TOWARD A PHYSICS-BASED MODEL OF POWER COEFFICIENT IN HORIZONTAL-AXIS WIND TURBINE

ĐỀ XUẤT MỘT MÔ HÌNH VẬT LÝ VỀ HỆ SỐ CÔNG SUẤT TUABIN GIÓ TRỤC NGANG

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ABSTRACT

Global energy is in a major shift from energy using fossil sources to renewable energy using more sustainable energy sources. The wind is the clean and inexhaustible one of renewable energy resources that is available in most parts of the world. In order for wind power generation to meet more ambitious targets around the world, it is necessary to understand all the physics behind processes of electrical generation from wind. In this paper, an analytical physics-based method is developed to estimate the generated power and coefficient of a horizontal-axis wind turbines (HAWT). A special point in this paper is the construction of power coefficients, which compare with coefficients of various wind generation systems, that fitted as functions from actual data measured from popular wind turbines in the world.

Keywords: *Wind power generation; horizontal-axis wind turbine, power coefficient.*

TÓM TẮT

Năng lượng toàn cầu đang trong giai đoạn thay đổi lớn từ năng lượng sử dụng các nguồn hóa thạch sang năng lượng tái tạo sử dụng các nguồn năng lượng bền vững hơn. Gió là một trong những nguồn năng lượng tái tạo sạch và vô tận có sẵn ở hầu hết các nơi trên thế giới. Để sản xuất điện gió đáp ứng các mục tiêu đầy tham vọng hơn trên toàn thế giới, cần phải hiểu tất cả yếu tố vật lý đằng sau các quá trình sản xuất điện từ gió. Trong bài báo này, một phương pháp dựa trên cơ sở phân tích vật lý được phát triển để tính công suất và hệ số công suất của tuabin gió trục ngang (HAWT). Một điểm đặc biệt trong bài báo này là việc xây dựng các hệ số công suất, có so sánh với các hệ số của các hệ thống phát điện gió khác nhau, được tổng quát hoá như các hàm từ dữ liệu được đo thực tế từ các tuabin gió phổ biến trên thế giới.

Từ khóa: *Sản xuất điện gió; tuabin gió trục ngang, hệ số công suất.*

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1. INTRODUCTION

The increasing growth in global electricity demand leads to environmental degradation, manifested by higher carbon dioxide (CO_2) emissions and increased greenhouse effect [1, 2]. Humans' high dependence on fossil fuels, their resulting involvement in "climate change," and their increasing costs have spurred interest in alternative forms of electricity generation. depends on this type of fuel [3]. These alternative forms are known as renewable energy sources and they now have led to diversification in energy production [4, 5]. Popular today of these new forms include wind energy and solar energy.

Although wind is a clean and inexhaustible resource available in most parts of the world, however, there are specific regions where this natural resource is abundant. Therefore, many different statistical methods have been developed to calculate the average wind speed, wind energy density and its load factor for a specific geographical area [6]. This allows the identification of suitable areas for the installation of wind farms of the desired capacity. Based on these methods, many projects have been implemented in many countries. Vietnam is no exception to this trend, aiming to comply with the Kyoto Protocol and achieve zero CO_2 emissions by 2050, and even has incentives to develop these alternative energy sources.

The process of converting energy from wind into electricity begins with the wind colliding with the turbine blades to rotate the generator. In fact, wind has a random nature, so the voltage amplitude and frequency generated by the generator change continuously. To regulate the varying voltages of the generator and supply the loads with amplitude and frequency values according to the regulations of a given country, an electronic system is used to perform this conversion [7, 8]. From there, to evaluate the performance of a wind turbine, one of the most important parameters is the power coefficient (C_p), established by the International Electrotechnical Commission (IEC) [9]. Power coefficient refers to the relationship between the actual produced power and the energy from the surrounding wind transmitted towards the turbine blades [10]. Power coefficient is one of the parameters needed to calculate the production efficiency of a wind turbine or a wind farm, along with parameters such as air turbulence, temperature and density. At the same time, it has been established that the power coefficient is one of the most important parameters that distinguish one wind turbine from another. Therefrom, many statistical data from many parts of the world are used to analyze all these parameters and make the design and operation of wind turbines more efficient [11, 12]. These statistics gave the mathematical functions used to represent the behavior of the power coefficient in relation to the specific speed and the blade's angle of wind attack, and then established the general formulas for the analysis and compare better.

This article focuses on the analysis of horizontal-axis and three-blades turbines because they are the most common types. This is structured as follows. Start by displaying the characteristics of a wind turbine in order to identify the most important parameters that determine its operation and give each turbine its particular characteristics in the wind energy conversion process. A physics-based analytical model is established based on the Weibull wind distribution. The analytical expression of the power coefficient was obtained. The following part is to determine the physical parameters in this expression of the power coefficient in comparison with fitted functions of the exponential type obtained from experimental data of popular wind turbines in the world. Finally, an evaluation discussion is conducted based on the actual cases presented in the reference data.

