VIBRATION SUPPRESSION CONTROL FOR 5-DOF VARYING-CABLE-LENGTH TOWER CRANES

ĐIỀU KHIỂN CHỐNG RUNG CHO CẦN TRỤC THÁP 5 BẬC TỰ DO CÓ CHIỀU DÀI CÁP THAY ĐỔI

Nguyen Huu Hai[1], Bui Tung Duong[2], Nguyen Thi Trang[2], Quach Duc Cuong[1], Duong Minh Duc[2,*]

DOI: http://doi.org/10.57001/huih5804.2024.302

ABSTRACT

Tower crane is widely used in various fields such as construction, industry, transportation, etc. The tower crane, by its physical nature, is a oscillating pendulum in space, thus the vibrations of the load are inevitable during operation, highlighting the necessity of researching vibration control methods for tower cranes. Most current studies on anti-vibration for tower cranes focus on the object with a fixed cable length and 4 degrees of freedom, while research on anti-vibration for 5 degrees of freedom tower cranes is still limited. This article presents a sliding mode control (SMC) method for vibration control of a 5 degrees of freedom tower crane with variable cable length. The control system, with high robustness, helps move the load along the desired trajectory while effectively suppressing vibrations. The feasibility and effectiveness of the proposed control method are tested through simulations.

***Keywords*:** *Tower cranes, sliding mode control, varying cable length.*

TÓM TẮT

Cần trục tháp giữ là một trong các thiết bị nâng hiện đại được sử dụng trong ngành xây dựng, đáp ứng nhu cầu vận chuyển vật liệu xây dựng một cách chính xác và hiệu quả. Vì cẩu tháp có bản chất vật lý là một hệ con lắc dao động trong không gian, do đó, khó tránh khỏi các rung động của tải trong quá trình vận hành. Điều này đặt ra yêu cầu trong việc nghiên cứu các phương pháp điều khiển chống rung cho cẩu tháp, nhằm giảm thiểu hiện tượng rung động và đảm bảo sự ổn định trong quá trình vận hành. Hầu hết các nghiên cứu chống rung cẩu tháp hiện nay tập trung vào đối tượng cẩu tháp có chiều dài dây cáp là cố định với 4 bậc tự do, các nghiên cứu về chống rung cho cẩu tháp 5 bậc tự do còn rất hạn chế. Bài báo này trình bày phương pháp điều khiển chống rung cho cần trục tháp 5 bậc tự do với chiều dài cáp thay đổi sử dụng phương pháp điều khiển trượt (sliding mode control - SMC). Bộ điều khiển có tính bền vững cao giúp di chuyển tải theo quỹ đạo mong muốn, đồng thời triệt tiêu rung động một cách hiệu quả. Hiệu quả của phương pháp đề xuất được kiểm nghiệm qua các mô phỏng.

Từ khóa: *Cần trục tháp, điều khiển trượt, chiều dài cáp thay đổi.*

[1]Faculty of Faculty of Electrical Engineering, Hanoi University of Industry, Vietnam
[2]School of Electrical and Electronic Engineering, Hanoi University of Science and Technology, Vietnam
[*]Email:duc.duongminh@hust.edu.vn
Received: 15/4/2024
Revised: 05/6/2024
Accepted: 27/9/2024

1. INTRODUCTION

A tower crane is a type of lifting device with a tower body that rapidly increases from tower sections to the height of the building, with a very large reach [1]. It is widely used for material transportation and assembly in factories, civil construction works, industrial construction, and hydroelectric projects, etc. [2]. However, reality presents major challenges to tower crane operations. Occupational accidents still occur regularly, serious safety accidents of cranes have caused casualties and property loss [3]. One of the main causes of this phenomenon is load vibration during operation. During the tower crane's operation, its movement causes vibrations inside the structure. The faster the crane moves, the greater the moment of inertia it generates [4]. In addition to moving with the crane, inertial forces will cause relative movement of the load with respect to the crane, causing swing angles of the load cable around its equilibrium position [5]. This phenomenon not only reduces work efficiency but also causes labor safety. Therefore, eliminating vibration of tower cranes during operation is extremely necessary.

In recent years, many studies have proposed anti-vibration control methods for tower cranes. The most notable studies include: robust control [6, 7], nonlinear control [8, 9], model predictive control [10, 11], adaptive control [12, 13], etc. In [14], a feedback control method for tower cranes using gain-scheduling technique has been proposed to eliminate load fluctuations, but the nonlinear characteristics of the system are not considered. In [15] an elastic model of tower crane is developed. Then, a two-degree-of-freedom control structure for tower cranes is proposed. The control system combines nonlinear flatness-based control and a linear quadratic regulator. In addition to feedback control methods, several feedforward control methods are also developed for tower cranes. In [16] , a command shaping approach for the radial motion of a tower crane has been proposed. In [17, 18], an optimal trajectory is considered for a tower crane. Most of the above studies have studied tower cranes with 4 degrees of freedom with fixed rope length. To ensure effective completion of transportation and lifting/lowering of loads, it is necessary to consider the change in rope length, therefore, the control method of the 5-degree-of-freedom tower crane needs to be studied further. In addition, the problem of controlling the nonlinear model of tower cranes also needs to be applied. Using a dynamic model close to reality will improve durability as well as the ability to resist disturbances when applied in practice [19].

Sliding mode control (SMC) is a robust and effective control strategy that has found widespread applications in various engineering domains because of its of robustness, fast response, and design simplicity. Therefore, this article proposes the sliding control method (SMC), which essentially uses intermittent feedback control law to enforce system stability, helping to effectively reduce load vibration. First, the system operates as a scaled down system compared to the original object, second, the movement on the sliding surface of the system makes the system insensitive to disturbances and model uncertainty [20]. The article uses a nonlinear dynamic model and considers the change in cable length, ensuring accuracy compared to the actual tower crane model. The results of the method solved the problem of planning the trajectory of a 5-degree-of-freedom tower crane with increasing/decreasing the load, controlling the crane rotation, moving the trolley and changing the cables. The oscillation angles of the load are eliminated almost completely quickly and reach zero before the crane stops completely.

2. TOWER CRANE DYNAMICAL MODEL

The tower crane model is described in the static coordinate system Oxyz and rotate coordinate system $O'x'y'z'$ as shown in Fig. 1. The system consists of a trolley with mass m_t, a load with mass m_c, and the moment of inertia of the tower body rotation J. There are 5 types of motions: translation of the trolley x, rotatio of the tower body γ, length variation motion of the cable l and swinging motion of the load cable due to oscillation angles θ_1 and θ_2. Thus, $\mathbf{q} = [\gamma, x, l, \theta_1, \theta_2]^T$ is chosen as the generalized coordinate vector. The control inputs include the pulling force u_t, the torque u_r and the control force for cable length adjustment u_l to move the trolley, rotate the tower body, and adjust the cable length to reach the destination from the initial position.

