MODEL OF INTERIOR BALLISTIC OF MORTAR BASED ON THERMODYNAMIC THEORY

MÔ HÌNH THUẬT PHÓNG TRONG CỦA PHÁO CỐI THEO LÝ THUYẾT NHIỆT ĐỘNG LỰC HỌC

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ABSTRACT

This paper presents a new approach to establishing the interior ballistic algorithm for mortar systems based on thermodynamic theory. Contrary to previous studies, the processes that occurred in the ignition charges and the gas mixture properties in the barrel were considered in this study. The numerical integration method has been used to solve the problem with high accuracy. Numerical calculation applied on 100mm mortar. The calculation result is the law of pressure in the combustion chamber and the law of movement of the projectile in the barrel. These results are compared with the manufacturer's data to evaluate the reliability of the mathematical model as well as the solution method. According to the comparison between the simulation and the manufacturer's data, the maximum errors of velocity of 3.27%. The results obtained from this study are an important scientific basis for improving and optimizing the design of mortar systems and weapons with similar structures.

Keywords: *Interior ballistic, mortar, thermodynamic.*

TÓM TẮT

Bài báo trình bày một phương pháp tiếp cận mới để thiết lập hệ phương trình thuật phóng trong của pháo cối theo lý thuyết nhiệt động lực học. Khác với các nghiên cứu trước đấy, quá trình xảy ra ở liều chính, liều phụ và tính chất của hỗn hợp khí trong nòng pháo cối đều được xem xét đến trong nghiên cứu này. Phương pháp tích phân số đã được sử dụng để giải bài toán thuật phóng trong của pháo cối với độ chính xác cao. Tính toán số được áp dụng trên pháo cối 100mm. Kết quả tính toán là quy luật của áp suất trong buồng đốt và quy luật chuyển động của đạn trong nòng theo thời gian. Các kết quả này được so sánh với dữ liệu công bố của nhà sản xuất để đánh giá độ tin cậy của mô hình toán học cũng như phương pháp tính toán. Từ kết quả so sánh có thể thấy rằng: sai số lớn nhất của vận tốc viên đạn khi rời nòng là 3,27%. Kết quả thu được từ nghiên cứu này là cơ sở khoa học tin cậy để cải tiến và tối ưu hóa kết cấu của hệ thống pháo cối và các loại vũ khí có cấu trúc tương tự.

Từ khóa: *Thuật phóng trong, pháo cối, nhiệt động lực học.*

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1. INTRODUCTION

Mortars are weapons with a long history of development. It has a simple structure and is often used to destroy targets obscured by its ability to fire at large angles of fire, giving the projectile a rainbow trajectory, see Figure 1. The interior ballistic algorithm is a fundamental problem when calculating weapon design and there are many different approaches to solving this problem. The theoretical approach to interior ballistics of conventional artillery guns [1, 2] still has certain limitations. Previous studies often considered the phenomena occurring in the barrel to include two fundamental thermodynamic processes: the process of expanding the combustion gas in the barrel with high temperature and pressure, the process generates work to push the projectile in motion; the process of injecting combustible gas through the gap between the projectile and the barrel into the environment [3, 4]. Here, the phenomena occurring in the ignition charges are ignored, considering the combustion of the ignition charges to be instantaneous. The ignition charges involved in the firing process are very complicated. First of all, the ignition charges are responsible for igniting the propellant, ensuring a reliable and stable propellant throughout the combustion process. In conventional artillery projectiles, the weight of the ignition charges accounts for a small percentage, so it can be ignored. However, it is so significant compared to the mortar's propellant weight that the effect of the ignition charges on the movement of the projectile in the barrel cannot be ignored. In other words, the ignition charges should be considered an important part of the total charges of the propellant.

When calculating the interior ballistics characteristics of the firing phenomenon, it should be noted that the combustible gas in the barrel as well as the combustible gas ejected from the gap between the projectile and the barrel is the sum of the mixture of the combustible gas of both the ignition and propelling charges. Including the influence of the ignition charges, the temperature and combustion gas pressure in the barrel will change, which is the cause of errors in the process of calculating the actual launch of mortars in previous studies [5, 6].

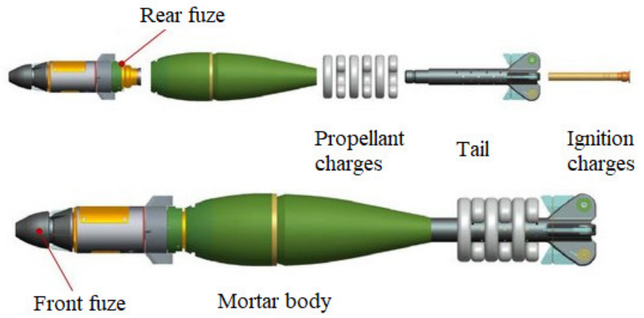

Figure 1. General structure of mortars

In this paper, the first principle of the thermodynamic system combined with thermodynamic theory and additional equations has been used to study the mortar's interior ballistics algorithm fully and completely close to the actual shooting. The firing phenomena of the ignition and the propellant are clearly described, and the complex properties of the gas mixture are fully considered. The results obtained from this study are the basis for the calculation and optimal design of mortar systems and artillery systems with similar structures.