2. THE EXPERIMENT-BASED MODELS

Wind is air movement in the Earth's atmosphere. Wind energy is the kinetic energy of air moving at speed v.

$$E_{kin} = \frac{1}{2} mv^2 \quad (1)$$

where, the mass of air m passing through a circular plane perpendicular to the wind direction during time t is

$$m = \rho V = \rho Avt \quad (2)$$

here, ρ the air density, V is the volume of air mass passing through the circular cross-section of the swept area covered by the blades of the turbine $A = \pi R^2$, and R is radius of rotor (Fig. 1).

Therefore, the kinetic energy E_{kin} and power P_w of the wind are

$$E_{kin} = \frac{1}{2} \rho Atv^3 \quad (3)$$

$$P_w = \frac{dE_{kin}}{dt} = \frac{1}{2} \rho Av^3 \quad (4)$$

It is worth noting that wind power increases with the third power of wind speed and therefore wind speed is one of the deciding factors when wanting to use wind energy.

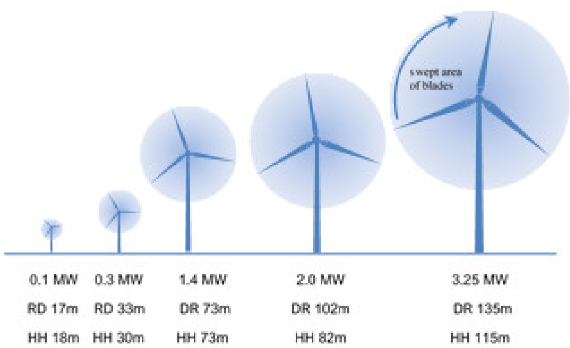

Fig. 1. The swept area of blades corresponding to size of wind turbines [13]

Wind power can be used, for example, through a wind turbine to generate electricity, but the energy produced is much less than the energy of the wind flow because of the speed of the wind behind a turbine. cannot be reduced to zero. In theory, a maximum of 59.3% of the energy existing in the wind can be extracted (called Betz's Law, discovered by Albert Betz in 1919 [14]). The value of the ratio between the power extracted from the wind and the wind power is called the power coefficient (C_p). Therefore, the power in the turbine (P_t) is expressed by

$$P_t = \frac{1}{2} \rho Av^3 C_p \quad (5)$$

The power coefficient (C_p) depends on several parameters, such as the type of turbine (horizontal or vertical axis), the number of blades (in the case of a horizontal axis), the specific speed or tip speed ratio ($\lambda = R\omega/v$, where ω is the rotational speed of the turbine), and the pitch angle of the blades (θ), see Fig. 2.

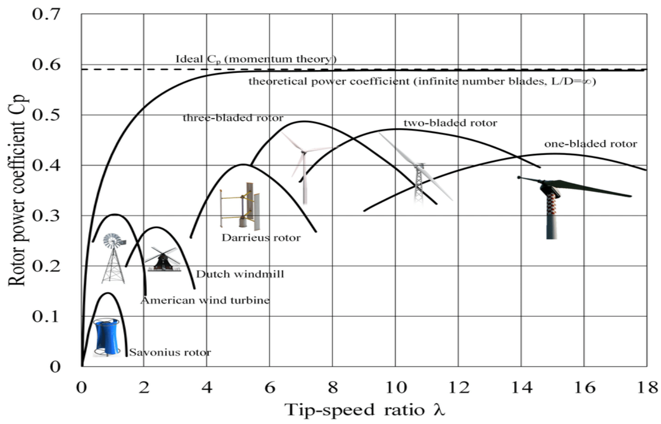

Fig. 2. The dependence of power coefficient on λ and types of wind turbine [15]

In fact, $C_p(\lambda, \omega)$ are different depending on the case of commercial wind turbines. Normally, manufacturers measure and provide documentation where the C_p behavior are plotted as graphs. These graphs, then, are used to obtain mathematical approximations (i.e. by fitting to functions) that allows us to understand their behavior in an analytical formula. These approximations are carried out by means of optimal numerical methods [16], almost are expressed through three mathematical forms:

- Polynomial functions
- Sinusoidal functions
- Exponential functions

The power coefficients based on polynomial functions depend only on λ, since θ is a constant, and they are used normally in low power wind turbines where control of θ is not applied. In addition, in some cases it is necessary to limit λ to make the power coefficient is viable and valid. For values of λ larger than the limit, this coefficient does not occur in a real turbine [17].