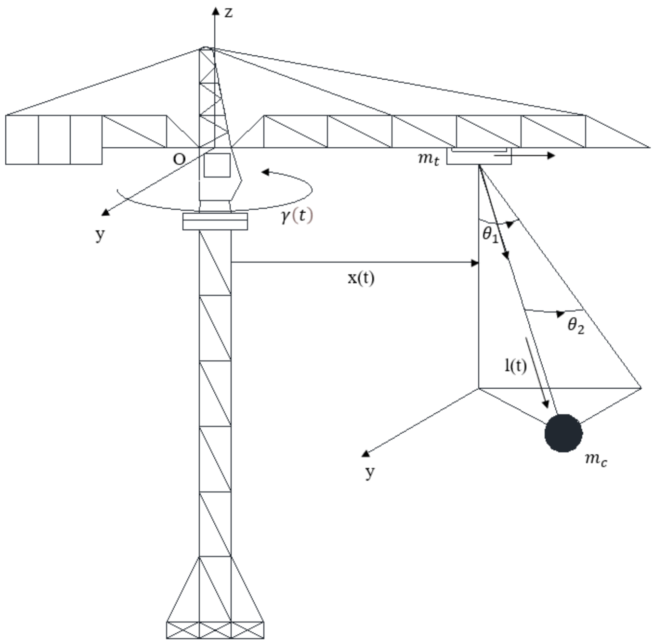


Fig. 1. Tower crane model

l: rope length; x: distance of the trolley from the tower axis; γ: rotation angle of the tower body; θ_1, θ_2: oscillation angles of the load; m_c, m_t: mass of the load and trolley

Using the Lagrange method and after computational steps, we obtain the dynamical model of the tower crane in matrix form as follows [3]:

$$\mathbf{M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = F} \quad (1)$$

where:

$\mathbf{M(q)}$ is the inertia symmetric matrix:

$$\mathbf{M}(\mathbf{q}) = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} \\ m_{21} & m_{22} & m_{23} & m_{24} & m_{25} \\ m_{31} & m_{32} & m_{33} & 0 & 0 \\ m_{41} & m_{42} & 0 & m_{44} & 0 \\ m_{51} & m_{52} & 0 & 0 & m_{55} \end{bmatrix} \quad (2)$$

The parameters of the elements of the matrix $\mathbf{M}(\mathbf{q})$:

$m_{11} = (m_t + m_c)x^2 + m_c l^2(\theta_1^2 + \theta_2^2) + 2m_C xl\theta_1 + J$;

$m_{12} = m_{21} = -m_c l\theta_2$; $m_{13} = m_{31} = m_C x\theta_2$;

$m_{14} = m_{41} = -m_c l^2\theta_2$; $m_{15} = m_{51} = m_c l(x + l\theta_1)$;

$m_{22} = (m_t + m_c)$; $m_{23} = m_{32} = m_C\theta_1$;

$m_{24} = m_{42} = m_c l$; $m_{25} = m_{52} = -m_C l\theta_1\theta_2$;

$m_{33} = m_c$; $m_{44} = m_{55} = m_c l^2$;

$\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is the centrifugal and Coriolis matrix. The elements of the matrix are represented as follows:

$$\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} c_{11} & 0 & c_{13} & c_{14} & c_{15} \\ c_{21} & 0 & c_{23} & c_{24} & c_{25} \\ c_{31} & c_{32} & 0 & c_{34} & c_{35} \\ c_{41} & 0 & c_{43} & c_{44} & c_{45} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} \end{bmatrix} \quad (3)$$

The parameters of the elements of the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$:

$C_{11} = 2(m_C + m_t)\dot{x}x + 2m_C lx\dot{\theta}_1$;

$c_{13} = (2m_c l\theta_2^2 + 2m_c l\theta_1^2 + 2m_c x\theta_1)\dot{\gamma} - 2m_c l\dot{\theta}_1\theta_2 + (2m_c l\theta_1 + 2m_c x)\dot{\theta}_2$;

$c_{14} = (2m_c xl + 2m_c l^2\theta_1)\dot{\gamma} + m_c l^2\dot{\theta}_1\theta_1\theta_2$;

$c_{15} = (2m_c l^2\theta_2 - 2m_c xl\theta_1\theta_2)\dot{\gamma} + 2m_c l^2\dot{\theta}_1\theta_2^2 - m_c lx\dot{\theta}_2\theta_2$;

$c_{21} = -[m_c l\theta_1 + (m_t + m_c)x]\dot{\gamma}$;

$c_{23} = -2m_c\dot{\gamma}\theta_2 + 2m_c\dot{\theta}_1 - 2m_c\dot{\theta}_2\theta_1\theta_2$;

$c_{24} = -2m_C l\dot{\theta}_2\theta_2 - m_C l\dot{\theta}_1\theta_2$;

$c_{25} = -2m_c l\dot{\gamma} - m_c l\dot{\theta}_2\theta_1$;

$c_{31} = -(m_c x\theta_1 + m_c l^2\theta_1^2 + m_c l\theta_2^2)\dot{\gamma}$;

$c_{32} = 2m_c\dot{\gamma}\theta_2$; $c_{34} = 2m_c l\theta_2\dot{\gamma}$;

$c_{35} = -2m_C l\dot{\gamma}\theta_1 - m_c l\dot{\theta}_2$; $c_{41} = -m_c l(x + l\theta_1)\dot{\gamma}$;

$c_{43} = -2m_c l\dot{\gamma}\theta_2 + 2m_c l\dot{\theta}_1$; $c_{44} = -2m_c l^2\theta_2\dot{\theta}_2$;

$c_{45} = -2m_c l^2\dot{\gamma}$; $c_{51} = m_c l(x\theta_1\theta_2 - l\theta_2)\dot{\gamma}$;

$c_{52} = 2m_c l\dot{\gamma}$; $c_{53} = 2m_c l\dot{\gamma}\theta_1$;

$c_{54} = 2m_c l^2\dot{\gamma} + m_c l^2\dot{\theta}_1\theta_2$; $c_{55} = 2m_c l\dot{l}$;

$\mathbf{G}(\mathbf{q})$ represents the gravitational component and $\mathbf{F}$ represents the control input.

$$\mathbf{G}(\mathbf{q}) = \begin{bmatrix} 0 \\ 0 \\ g_3 \\ g_4 \\ g_5 \end{bmatrix}, \mathbf{F} = \begin{bmatrix} u_r \\ u_t \\ u_l \\ 0 \\ 0 \end{bmatrix} \quad (4)$$

The elements of the gravitational component $\mathbf{G}(\mathbf{q})$ are: $g_3 = -m_c l\theta_1^2 + m_c g$; $g_4 = m_c gl\theta_1$; $g_5 = m_c gl\theta_2$;