2. ESTABLISHING A MATHEMATICAL MODEL

In the process of building a mathematical model of the interior ballistics problem for mortar systems, the description of the main working stages of mortar systems should be done carefully, this ensures that the model is established closely with the most realistic process. The processes that occur when firing mortars are divided into four basic stages as follows [2]:

Stage 1: The burning period of the ignition charges inside the tube before the fire holes are opened. The characteristics of this period are only the ignited ignition charges, the combustible gas is produced in a constant volume, the propellant charges are not ignited, and the projectile has not moved.

Stage 2: Starting from the end of the first period until the propellant burns out. The combustible gas is generated by the combustion of the ignition and the propellant, this process increases the pressure inside the barrel to the maximum pressure value pm and then decreases, the speed of the projectile increases gradually and the burning gas continuously erupts through the gap between the projectile and the barrel.

Stage 3: Starting from when the propellant burns out to the time when the centering belt of the projectile exits the barrel. The process of gas expansion can be considered adiabatic.

Stage 4: This is the final effect stage of the combustible gas after the centering belt of the projectile leaves the barrel. For mortar systems, the final effect of the combustible gas occurs in a very short time, its influence on the change in speed of the projectile is very small. Therefore, setting up the interior ballistics problem in mortars often skips this stage.

To establish the interior ballistics problem of the mortar system, some basic assumptions are used as follows [3-7]:

- The propellant is considered to burn according to the laws of geometry;
- The fire law of propellant is linear;
- Combustible gas pressure in the barrel is the average pressure;
- The shape characteristics of the propellant are unchanged;
- All extra work is included in the projectile's aggravation factor φ;
- Combustible gas ejected through the gap between the projectile and the barrel is a stable unidirectional flow.

The mortar system consists of 2 combustion chambers, the first chamber is the tube containing the ignition charges, the chamber has an initial volume of W_{01}, the second chamber is the cavity containing the propellant charges, this chamber has an initial volume of W_{02}, see Figure 2. In this study, the index variable "1" corresponds to the first chamber, the index variable "2" corresponds to the second chamber, and the index variable "s" corresponds to the gas mixture formed from the ignition charges and propelling charges.

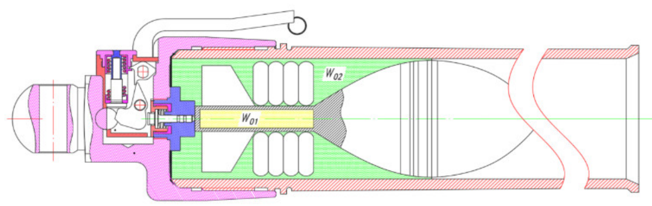

Figure 2. Structure diagram of mortar system

2.1. Description of phenomena in the tube containing the ignition charges

In a tube containing a charge of ignition, the initial volume is constant W_{01}, and the initial mass of the propellant is ω_1. Phenomena occurring in this tube are described by the following equations [8-11]:

a) Equations of combustion and gas generation:

$$\frac{dz_1}{dt}=\begin{cases}\dfrac{p_1}{I_{k1}} & \text{when } 0<e\le e_{11}\\ 0 & \text{when } e>e_{11}\end{cases} \tag{1}$$

$$\frac{d\psi_1}{dt}=\chi_1\left(1+2\lambda_1.z_1+3\mu_1.z_1^2\right)\frac{dz_1}{dt} \tag{2}$$

where: p_1 is the combustion gas pressure in the tube; ψ_1 is the relative burned mass of the propellant; I_{k1} is the total impulse of the combustion gas pressure in the tube; z_1 is the relative burned thickness of the propellant; e_{11} is the thickness of the propellant in the ignition charges; e is the burned thickness of the propellant at the time of calculation; χ_1, λ_1, μ_1 are the shape factor of the propellant of the ignition charges.

b) The equation describes the law of combustion gas pressure in the tube containing the ignition charges

The equation describing the law of combustion gas pressure in the tube containing the ignition charges is determined based on the basic state equation of the gas.

