Quý Đôn