3. DESIGNING OF SLIDING MODE CONTROLLER

3.1.Controller designing

The purpose of the sliding mode controller (SMC) is to control the state trajectories $\mathbf{q}_1 = [\gamma \;\; x \;\; l]^T$ to achieve desired values $\mathbf{q}_{1d} = [\gamma_d \;\; x_d \;\; l_d]^T$ and to drive the oscillation angle of the load $\mathbf{q}_2 = [\theta_1 \;\; \theta_2]^T$ towards $\mathbf{q}_{2d} = [0 \;\; 0]^T$. A sliding surface first is constructed such that the state trajectories are attracted towards it, then a control law is designed to drive all states to the equilibrium point. Before designing the controller, the equation (1) is rewritten as follows:

$$\mathbf{M}_{11}(\mathbf{q})\ddot{\mathbf{q}}_1 + \mathbf{M}_{12}(\mathbf{q})\ddot{\mathbf{q}}_2 + \mathbf{C}_{11}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_1 + \mathbf{C}_{12}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_2 + \mathbf{G}_1(\mathbf{q}) = \mathbf{F}_1 \quad (5)$$

$$\mathbf{M}_{21}(\mathbf{q})\ddot{\mathbf{q}}_1 + \mathbf{M}_{22}(\mathbf{q})\ddot{\mathbf{q}}_2 + \mathbf{C}_{21}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_1 + \mathbf{C}_{22}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_2 + \mathbf{G}_2(\mathbf{q}) = 0 \quad (6)$$

where:

$$\mathbf{M}_{11}(\mathbf{q}) = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix};$$

$$\mathbf{M}_{12}(\mathbf{q}) = \begin{bmatrix} m_{15} & m_{14} \\ m_{25} & m_{24} \\ m_{35} & m_{34} \end{bmatrix};$$

$$\mathbf{M}_{21}(\mathbf{q}) = \begin{bmatrix} m_{51} & m_{52} & m_{53} \\ m_{41} & m_{42} & m_{43} \end{bmatrix};$$

$$\mathbf{M}_{22}(\mathbf{q}) = \begin{bmatrix} m_{55} & m_{54} \\ m_{45} & m_{44} \end{bmatrix};$$

$$\mathbf{C}_{11}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix};$$

$$\mathbf{C}_{12}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} c_{15} & c_{14} \\ c_{25} & c_{24} \\ c_{35} & c_{34} \end{bmatrix};$$

$$\mathbf{C}_{21}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} c_{51} & c_{52} & c_{53} \\ c_{41} & c_{42} & c_{43} \end{bmatrix};$$

$$\mathbf{C}_{22}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} c_{55} & c_{54} \\ c_{45} & c_{44} \end{bmatrix};$$

$$\mathbf{G}_1(\mathbf{q}) = \begin{bmatrix} 0 \\ 0 \\ g_3 \end{bmatrix}; \quad \mathbf{G}_2(\mathbf{q}) = \begin{bmatrix} g_5 \\ g_4 \end{bmatrix}; \quad \mathbf{F}_1 = \begin{bmatrix} u_r \\ u_t \\ u_l \end{bmatrix};$$

Equations (5) and (6) are transformed into the following form:

$$\ddot{\mathbf{q}}_1 = \mathbf{M}_{11}^{-1}(\mathbf{q})(-\mathbf{M}_{12}(\mathbf{q})\ddot{\mathbf{q}}_2 - \mathbf{C}_{11}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_1 - \mathbf{C}_{12}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_2 - \mathbf{G}_1(\mathbf{q}) + \mathbf{F}_1) \quad (7)$$

$$\ddot{\mathbf{q}}_2 = \mathbf{M}_{22}^{-1}(\mathbf{q})\big(-\mathbf{M}_{21}(\mathbf{q})\ddot{\mathbf{q}}_1 - \mathbf{C}_{21}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_1 - \mathbf{C}_{22}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_2 - \mathbf{G}_2(\mathbf{q})\big) \quad (8)$$

It should be noted that $\mathbf{M}_{11}(\mathbf{q})$ and $\mathbf{M}_{22}(\mathbf{q})$ are positive definite matrices with parameters $l > 0$, $\theta_1 < \frac{\pi}{2}$

và $\theta_2 < \frac{\pi}{2}$. By substituting (8) into (5), we can derive the following:

$$\mathbf{M}_3(\mathbf{q})\ddot{\mathbf{q}}_1 + \mathbf{C}_1(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_1 + \mathbf{C}_2(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_2 + \mathbf{G}_3(\mathbf{q}) = \mathbf{F}_1 \quad (9)$$

where:

$$\mathbf{M}_3(\mathbf{q}) = \mathbf{M}_{11}(\mathbf{q}) - \mathbf{M}_{12}(\mathbf{q})\mathbf{M}_{22}^{-1}(\mathbf{q})\mathbf{M}_{21}(\mathbf{q})$$

$$\mathbf{C}_1(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{C}_{11}(\mathbf{q},\dot{\mathbf{q}}) - \mathbf{M}_{12}(\mathbf{q})\mathbf{M}_{22}^{-1}(\mathbf{q})\mathbf{C}_{21}(\mathbf{q},\dot{\mathbf{q}})$$

$$\mathbf{C}_2(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{C}_{12}(\mathbf{q},\dot{\mathbf{q}}) - \mathbf{M}_{12}(\mathbf{q})\mathbf{M}_{22}^{-1}(\mathbf{q})\mathbf{C}_{22}(\mathbf{q},\dot{\mathbf{q}})$$

$$\mathbf{G}_3(\mathbf{q}) = \mathbf{G}_1 - \mathbf{M}_{12}(\mathbf{q})\mathbf{M}_{22}^{-1}(\mathbf{q})\mathrm{G}_2(\mathbf{q})$$

The mathematical model (9) will be used to design the tower crane controller. The designing steps are as follows.

Step 1: define the sliding surface as:

$$\mathbf{w} = \begin{bmatrix} \mathrm{w}_1 \\ \mathrm{w}_2 \\ \mathrm{w}_3 \end{bmatrix} = \dot{\mathbf{e}}_1 + \boldsymbol{\mu}_1\mathbf{e}_1 + \boldsymbol{\alpha}\dot{\mathbf{e}}_2 + \boldsymbol{\mu}_2\mathbf{e}_2 \quad (10)$$

where $\mathbf{e}_1 = \mathbf{q}_1 - \mathbf{q}_{1d} = \begin{bmatrix} \gamma - \gamma_d \\ x - x_d \\ l - l_d \end{bmatrix}$; $\mathbf{e}_2 = \mathbf{q}_2 - \mathbf{q}_{2d} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$ are signal errors.

$\boldsymbol{\mu}_1$; $\boldsymbol{\mu}_2$; $\boldsymbol{\alpha}$ are controller coefficient matrix, with:

$$\boldsymbol{\mu}_1 = \begin{bmatrix} \mu_{11} & 0 & 0 \\ 0 & \mu_{12} & 0 \\ 0 & 0 & \mu_{13} \end{bmatrix}; \quad \boldsymbol{\mu}_2 = \begin{bmatrix} \mu_{21} & 0 \\ 0 & \mu_{22} \\ 0 & 0 \end{bmatrix};$$

$$\boldsymbol{\alpha} = \begin{bmatrix} a_{21} & 0 \\ 0 & a_{22} \\ 0 & 0 \end{bmatrix};$$

Derivative of (10) one can obtain:

$$\begin{aligned} \dot{\mathbf{w}} &= \ddot{\mathbf{e}}_1 + \boldsymbol{\mu}_1\dot{\mathbf{e}}_1 + \boldsymbol{\alpha}\ddot{\mathbf{e}}_2 + \boldsymbol{\mu}_2\dot{\mathbf{e}}_2 \\ &= \ddot{\mathbf{q}}_1 + \boldsymbol{\mu}_1\dot{\mathbf{q}}_1 + \boldsymbol{\alpha}\ddot{\mathbf{q}}_2 + \boldsymbol{\mu}_2\dot{\mathbf{q}}_2 \end{aligned} \quad (11)$$