$$p_1.V_1=m_1.R_1.T_1\rightarrow p_1=\frac{m_1.R_1.T_1}{V_1} \tag{3}$$

where: T_1 is the temperature of combustion gas in the tube; R_1 is gas constant; m_1 is the mass of gas burned in the tube; V_1 is the free volume of gas in the tube; W_{01} is the volume of the initial combustion space; $\frac{\omega_1}{\delta_1}.(1-\psi_1)$ the volume of unburnt propellant; $m_1 \cdot \alpha_1$ is the volume of combustion gas [1], α_1 is the co-volume coefficient of combustion gases. The free volume of combustion gas in the tube is determined by the following expression:

$$V_1=W_{01}-\frac{\omega_1}{\delta_1}.(1-\psi_1)-m_1.\alpha_1 \tag{4}$$

Because the combustion gas is transmitted through the barrel in a certain amount $m_{out,1}$, Therefore, the mass of combustion gas in the tube containing the ignition charges is changed and is determined by the following formula:

$$m_1=m_{in,1}-m_{out,1}=\psi_1\omega_1-m_{out,1} \tag{5}$$

c) The equation describes the change in the temperature of the combustible gas in the tube containing the ignition charges

From the equation of the first law of thermodynamics for the case of an open thermodynamic system and ignoring heat transfer (dQ = 0), the equation describing the law of changing the temperature of combustion gas in the bore is determined as follows, see [8, 9]:

$$\frac{dH_1}{dt}=\frac{dU_1}{dt}+p_1.\frac{dV_1}{dt} \tag{6}$$

Since no work is done in the tube containing the ignition charges, that is $p_1.\frac{dV_1}{dt}=0$, then equation (6) is rewritten as:

$$\frac{dH_1}{dt}=\frac{dH_{in,1}}{dt}-\frac{dH_{out,1}}{dt}=\frac{dU_1}{dt} \tag{7}$$

Enthalpy is determined using the following expression [9]:

$$\begin{cases}\dfrac{dH_{in,1}}{dt}=c_{p1}.T_{v1}.\dfrac{dm_{in,1}}{dt}=c_{p1}.T_{v1}.\omega_1.\dfrac{d\psi_1}{dt}\\ \dfrac{dH_{out,1}}{dt}=c_{p1}.T_1.\dfrac{dm_{out,1}}{dt}=c_{p1}.T_1.\dot{m}_{out,1}\end{cases} \tag{8}$$

The change in internal energy of combustion gas is determined according to the following formula [9]:

$$\frac{dU_1}{dt}=c_{v1}.\left[T_1.\left(\omega_1.\frac{d\psi_1}{dt}-\dot{m}_{out,1}\right)+m_1\frac{dT_1}{dt}\right] \tag{9}$$

The equations (8), (9) are substituted into equation (7), after the transformation we get:

$$\frac{dT_1}{dt}=\frac{1}{m_1}.\left[\left(\frac{c_{p1}}{c_{v1}}.T_{v1}-T_1\right).\omega_1.\frac{d\psi_1}{dt}-\left(\frac{c_{p1}}{c_{v1}}-1\right)\dot{m}_{out,1}.T_1\right] \tag{10}$$

where: c_{p1} is the isobaric specific heat of a combustion gas; c_{v1} is the isovolumetric specific heat of the combustion gas, T_{v1} is the combustion temperature of the propellant in the ignition charges; T_1 is the temperature of the combustion gas in the tube containing the ignition charges.

- In case the pressure of combustible gas in the tube containing the ignition charges is greater than the pressure of the combustible gas in the barrel ($p_1 > p_2$), the combustible gas will flow from the tube containing the ignition charges into the barrel. The expression to determine gas flow is as follows [4, 10]:

$$\dot{m}_{out,1}=\begin{cases}\mu_1.\xi_1.\dfrac{p_1A_1}{\sqrt{R_1T_1}}.\left(\dfrac{2}{\kappa_{c1}+1}\right)^{\frac{1}{\kappa_{c1}-1}}.\sqrt{\dfrac{2\kappa_{c1}}{\kappa_{c1}+1}} & \text{when } \dfrac{p_1}{p_2}\ge\left(\dfrac{\kappa_{c1}+1}{2}\right)^{\frac{\kappa_{c1}}{\kappa_{c1}-1}}\\ \mu_1.\xi_1.\dfrac{p_1A_1}{\sqrt{R_1T_1}}.\sqrt{\dfrac{2\kappa_{c1}}{\kappa_{c1}-1}\left[\left(\dfrac{p_2}{p_1}\right)^{\frac{2}{\kappa_{c1}}}-\left(\dfrac{p_2}{p_1}\right)^{\frac{\kappa_{c1}+1}{\kappa_{c1}}}\right]} & \text{when } 1<\dfrac{p_1}{p_2}<\left(\dfrac{\kappa_{c1}+1}{2}\right)^{\frac{\kappa_{c1}}{\kappa_{c1}-1}}\end{cases} \tag{11}$$

- In case the combustion gas pressure in the tube containing the ignition charges is less than the combustion gas pressure in the barrel ($p_1 < p_2$), the combustion gas will flow from the barrel into the tube containing the ignition charges. The expression to determine gas flow is as follows [4, 10]:

$$\dot{m}_{out,1}=\begin{cases}-\mu_1.\xi_1.\dfrac{p_2A_1}{\sqrt{R_{2,s}T_2}}.\left(\dfrac{2}{\kappa_{c2,s}+1}\right)^{\frac{1}{\kappa_{c2,s}-1}}.\sqrt{\dfrac{2\kappa_{c2,s}}{\kappa_{c2,s}+1}} & \text{when } \dfrac{p_2}{p_1}\geq\left(\dfrac{\kappa_{c2,s}+1}{2}\right)^{\frac{\kappa_{c2,s}}{\kappa_{c2,s}-1}} \\ -\mu_1.\xi_1.\dfrac{p_2A_1}{\sqrt{R_{2,s}T_2}}.\sqrt{\dfrac{2\kappa_{c2,s}}{\kappa_{c2,s}-1}\left[\left(\dfrac{p_1}{p_2}\right)^{\frac{2}{\kappa_{c2,s}}}-\left(\dfrac{p_1}{p_2}\right)^{\frac{\kappa_{c2,s}+1}{\kappa_{2,s}}}\right]} & \text{when } 1<\dfrac{p_2}{p_1}<\left(\dfrac{\kappa_{c2,s}+1}{2}\right)^{\frac{\kappa_{c2,s}}{\kappa_{c2,s}-1}}\end{cases} \quad (12)$$

- In case the combustion gas pressure in the tube containing the ignition charges is equal to the combustion gas pressure in the barrel ($p_1 = p_2$):

$$\dot{m}_{out,1}=0 \quad (13)$$

where: μ_1 is the flow loss coefficient of combustion gas; A_1 is the total cross-sectional area of the fire propagation holes; ξ_1 is the flow correction factor, which depends on the flow surface and is determined experimentally; κ_{c1} is the adiabatic exponent index of the combustion gas in the tube containing the ignition charges; $\kappa_{c2,s}$ is the adiabatic exponent index of the combustion gas mixture in the barrel.

Combining equations (1), (2), (3), (5), (10), (11), (12), and (13), the differential equation describes the state in the tube containing the ignition charges is determined as follows:

$$\begin{cases}\dfrac{dz_1}{dt}=\dfrac{p_1}{I_{k1}} \\ \dfrac{d\psi_1}{dt}=\chi_1(1+2\lambda_1.z_1+3\mu_1.z_1^2)\dfrac{dz_1}{dt} \\ \dfrac{dT_1}{dt}=\dfrac{1}{m_1}.\left[\left(\dfrac{c_{p1}}{c_{v1}}.T_{v1}-T_1\right).\omega_1.\dfrac{d\psi_1}{dt}-\left(\dfrac{c_{p1}}{c_{v1}}-1\right)\dot{m}_{out,1}.T_1\right] \\ p_1=\dfrac{m_1R_1T_1}{W_{01}-\dfrac{\omega_1}{\delta_1}.(1-\psi_1)-m_1.\alpha_1} \\ \dfrac{dm_{out,1}}{dt}=\dot{m}_{out,1} \\ m_1=\psi_1.\omega_1-m_{out,1}\end{cases} \quad (14)$$

2.2. Describe the phenomena in the barrel

a) Equations of combustion and gas generation

$$\frac{dz_2}{dt}=\begin{cases}\dfrac{p_2}{I_{k2}} & \text{when } 0<e\leq e_{12} \\ 0 & \text{when } e>e_{12}\end{cases} \quad (15)$$

$$\frac{d\psi_2}{dt}=\chi_2(1+2\lambda_2.z_2+3\mu_2.z_2^2)\frac{dz_2}{dt} \quad (16)$$

where: p_2 is the combustion gas pressure in the barrel; ψ_2 is the relative burning mass of the propellant; I_{k2} is the total momentum of combustion gas pressure in the barrel; z_2 is the relative burn thickness; e_{12} is the thickness of the propellant of the propelling charges; e is the burned thickness of the propellant up to the time of review; χ_2, λ_2, μ_2 are the shape factor of the propellant in the propelling charges.

b) The equation describes the law of combustion gas pressure in propelling charges

The equation representing the law of combustion gas pressure in the barrel is determined based on the equation of the state of combustible gas in the barrel.

$$p_2.V_2=m_2.R_{2,s}.T_2\rightarrow p_2=\frac{m_2.R_{2,s}.T_2}{V_2} \quad (17)$$

c) The equations of motion of the projectile

$$\frac{dv}{dt}=\frac{(s-A_2).p_2}{\varphi.m_q} \quad (18)$$

$$\frac{dl}{dt}=v \quad (19)$$

where: s is the cross-sectional area of the barrel; A_2 is area of gap between projectile and barrel; l is the distance of movement of the projectile in the barrel; m_q is the mass of the projectile; v is the moving speed of the projectile in the barrel; φ is the second work coefficient.

d) The equation describes the law of changing the combustion gas temperature in the barrel