Step 2: set $\dot{\mathbf{w}} = -\mathrm{K}.\operatorname{sgn}(\mathrm{w})$, then, the control input is obtained as

$$\begin{aligned} \mathbf{U} &= \mathbf{C}_1(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_1 + \mathbf{G}_3(\mathbf{q}) \\ &-\mathbf{M}_3(\mathbf{q})\left(\mathbf{I}_3 - \boldsymbol{\alpha}\mathbf{M}_{22}^{-1}(\mathbf{q})\mathbf{M}_{21}(\mathbf{q})\right)^{-1} \times \\ &\begin{bmatrix} (\boldsymbol{\mu}_1 - \boldsymbol{\alpha}\mathbf{M}_{22}^{-1}(\mathbf{q})\mathbf{C}_{21}(\mathbf{q},\dot{\mathbf{q}}))\dot{\mathbf{q}}_1 + (\boldsymbol{\mu}_2 \\ -\boldsymbol{\alpha}\mathbf{M}_{22}^{-1}(\mathbf{q})\mathbf{C}_{22}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_2 - \boldsymbol{\alpha}\mathbf{M}_{22}^{-1}(\mathbf{q})\mathbf{G}_2(\mathbf{q}) \end{bmatrix} \\ &-\mathbf{K}.\operatorname{sgn}(\mathbf{w}) \end{aligned} \quad (12)$$

Comment 1: In (16), $\mathbf{K} = \operatorname{diag}(\mathrm{K}_1, \mathrm{K}_2, \mathrm{K}_3)$ is positive definite matrix ensures the sliding surface converges to 0. Furthermore, $\mathbf{K}$, $\boldsymbol{\mu}_1$, $\boldsymbol{\mu}_2$ are selected to stabilize the system and to make the sliding state trajectory converge to the equilibrium point as fast as possible [21]. Sign function sgn ($\mathbf{w}$) causes chattering around the surface of the state trajectory. To minimize this, a saturation function $\operatorname{sat}(\mathbf{w}) = [\operatorname{sat}(\mathrm{w}_1) \quad \operatorname{sat}(\mathrm{w}_2) \quad \operatorname{sat}(\mathrm{w}_3)]^{\mathrm{T}}$ described in equation (13) as follows to replace the sign function:

$$\operatorname{sat}(\mathrm{w}_i) = \begin{cases} +1 & \text{if } \frac{\mathrm{w}_i}{\varepsilon} > 1 \\ \frac{\mathrm{w}_i}{\varepsilon} & \text{if } -1 < \frac{\mathrm{w}_i}{\varepsilon} < 1 \\ -1 & \text{if } \frac{\mathrm{w}_i}{\varepsilon} < -1 \end{cases} \quad (13)$$

3.2. Stability Analysis of the Proposed Controller

Control scheme (12) must satisfy two requirements: the controller forces the state trajectories to approach the sliding surface after entering it (necessary condition), and the control law pulls the state trajectories on it towards the equilibrium point (sufficient condition). By choosing $\boldsymbol{\alpha}$ such that there exists the inverse of the matrix $\left(\mathbf{I}_3 - \boldsymbol{\alpha}\mathbf{M}_{22}^{-1}(\mathbf{q})\mathbf{M}_{21}(\mathbf{q})\right)$, the control law (12) ensures that all trajectories of the system slide to the surface. However, trajectories may not always converge to the equilibrium point when they are on the surface. If the linear surface (10) and the two accelerations (6) are stable, then stability on the surface will be established [21].

To show the system stability the following Lyapunov candidate function is considered:

$$\mathrm{V} = \frac{1}{2}\mathbf{w}^{\mathrm{T}}.\mathbf{w} \geq 0 \quad (14)$$

Taking the derivative, we obtain:

$$\dot{\mathrm{V}} = \mathbf{w}^{\mathrm{T}}\dot{\mathbf{w}} \quad (15)$$

Substituting the expressions from equations (7), (8), and (12) into equation (15) and simplifying, we obtain:

$$\dot{\mathrm{V}} = -\mathbf{w}^{\mathrm{T}}\mathbf{K}\operatorname{sat}(\mathbf{w}) \quad (16)$$

It is clear that $\dot{\mathrm{V}} \leq 0$ for all positive definite constants $\mathbf{K}$. By using the Barbalat's lemma, the values of $[\mathrm{w}_1 \quad \mathrm{w}_2 \quad \mathrm{w}_3]^{\mathrm{T}}$ will tend to $[0 \quad 0 \quad 0]^{\mathrm{T}}$. Therefore, the sliding surface is asymptotically stable, ensuring the condition for stable convergence of the sliding surface. The value of $\boldsymbol{\alpha}$ is chosen such that the inverse of the expression for $(\mathbf{I}_3 - \boldsymbol{\alpha}\mathbf{M}_{22}^{-1}(\mathbf{q})\mathbf{M}_{21}(\mathbf{q}))$ exists. However, stable sliding surface does not lead to stability of the system's output. By using equations (8), (11), and (12), we obtain:

$$\begin{aligned} \ddot{\mathbf{q}}_2 &= -\mathbf{M}_{22}^{-1}(\mathbf{q})\begin{pmatrix} \mathbf{M}_{21}(\mathbf{q})\ddot{\mathbf{q}}_1 + \mathbf{C}_{21}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_1 \\ +\mathbf{C}_{22}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_2 + \mathbf{G}_2(\mathbf{q}) \end{pmatrix} \\ &= -\mathbf{M}_{22}^{-1}(\mathbf{q})\begin{pmatrix} \mathbf{M}_{21}(\mathbf{q})\mathbf{M}_3^{-1}(\mathbf{q})\begin{pmatrix} \mathbf{u} - \mathbf{C}_1(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_1 \\ -\mathbf{G}_3(\mathbf{q}) \end{pmatrix} \\ + \mathbf{C}_{21}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_1 + \mathbf{C}_{22}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_2 \\ +\mathbf{G}_2(\mathbf{q}) \end{pmatrix} \\ &= -\boldsymbol{M}_{22}^{-1}(\boldsymbol{q})(-\boldsymbol{M}_{21}(\boldsymbol{q})\boldsymbol{M}_3^{-1}(\boldsymbol{q})\boldsymbol{M}_3(\boldsymbol{q}) \\ &\quad \left(\boldsymbol{I}_3 - \boldsymbol{\alpha}\boldsymbol{M}_{22}^{-1}(\boldsymbol{q})\boldsymbol{M}_{21}(\boldsymbol{q})\right)^{-1} \\ &\quad \times ((\boldsymbol{\mu}_1\text{-}\ \boldsymbol{\alpha}\boldsymbol{M}_{22}^{-1}(\boldsymbol{q})\boldsymbol{C}_{21}(\boldsymbol{q},\dot{\boldsymbol{q}}))\dot{\boldsymbol{q}}_1 \\ &\quad + (\boldsymbol{\mu}_2\text{-}\boldsymbol{\alpha}\boldsymbol{M}_{22}^{-1}(\boldsymbol{q})\boldsymbol{C}_{22}(\boldsymbol{q},\dot{\boldsymbol{q}}))\dot{\boldsymbol{q}}_2 \\ &\quad -\boldsymbol{\alpha}\boldsymbol{M}_{22}^{-1}(\boldsymbol{q})\boldsymbol{G}_2(\boldsymbol{q})) \\ &\quad +\boldsymbol{C}_{21}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}_1 + \boldsymbol{C}_{22}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}_2 + \boldsymbol{G}_2(\boldsymbol{q})) \end{aligned} \quad (17)$$