When the ignition charges are ignited, the combustion gas pressure in the tube containing the ignition charges increases very quickly. When the combustion gas pressure is large enough, the paper tube is broken at the places where the fire transmission holes on the tube contain the ignition charges. The burning gas in the tube containing the ignition charges is ejected through the flame transfer holes into the barrel with volume W_{02} and gas flow $\dot{m}_{in,12}=\dot{m}_{out,1}=0$. Then the propellant of the propelling charges is burned. Most of the combustible energy propels the projectile to move with velocity v and some of the combustible gas escapes through the gap between the barrel and the projectile, which has a total area A_2. Equation (6) is written in the following form:

$$\frac{dH_2}{dt}=\frac{dH_{in,12}}{dt}+\frac{dH_{in,2}}{dt}-\frac{dH_{out,2}}{dt}=\frac{dU_2}{dt}+p_2.\frac{dV_2}{dt} \quad (20)$$

The mass of the gas mixture in the barrel m_2 is determined by the following formula:

$$m_2=m_{in,12}+m_{in,2}-m_{out,2}=m_{out,1}+\psi_2.\omega_2-m_{out,2} \quad (21)$$

$$\frac{dm_2}{dt}=\dot{m}_{out,1}+\omega_2.\frac{d\psi_2}{dt}-\dot{m}_{out,2} \quad (22)$$

The Entapy stream components are determined using the following expression [8]:

$$\begin{cases}\dfrac{dH_{in,12}}{dt}=c_{p1}.T_1.\dfrac{dm_2}{dt}=c_{p1}.T_1.\dot{m}_{out,1}\\ \dfrac{dH_{in,2}}{dt}=c_{p2}.T_{v2}.\omega_2.\dfrac{d\psi_2}{dt}\\ \dfrac{dH_{out,2}}{dt}=c_{p2,s}.\dot{m}_{out,2}T_2\end{cases} \quad (23)$$

The instantaneous change in internal energy of the gas mixture in the barrel is given by the following expression:

$$\frac{dU_2}{dt}=c_{v2,s}.\left(T_2.\frac{dm_2}{dt}+m_2.\frac{dT_2}{dt}\right)=c_{v2,s}.\left[T_2.\left(\dot{m}_{out,1}+\omega_2.\frac{d\psi_2}{dt}-\dot{m}_{out,2}\right)+m_2.\frac{dT_2}{dt}\right] \quad (24)$$

$$\dot{m}_{out,2}=\begin{cases}\mu_2.\xi_2.\dfrac{p_2A_2}{\sqrt{R_{2,s}T_2}}.\left(\dfrac{2}{\kappa_{c2,s}+1}\right)^{\frac{1}{\kappa_{c2,s}-1}}.\sqrt{\dfrac{2\kappa_{c2,s}}{\kappa_{c2,s}+1}} & \text{when } \dfrac{p_2}{p_{kq}}\geq\left(\dfrac{\kappa_{c2,s}+1}{2}\right)^{\frac{\kappa_{c2,s}}{\kappa_{c2,s}-1}}\\ \mu_2.\xi_2.\dfrac{p_2A_2}{\sqrt{R_{2,s}T_2}}.\sqrt{\dfrac{2\kappa_{c2,s}}{\kappa_{c2,s}-1}\left[\left(\dfrac{p_{kq}}{p_2}\right)^{\frac{2}{\kappa_{c2,s}}}-\left(\dfrac{p_{kq}}{p_2}\right)^{\frac{\kappa_{c2,s}+1}{\kappa_{2,s}}}\right]} & \text{when } 1<\dfrac{p_2}{p_{kq}}<\left(\dfrac{\kappa_{c2,s}+1}{2}\right)^{\frac{\kappa_{c2,s}}{\kappa_{c2,s}-1}}\\ 0 & \text{when } p_2=p_{kq}\end{cases} \quad (25)$$

where: μ_2 is the flow loss coefficient of combustion gas; A_2 is the total cross-sectional area of the gap between the projectile and the barrel; ξ_2 is the flow correction factor, which depends on the flow surface and is determined experimentally; p_{kq} is the atmospheric pressure.

The free volume of combustion gas in the barrel (V_2) is determined as the difference between the volume of the initial combustion space W_{02} and the volume of unburned propellant $\frac{\omega_2}{\delta_2}.(1-\psi_2)$ and the volume of produced gas $\alpha_{2,s}{\cdot}m_2$, see Figure 3 [1]. In addition, this free volume is also increased due to the movement of the projectile in the barrel. If the projectile moves a distance l, this gain will be equal to the value s·l. Then, the free volume of the second compartment is determined as follows:

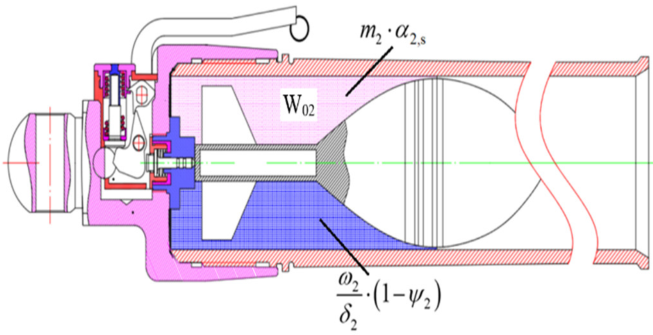

Figure 3. Free volume of combustion gases in the barrel

$$V_2=W_{02}-\frac{\omega_2}{\delta_2}.(1-\psi_2)-m_2.\alpha_{2,s}+s.l \quad (26)$$

$$\frac{dV_2}{dt}=\frac{\omega_2}{\delta_2}.\frac{d\psi_2}{dt}-\alpha_{2,s}.\frac{dm_2}{dt}+s.\frac{dl}{dt}=\frac{\omega_2}{\delta_2}.\frac{d\psi_2}{dt}-\alpha_{2,s}.\frac{dm_2}{dt}+s.v \quad (27)$$

Substitute equations (23), (24), and (27) into equation (20), and after transformation, the equation describing the law of changing combustion gas temperature in the barrel is determined:

$$\frac{dT_2}{dt}=\frac{1}{m_2}.\begin{bmatrix}\left(\dfrac{c_{p2}}{c_{v2,s}}.T_{v2}-T_2\right).\omega_2.\dfrac{d\psi_2}{dt}\\ -\left(\dfrac{c_{p1}}{c_{v2,s}}.T_1-T_2\right)\dot{m}_{out,1}\\ -\left(\dfrac{c_{p2,s}}{c_{v2,s}}-1\right)\dot{m}_{out,2}.T_2-\dfrac{p_2}{c_{v2,s}}.\dfrac{dV_2}{dt}\end{bmatrix} \quad (28)$$

Combining the above equations, the system of differential equations describing the state of the combustible gas in the barrel is determined as follows:

$$\begin{cases} \dfrac{dz_2}{dt} = \dfrac{p_2}{I_{k2}} \\ \dfrac{d\psi_2}{dt} = \chi_2(1 + 2\lambda_2.z_2 + 3\mu_2.z_2^2)\dfrac{dz_2}{dt} \\ \dfrac{dT_2}{dt} = \dfrac{1}{m_2}.\begin{bmatrix} \left(\dfrac{c_{p2}}{c_{v2,s}}.T_{v2} - T_2\right).\omega_2.\dfrac{d\psi_2}{dt} \\ -\left(\dfrac{c_{p1}}{c_{v2,s}}.T_1 - T_2\right)\dot{m}_{out,1} \\ -\left(\dfrac{c_{p2,s}}{c_{v2,s}} - 1\right)\dot{m}_{out,2}.T_2 \\ -\dfrac{p_2}{c_{v2,s}}.\dfrac{dV_2}{dt} \end{bmatrix} \\ \dfrac{dm_{out,2}}{dt} = \dot{m}_{out,2} \\ \dfrac{dm_2}{dt} = \dot{m}_{out,1} + \omega_2.\dfrac{d\psi_2}{dt} - \dot{m}_{out,2} \\ \dfrac{dV_2}{dt} = \dfrac{\omega_2}{\delta_2}.\dfrac{d\psi_2}{dt} - \alpha_{2,s}.\dfrac{dm_2}{dt} + s.v \\ \dfrac{dv}{dt} = \dfrac{(s - A_2).p_2}{\varphi.m_q} \\ p_1 = \dfrac{m_2 R_{2,s}.T_2}{V_2} \\ \dfrac{dl}{dt} = v \end{cases} \quad (29)$$

In the above equation, the variables are the characteristics of the gas mixture in the barrel. They are defined as follows [9]: $\varepsilon = \dfrac{\psi_2.\omega_2}{\psi_2.\omega_2 + m_{out,1}}$;

The powder force of the mixture: $f_{2,s} = \varepsilon.f_2 + (1 - \varepsilon).f_1$;

Covolume coefficient of gas mixture:

$\alpha_{2,s} = \varepsilon.\alpha_2 + (1 - \varepsilon).\alpha_1$;

Isostatic specific heat: $c_{p2,s} = \varepsilon.c_{p2} + (1 - \varepsilon).\ c_{p1}$;

Isovolumetric specific heat: $c_{v2,s} = \varepsilon.c_{v2} + (1 - \varepsilon).\ c_{v1}$;

Gas constant: $R_{2,s} = c_{p2,s} - c_{v2,s}$;

Adiabatic exponent index of combustion gas:

$\kappa_{c2,s} = \varepsilon.\kappa_{c2} + (1 - \varepsilon).\ \kappa_{c1}$.

3. SIMULATION RESULTS AND DISCUSSION

The interior ballistics model in the mortar system is described by the equations (14) and (29) which can be applied to any mortar. To verify the mathematical model just built above, numerical calculation is applied to the 100mm mortar of Vietnam, see Figure 4. Due to the large volume of input parameters, only some basic parameters are mentioned here, see Table 1 and Table 2 [12].

Figure 4. The 100mm mortar of Vietnam

Table 1. Input parameters of the propellant of the ignition charges

Parameters	Symbol	Value	Unit
Interior ballistics parameter of ignition charges			
Mass of propellant	ω_1	0.02	kg
Powder force	f_1	$1.06*10^6$	J/kg
Gas constant	R_1	354.2	J/kg·K
Fire temperature of the propellant	T_{v1}	2996	K
Density of propellant	δ_1	1600	kg/m^3
Covolume of combustion gas	α_1	$8.918*10^{-4}$	m^3/kg
Total momentum of combustion gas pressure	I_{k1}	$1.722*10^5$	Pa·s
Adiabatic exponent index of combustion gas	κ_{c1}	1.0633	-
Pressure to open the fire tranfer hole	p_{01}	9.65	MPa
Propellant thickness	$2e_{11}$	0.00035	m
Shape factor of the propellant	χ_1	1.2369	-
	$\chi_1\lambda_1$	-0.2378	-
	$\chi_1\mu_1$	$8.1667*10^{-4}$	-

Table 2. Input parameters of propelling charges

Parameters	Symbol	Value	Unit
Initial volume of the combustion chamber	W_{02}	$1.83*10^{-3}$	m^3
Mass of the projectile	m_q	8.0	kg

Displacement of projectiles moving in the barrel	l	1.096	m
Cross-sectional area of the barrel	s	0.00785	m^2
Area of gap between projectile and barrel	A_2	$9.4*10^{-6}$	m^2
Interior ballistics parameter of propelling charges			
Mass of propellant	ω_2	0.175	kg
Powder force	f_2	$1.0705*10^6$	J/kg
Gas constant	R_2	328.4	J/kg·K
Fire temperature of the propellant	T_{v2}	3254	K
Density of propellant	δ_2	1550	kg/m3
Covolume of combustion gas	α_2	$0.8838*10^{-3}$	m^3/kg
Total momentum of combustion gas pressure	I_{k2}	$2.69*10^7$	Pa·s
Adiabatic exponent index of combustion gas	κ_{c2}	1.15	-
Propellant thickness	$2e_{12}$	0.0003	m
Shape factor of the propellant	χ_2	1.0122	-
	$\chi_2\lambda_2$	-1.0122	-
	$\chi_2\mu_2$	0	-

The system of equations (29) is solved with the following initial conditions: $t = 0$: $z_0 = \psi_0 = 0$; $p_0 = p_{moi}$; $m_{out,1} = m_{out,2} = 0$; $T = 294K$, $v = 0$; $l = 0$.

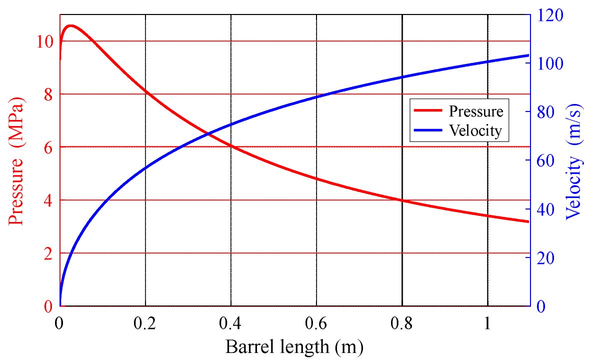

Figure 5. Pressure and velocity graph versus barrel length with 01 propelling charge

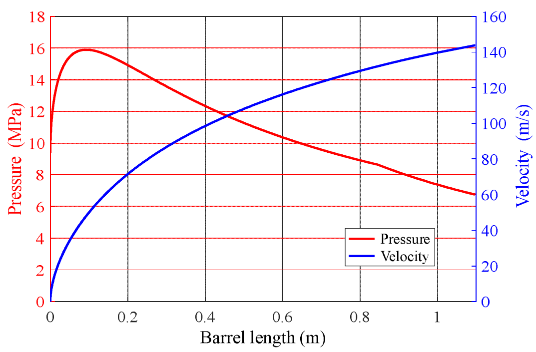

Figure 6. Pressure and velocity graph versus barrel length with 02 propelling charges

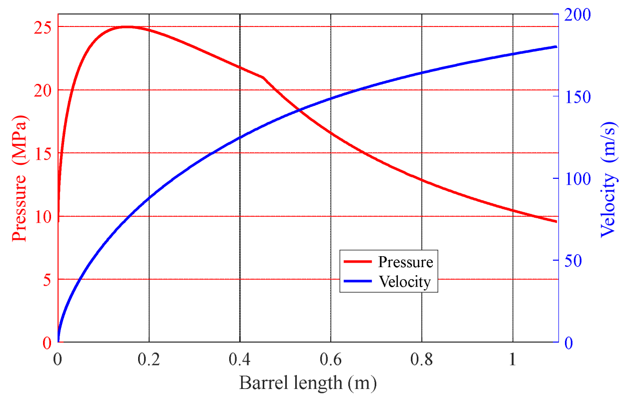

Figure 7. Pressure and velocity graph versus barrel length with 03 propelling charges