Alternatively, it can be summarized as follows:

$$\ddot{\mathbf{q}}_2 = \mathbf{P}_1(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_1 + \mathbf{P}_2(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_2 + \mathbf{P}_3(\mathbf{q}) \quad (18)$$

where

$$\mathbf{P}_1(\mathbf{q},\dot{\mathbf{q}}) = (\mathbf{M}_{22}^{-1}\mathbf{M}_{21}(\mathbf{q})\left(\mathbf{I}_3 - \boldsymbol{\alpha}\mathbf{M}_{22}^{-1}(\mathbf{q})\mathbf{M}_{21}(\mathbf{q})\right)^{-1} \times (\boldsymbol{\mu}_1 \text{-} \boldsymbol{\alpha}\mathbf{M}_{22}^{-1}(\mathbf{q})\mathbf{C}_{21}(\mathbf{q},\dot{\mathbf{q}})) \text{ - } \mathbf{M}_{22}^{-1}(\mathbf{q})\mathbf{C}_{21}(\mathbf{q},\dot{\mathbf{q}}))$$

$$\mathbf{P}_2(\mathbf{q},\dot{\mathbf{q}}) = (\mathbf{M}_{22}^{-1}\mathbf{M}_{21}(\mathbf{q})\left(\mathbf{I}_3 - \boldsymbol{\alpha}\mathbf{M}_{22}^{-1}(\mathbf{q})\mathbf{M}_{21}(\mathbf{q})\right)^{-1} \times (\boldsymbol{\mu}_1 \text{-} \boldsymbol{\alpha}\mathbf{M}_{22}^{-1}(\mathbf{q})\mathbf{C}_{22}(\mathbf{q},\dot{\mathbf{q}})) \text{ - } \mathbf{M}_{22}^{-1}(\mathbf{q})\mathbf{C}_{22}(\mathbf{q},\dot{\mathrm{q}}))$$

$$\mathbf{P}_3(\mathbf{q}) = -(\mathbf{M}_{22}^{-1}\mathbf{M}_{21}(\mathbf{q})\left(\mathbf{I}_3 - \boldsymbol{\alpha}\mathbf{M}_{22}^{-1}(\mathbf{q})\mathbf{M}_{21}(\mathbf{q})\right)^{-1} \times \boldsymbol{\alpha}\mathbf{M}_{22}^{-1}(\mathbf{q}) + \mathbf{M}_{22}^{-1}(\mathbf{q}))\mathbf{G}_2(\mathbf{q})$$

Because the controller obtained in equation (12) must ensure that all state variables of the system reach the sliding surface $\mathbf{w} = 0$ within a finite time, we consider the case where trajectories reach the sliding surface $\mathbf{w} = 0$. Using equation (10) with $\dot{\mathbf{q}}_{1d} = 0, \mathbf{w} = 0$ we have:

$$\dot{\mathbf{q}}_1 + \boldsymbol{\mu}_1(\mathbf{q}_1 - \mathbf{q}_{1d}) + \boldsymbol{\alpha}\dot{\mathbf{q}}_2 + \boldsymbol{\mu}_2\mathbf{q}_2 = 0 \quad (19)$$

Equivalent to:

$$\dot{\mathbf{q}}_1 = -\boldsymbol{\mu}_1(\mathbf{q}_1 - \mathbf{q}_{1d}) - \boldsymbol{\alpha}\dot{\mathbf{q}}_2 - \boldsymbol{\mu}_2\mathbf{q}_2 \quad (20)$$

Let's denote the state variables as follows:

$$\mathbf{x} = [\mathbf{x}_1 \quad \mathbf{x}_2 \quad \mathbf{x}_3]^{\mathrm{T}} = [\mathbf{q}_2 \quad \dot{\mathbf{q}}_2 \quad \mathbf{q}_1 - \mathbf{q}_{1d}]^{\mathrm{T}} \quad (21)$$

Then,

$$\dot{\mathbf{x}}_1 = \dot{\mathbf{q}}_2 = \mathbf{x}_2 \quad (22)$$

$$\dot{\mathbf{x}}_2 = \mathbf{P}_1(\mathbf{x})\dot{\mathbf{x}}_3 + \mathbf{P}_2(\mathbf{x})\mathbf{x}_2 + \mathbf{P}_3(\mathbf{x}) \quad (23)$$

$$\dot{\mathbf{x}}_3 = -\boldsymbol{\mu}_1\mathbf{x}_3 - \boldsymbol{\alpha}\mathbf{x}_2 - \boldsymbol{\mu}_2\mathbf{x}_1 \quad (24)$$

Substitute (24) into (23):

$$\begin{aligned}\dot{\mathbf{x}}_2 &= \mathbf{P}_1(\mathbf{x})(-\boldsymbol{\mu}_1\mathbf{x}_3 - \boldsymbol{\alpha}\mathbf{x}_2 - \boldsymbol{\mu}_2\mathbf{x}_1) + \mathbf{P}_2(\mathbf{x})\mathbf{x}_2 + \mathbf{P}_3(\mathbf{x}) \\ &= -\mathbf{P}_1(\mathbf{x})\boldsymbol{\mu}_2\mathbf{x}_1 - \mathbf{P}_1(\mathbf{x})\boldsymbol{\alpha}\mathbf{x}_2 - \mathbf{P}_1(\mathbf{x})\boldsymbol{\mu}_1\mathbf{x}_3 + \mathbf{P}_2(\mathbf{x})\mathbf{x}_2 + \mathbf{P}_3(\mathbf{x}) \quad (25) \\ &= -\mathbf{P}_1(\mathbf{x})\boldsymbol{\mu}_2\mathbf{x}_1 + (\mathbf{P}_2(\mathbf{x}) - \mathbf{P}_1(\mathbf{x})\,\boldsymbol{\alpha})\mathbf{x}_2 - \mathbf{P}_1(\mathbf{x})\boldsymbol{\mu}_1\mathbf{x}_3 + \mathbf{P}_3(\mathbf{x}) \\ &= \mathbf{h}(\mathbf{x})\end{aligned}$$

From (21)-(25) one can write:

$$\dot{\mathbf{x}} = \begin{bmatrix} \mathbf{x}_2 \\ \mathbf{h}(\mathbf{x}) \\ -\boldsymbol{\mu}_1\mathbf{x}_3 - \boldsymbol{\alpha}\mathbf{x}_2 - \boldsymbol{\mu}_2\mathbf{x}_1 \end{bmatrix} = \mathbf{f}(\mathbf{x}) \quad (26)$$

Linearize the system at equilibrium point $\mathbf{x} = 0$ (or $\mathbf{q} = \mathbf{q}_d$), we can derive a corresponding linear system as follows:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} \quad (27)$$

where:

$$\mathbf{A} = \left.\frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}}\right|_{x=0} = \left.\begin{bmatrix} 0_{2\times2} & \mathbf{I}_{2\times2} & 0_{2\times3} \\ \dfrac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}_1} & \dfrac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}_2} & \dfrac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}_3} \\ -\boldsymbol{\mu}_2 & -\boldsymbol{\alpha} & -\boldsymbol{\mu}_1 \end{bmatrix}\right|_{x=0} = \begin{bmatrix} 0_{2\times2} & \mathbf{I}_{2\times2} & 0_{2\times3} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \mathbf{A}_{23} \\ -\boldsymbol{\mu}_2 & -\boldsymbol{\alpha} & -\boldsymbol{\mu}_1 \end{bmatrix}$$

After some computational steps, we obtain the matrices $\mathbf{A}_{21}, \mathbf{A}_{22}, \mathbf{A}_{23}$ as:

$$\mathbf{A}_{21} = -\begin{bmatrix} \dfrac{\mu_{11}\mu_{21} + g}{l_d - \alpha_{21}} & 0 \\ 0 & \dfrac{\mu_{12}\mu_{22} + g}{l_d - \alpha_{22}} \end{bmatrix} \quad \mathbf{A}_{22} = -\begin{bmatrix} \dfrac{\mu_{11}\alpha_{21} - \mu_{21}}{l_d - \alpha_{21}} & 0 \\ 0 & \dfrac{\mu_{12}\alpha_{22} - \mu_{22}}{l_d - \alpha_{22}} \end{bmatrix}$$

$$\mathbf{A}_{23} = -\begin{bmatrix} \dfrac{\lambda_{11}^2}{l_d - \alpha_{21}} & 0 & 0 \\ 0 & \dfrac{\lambda_{12}^2}{l_d - \alpha_{22}} & 0 \end{bmatrix}$$

For the system $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ to be stable, the linearized matrix $\mathbf{A}$ must be Hurwitz. The characteristic polynomial of $\mathbf{A}$:

$$\begin{aligned}&\det(s\mathbf{I}_7 - \mathbf{A}) \\ &= \det\left[\begin{array}{cc|c} s\mathbf{I}_2 & -\mathbf{I}_2 & 0_{2\times3} \\ -\mathbf{A}_{21} & s\mathbf{I}_2 - \mathbf{A}_{22} & -\mathbf{A}_{23} \\ \hline \boldsymbol{\mu}_2 & \boldsymbol{\alpha} & s\mathbf{I}_3 + \boldsymbol{\mu}_1 \end{array}\right] \\ &= |s\mathbf{I}_3 + \boldsymbol{\mu}_1| \left| \begin{bmatrix} s\mathbf{I}_2 & -\mathbf{I}_2 \\ -\mathbf{A}_{21} & s\mathbf{I}_2 - \mathbf{A}_{22} \end{bmatrix} - \begin{bmatrix} 0_{2\times3} \\ -\mathbf{A}_{23} \end{bmatrix} (s\mathbf{I}_3 + \boldsymbol{\mu}_1)^{-1}[\boldsymbol{\mu}_2 \quad \boldsymbol{\alpha}] \right| \\ &= (s + \lambda_{13})\left(s^3 + \frac{(\mu_{12}l_d - \mu_{22})}{(l_d - \alpha_{22})}s^2 + \frac{g}{(l_d - \alpha_{22})}s + \frac{g\mu_{12}}{(l_d - \alpha_{22})} \right) \\ &\quad \times \left(s^3 + \frac{(\mu_{11}l_d - \mu_{21})}{(l_d - \alpha_{21})}s^2 + \frac{g}{(l_d - \alpha_{21})}s + \frac{g\mu_{11}}{(l_d - \alpha_{21})} \right) \\ &= (s + \lambda_{13})P(s).Q(s)\end{aligned} \quad (28)$$

where

$$P(s) = \left(s^3 + \frac{(\mu_{12}l_d - \mu_{22})}{(l_d - \alpha_{22})}s^2 + \frac{g}{(l_d - \alpha_{22})}s + \frac{g\mu_{12}}{(l_d - \alpha_{22})} \right)$$

$$Q(s) = \left(\begin{array}{c} s^3 + \dfrac{(\mu_{11} l_d - \mu_{21})}{(l_d - \alpha_{21})} s^2 \\ + \dfrac{g}{(l_d - \alpha_{21})} s + \dfrac{g\mu_{11}}{(l_d - \alpha_{21})} \end{array} \right)$$

The sufficient condition for stabilizing the sliding surface is that the functions $P(s)$ and $Q(s)$ must be Hurwitz. Then,

$$\mu_{21} < (l_d - 1)\mu_{11};\ \mu_{22} < (l_d - 1)\mu_{12};\ \alpha_{21} < l_d;\ \alpha_{22} < l_d \qquad (29)$$

Comment 2: Since the system (27) is asymptotically stable, $\mathbf{x}$ converges to 0. Therefore, from (21), $\mathbf{q}_1$ converges to $\mathbf{q}_{1d}$, and $\mathbf{q}_2$, $\dot{\mathbf{q}}_2$ converge to 0. Additionally, from equation (20), it is observed that $\dot{\mathbf{q}}_1$ converges to 0. Thus, the necessary condition has been satisfied, and the control law (12) ensures that the system states reach the sliding surface $\mathbf{w} = 0$. The constraints given in equation (28) are sufficient conditions for $\mathbf{q}_1$, $\dot{\mathbf{q}}_1$, $\mathbf{q}_2$ và $\dot{\mathbf{q}}_2$ to reach the desired values.