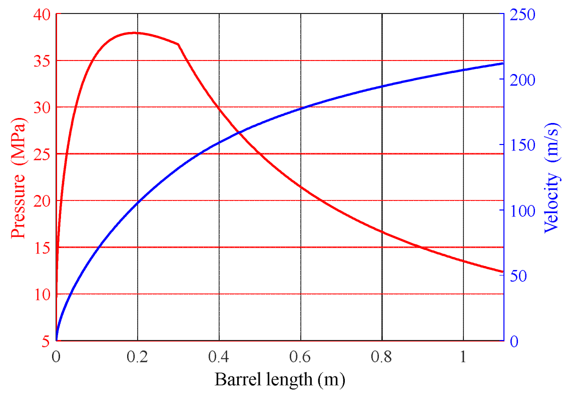

Figure 8. Pressure and velocity graph versus barrel length with 04 propelling charges

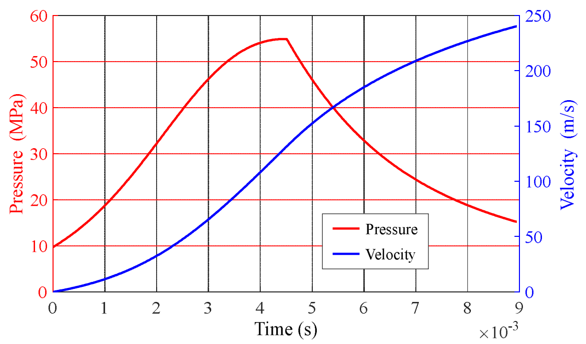

Figure 9. Pressure and velocity versus time graph

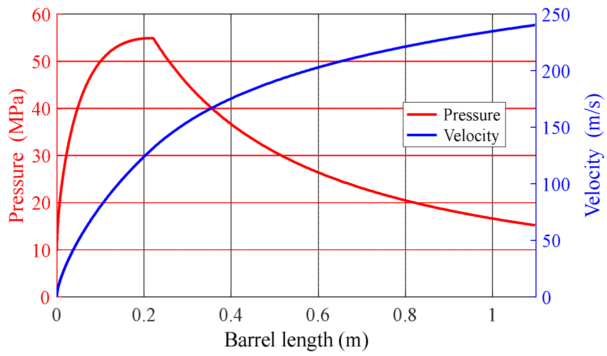

Figure 10. Pressure and velocity graph versus barrel length

The 4th Runge-Kutta numerical integration method programmed in the Matlab 2022b programming environment is used to solve the system of equations (29). Some typical results are given as follows, see Figures 5 ÷ 8.

The graph shows the dependence of the bullet's velocity and the gas pressure in the barrel on the time (Figure 9) and on the barrel length (Figure 10) when firing with the maximum propelling charges (with 05 propelling charges). Results of solving the maximum propelling charges at some special times are listed in Table 3.

Table 3. Results of solving the maximum propelling charges

	P (MPa)	V (m/s)	l (m)	t (s)
The time when the propellant burns out	55.98	131.59	0.221	0.00451
Time to reach maximum pressure P_{max}	55.99	129.75	0.216	0.00447
The time the bullet moves to the muzzle	15.51	241.82	1.096	0.00895

The theoretical calculation results show: When the number of propelling charges increases, the gas pressure in the barrel and the velocity of the bullet also increases, this is completely reasonable, see Figures 5 ÷ 8. The results in the case of the maximum propelling charges: The velocity of the bullet leaving the barrel is 241.82m/s (see Figures 7, 8 and Table 3), this is completely suitable compared to the manufacturer's parameters of 250m/s [12], error 3.27%. Therefore, it can be confirmed that the interior ballistics problem model for the mortar system built above is completely suitable and reliable. This mathematical model can be used to study subsequent problems such as choosing a reasonable structure and optimally designing the structure of the mortar system.

4. CONCLUSION

In this study, a thermodynamic mathematical model was developed to solve the interior ballistics algorithm problem for mortar systems. With the obtained calculation results, some conclusions are made as follows:

- A new mathematical model accurately and completely describes the shooting phenomena occurring in the ignition tube and in the gun barrel;

- The numerical integration method has been applied to solve the problem with high accuracy;

- The mathematical model and solution method in this study are established for the general case, it can be applied to specific cases with similar structures. The simulation results obtained for 100mm mortar are very consistent with the manufacturer's announced results, the error is within the allowable limit (< 10%);

- This research result is an important theoretical basis for the calculation and optimal design of mortar systems as well as weapons systems with similar structures.

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