4. SIMULATION RESULTS

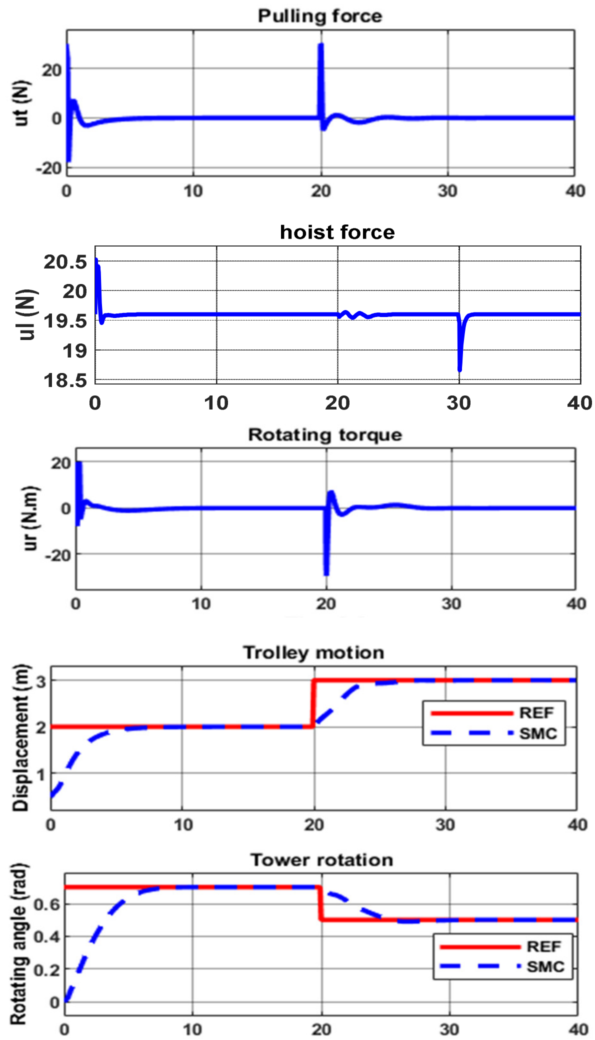
The parameters of the tower crane model and the controller are provided as follows: $x_0 = 0.5$m, $\gamma_0 = 0$rad, $l_0 = 0.5$m, $m_t = 8$kg, $m_c = 2$kg, $g = 9.8$m/s², $J = 6.8\text{kg.m}^2$, $\mu_{11} = 0.5$, $\mu_{12} = 0.5$, $\mu_{13} = 0.5$, $\mu_{21} = -3$, $\mu_{22} = -3$, $\alpha_{21} = 0.12$, $\alpha_{22} = 0.12$, $K_1 = 20$, $K_2 = 30$, $K_3 = 20$. To demonstrate the stability and robustness advantages of the controller, the system response is simulated for two cases. First case: In the initial 20 seconds, the trolley is controlled to move 2 meters, and the crane is controlled to rotate to 0.7 radians. In the subsequent 20 seconds, the setpoint for the trolley is changed to 3 meters, and the setpoint for the crane rotation is changed to 0.5 radians. In the first 30 seconds, the cable length is continuously changed to 4 meters and then kept constant for the last 10 seconds. Second case: The payload mass m_c is increased to 200% ($m_c = 3$kg), while keeping all other system parameters the same as in the first case. Through the simulation results, it can be observed that:

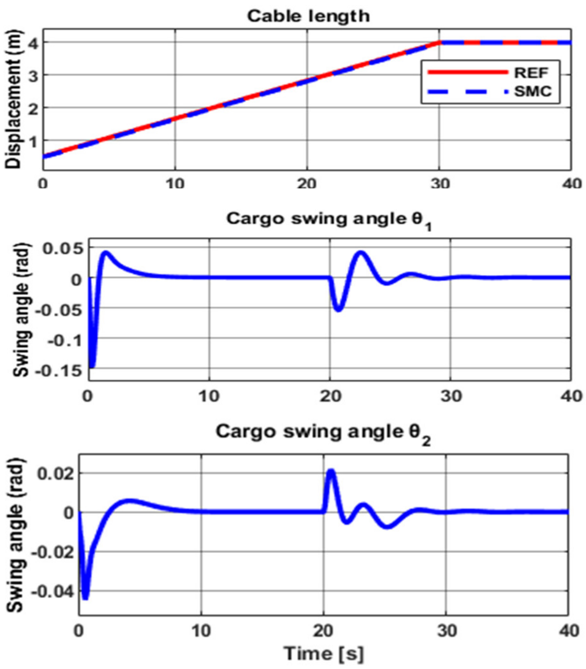
In case 1: As depicted in Fig. 2(a), under the influence of the proposed controller, the trolley, crane, and cable length can closely track the reference trajectories and achieve the desired positions accurately, while effectively attenuating load oscillations. The stability of the system is also verified when simultaneously changing the setpoints of all three movements. Specific results:

▪ In the first 20 seconds, the responses of the trolley and crane rotation achieve their desired values at t = 5.7s, t = 6.8s, respectively; the cable length response closely follows the setpoint signal. The angles θ_1, θ_2 converge to 0 radians at t = 6s and 8s, respectively, with the peak amplitudes of the maximum angle of θ_1 is 0.146 radians and θ_2 is 0.045 radians.

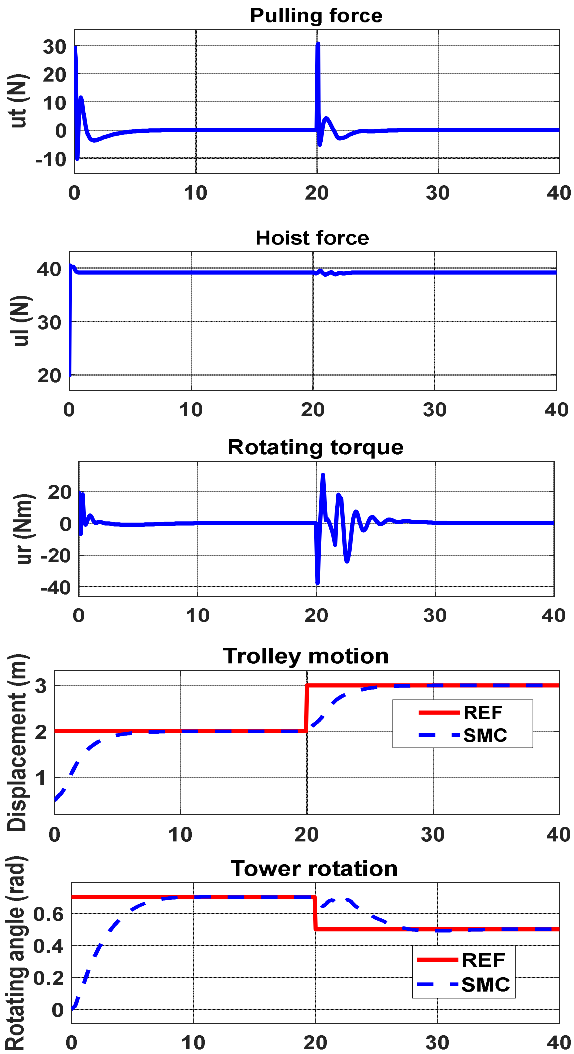
▪ In the next 20 seconds, when the oscillations of all three movements converge, the responses of the trolley and crane rotation achieve their desired values at t = 5.9s and t = 7.1s, respectively; the cable length response closely follows the setpoint signal. The angles θ_1, θ_2 converge to 0 radians at t = 8s and t = 8.2s, respectively, with the peak amplitudes of θ_1 is 0.054 radians and of θ_2 is 0.021 radians.

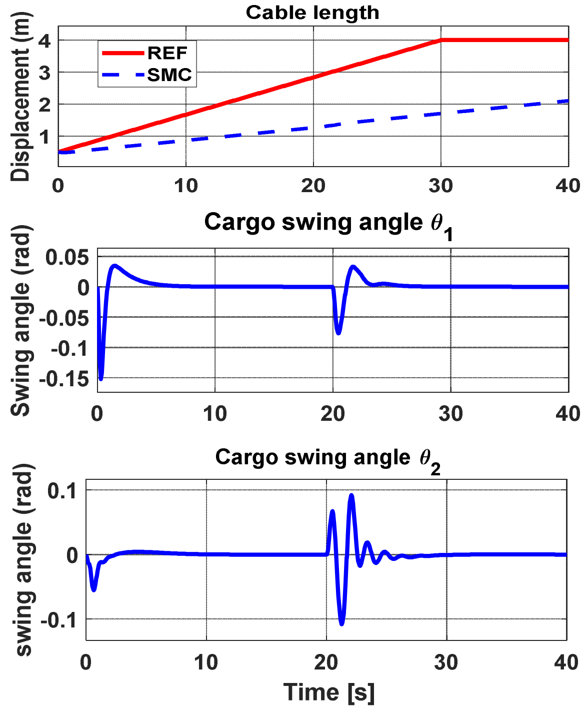
With this controller, the control variables follow the reference trajectory accurately and achieve the desired position, minimizing load fluctuations. Additionally, the system's stability is demonstrated when all three movements are changed simultaneously. From this, it can be concluded that the controller demonstrates good stability capabilities.





(a)





(b)

Fig. 2. Simulation results (a) payload mass $m_c = 2$kg (b) payload mass $m_c = 4$kg

In case 2: In practical applications, the effectiveness of a controller not only relies on its stability but also its robustness in handling parameter uncertainties, such as uncertain payload mass. To evaluate the robustness of the system, a simulation was conducted to examine the system's response when the payload mass is increased by 50% and 100% compared to the original payload. When the payload mass is increased by 50% (m_c = 3kg), the system response is almost the same with the case of original payload. This shows the robustness of the SMC controller. When the payload mass is increased by 100% (m_c = 4kg), as shown in Fig. 2b, there are some differences with the case of original payload. The system is still stable, however, the performance is decreased significantly. The payload swing angle increases and the position tracking of cable length is reduced. It means that the system exhibits a satisfactory level of robustness against parameter variations in a limited range. When the payload mass is increased to m_c = 5kg, the system becomes unstable. Thus, the adaptation of system parameters should be considered carefully.

Comment 3: The simulation parameters are chosen from the experimental tower crane that has been building in the author's laboratory. The controller parameters are selected by trial and error method to obtain good tracking, less payload vibration, no overshoot, and the control input is in the range from -30N to 30N for translation motion and -30Nm to 30Nm for rotation motion. The simulation results demonstrate that the tower crane system with the proposed controller performs well with uncertain parameters and unknown variations to some extent. Currently, we are unable to conduct hardware experiments due to testing constraints. In future work, we will continue to deploy hardware experiments to further verify the robustness of the proposed controller.

5. CONCLUSION

The paper presents a sliding mode controller for an under-actuated 5-degree-of-freedom tower crane with varying rope lengths. The system has been proven to be stable with the proposed controller. The position tracking for the trolley and rope length are achieved. Moreover, the variant frequency vibration of the payload is suppressed. The simulation also shows the robustness and effectiveness of the proposed controller.

In the future, developing improved controllers will be an important research direction in the presence of uncertain disturbances. Improving the efficiency of the algorithm and further minimizing transport time and energy consumption of the system will be considered. Additionally, the influence of uncertainty and disturbances on the system will also be considered to further enhance the system's robustness.

REFERENCES

[1]. Nguyen Van Hung, et. al., *Construction machines*. Science and Technics Publishing House, Hanoi, 2021. (in Vietnamese)

[2]. https://buildtech.com.vn/ctevn/index.php/gioi-thieu/415-gioi-thieu-cau-thap.

[3]. Zhaoqing Liu, et. al., *Multi-objective trajectory planning for 5-DOF underactuated tower cranes with state constraints*. Science China Press, 2022.

[4]. Ho Viet Long, Duong Minh Duc, "Anti-vibration control for tower cranes," in *the 3rd Vietnam International Conference and Exhibition on Control and Automation*, 2015. (in Vietnamese)

[5]. Nguyen Thuc Trang, Le Duc Long, "Researching dynamics of cable system for tower crane type horizontal jib during lifting and reaching changes," *Transport Magazine*, 2016. (in Vietnamese)

[6]. A.T. Le, S.G. Lee, "3D cooperative control of tower cranes using robust adaptive," *Journal of the Franklin Institute*, 354(18), 2017.

[7]. T.S. Wu, M.A. Karkoub, H. Wang, H.S. Chen, T.H. Chen, "Robust tracking control of MIMO underactuated nonlinear systems with dead-zone band and delayed uncertainty using an adaptive fuzzy control," *IEEE Transactions on Fuzzy Systems*, 25, 2017.

[8]. Y. Qian, Y. Fang, "Switching logic-based nonlinear feedback control of offshore ship-mounted tower cranes: A disturbance observer-based approach," *IEEE Transactions on Automation Science and Engineering*, 2019.

[9]. N. Sun, Y. Wu, H. Chen, Y. Fang, "Antiswing cargo transportation of underactuated tower crane systems by a nonlinear controller embedded with an integral term," *IEEE Transactions on Automation Science and Engineering*, 16(3), 2019.

[10]. Š. Ileš, J. Matuško, F. Kolonić, "Sequential distributed predictive control of a 3D tower crane," *Control Engineering Practice,* 79, 2018.

[11]. M. Bock, A. Kugi, "Real-time nonlinear model predictive path-following control of a laboratory tower crane," *IEEE Transactions on Control Systems Technology*, 22(4), 2014.

[12]. M. Zhang, Y. Zhang, B. Ji, C. Ma, X. Cheng, "Adaptive sway reduction for tower crane systems with varying cable lengths," *Automation in Construction*, 119, 2020.

[13]. H. Chen, Y. Fang, N. Sun, "An adaptive tracking control method with swing suppression for 4-DOF tower crane systems," *Mechanical Systems and Signal Processing*, 123, 2019.

[14]. Hanafy M. Omar, Ali H. Nayfeh, *Simple adaptive Feedback Controller for the Tower*. Engineering Science and Mechanics Virginia polytechnic Institute and State University, 2001.

[15]. Florentin Rauscher và Oliver Sawodny, "Modeling and Control of Tower Cranes With Elastic Structure," *IEEE Transactions on Control Systems Technology*, 2020.

[16]. David Blackburn, et. al. , "Radial-motion assisted command shapers for nonlinear tower crane rotational slewing," *Control Engineering Practice*, 2010.

[17]. G. Boschetti, et. al., "Moving the suspended load of an overhead crane along a pre-specified path: A non-time based approach," *Robotics & Computer-Integrated Manufacturing*, 30, 3, 2014.

[18]. José Luis Jaime, Fabrizio Leonardi, "Minimum-time anti-swing motion planning of cranes using linear programming," *Optimal Control Applications and Methods*, 2013.

[19]. Zhuoqing Liu, Ning Sun, Yiming Wu, Xin Xin, Yongchun Fang, "Nonlinear Sliding Mode Tracking Control of Underactuated Tower," *International Journal of Control, Automation and Systems,* 19, 2021.

[20]. Tran Van Hao, *Control of an inverted pendulum.* Master's thesis, Ho Chi Minh City University of Technology, 2017.

[21]. Naif B. Almutairi, Mohamed Zribi, "Sliding Mode Control of a Three-dimensional Overhead Crane," *Journal of Vibration and Control*, 15, 11, 8-9, 2009.

THÔNG TIN TÁC GIẢ

Nguyễn Hữu Hải[1], Bùi Tùng Dương[2], Nguyễn Thị Trang[2], Quách Đức Cường[1], Dương Minh Đức[2]

[1]Khoa Điện, Trường Đại học Công nghiệp Hà Nội

[2]Trường Điện - Điện tử, Đại học Bách khoa Hà Nội