For power coefficients based on sinusoidal functions, there is also a disadvantage that the power coefficient graph does not describe the real behavior of the wind turbine, so it is necessary to limit the value λ for each power coefficient. Additionally, some power coefficients have a problem for high values of both λ and θ, because, as the C_p value decreases, it increases again, which does not happen with real wind turbines. In some other cases, the power coefficients present an adequate behavior, but they tend to infinity through the variation of λ and θ [18].

Regarding the coefficients based on exponential functions, significant variety is observed, but all start at zero for a θ of zero, which does not occur in most cases of the coefficients based on polynomial and sinusoidal functions. Moreover, they all reach a maximum value and then descend until reaching zero at different values of θ, thus achieving the real behavior of a turbine. In addition, the behavior observed for the coefficient of torque is also very close to reality [19-26].

All power coefficients expressed in exponential form are most suitable for real wind turbines and are grouped to the general equation which is represented by

$$C_p(\lambda,\theta) = c_0\left(\frac{c_1}{\tilde{\lambda}} - c_2\theta - c_3\theta\tilde{\lambda} - c_4\tilde{\lambda}^{c_5} - c_6\right)e^{-\frac{c_7}{\tilde{\lambda}}} + c_8\lambda \quad (6)$$

where

$$\frac{1}{\tilde{\lambda}} = \frac{1}{\lambda + c_9\theta + c_{10}} - \frac{c_{11}}{1+\theta^3} \quad (7)$$

and the parameters c_i (i = 0, 1, 2, … 11) of this expression are identified by the papers [19-26] and summarized in [27]. They are shown in Table 1.

Table 1. Constants of the power coefficients in the exponential functions

Refs	c_0	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}
[19]	0.5	116	0	0.4	0	0	5	21	0	0.08	0	0.035
[20]	0.5	116	0.4	0	0	0	5	21	0	0	0.088	0.035
[21]	0.518	116	0.4	0	0	0	5	21	0.007	0.08	0	0.035
[22]	0.22	116	0.4	0	0	0	5	12.5	0	0.08	0	0.035
[23]	0.5	72.5	0.4	0	0	0	5	13.13	0	0.08	0	0.035
[24]	0.73	151	0.58	0	0.002	2.14	13.2	18.4	0	0.02	0	0.003
[25]	0.44	125	0.4	0	0	0	6.94	17.05	0	0.08	0	0.001
[26]	1	110	0.4	0	0.002	2.2	9.6	18.4	0	0.02	0	0.03

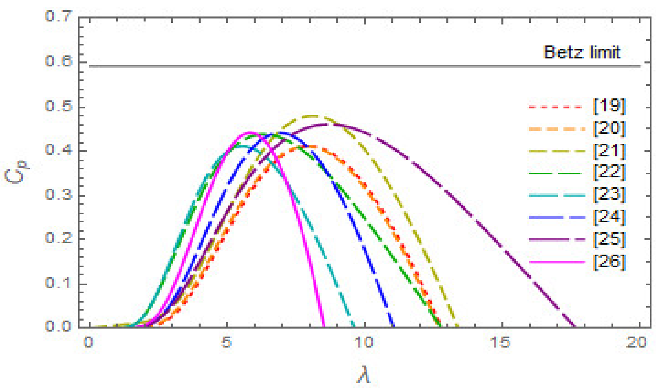

Fig. 3. The power coefficients based on an exponential function of [19-26]

These eight power coefficients as the functions depending on λ with θ equal to zero are shown in Fig. 3. It can be seen that none of the coefficients exceed the Betz limit. All the power coefficients have adequate compression because it starts from zero and increases its value until reaching a maximum value and then decreases to zero as λ increases.

All above functions describing the power coefficient are fitted from experimental data. Below we will try to find this function based on physics and the meaning of the parameters.

3. THE PHYSICS-BASED MODEL

Based on physics, the received power in the turbine (P_t) can be expressed by using an exponential cumulative distribution to the useful wind speed v as follows

$$P_t(v) = \int_0^v P_w(u)f(u)du \quad (8)$$

where

$$P_w(u) = \frac{1}{2}\rho A u^3 \quad (9)$$

is the wind power and

$$f(u) = \frac{1}{\zeta}e^{-\frac{u-\mu}{\zeta}} \quad (10)$$

is the exponential probability density function. Here, ζ is scale factor, and μ is location parameter. Taking the integral (6), we obtain:

$$P_t(v) = \frac{1}{2}\rho A v^3 C_p \quad (11)$$

where

$$C_p = e^{\frac{\mu}{\zeta}}\left[6\frac{\zeta^3}{v^3} - e^{-\frac{v}{\zeta}}\left(1 + 3\frac{\zeta}{v} + 6\frac{\zeta^2}{v^2} + 6\frac{\zeta^3}{v^3}\right)\right] \quad (12)$$

Putting $\zeta = v_t/\chi$, $v_t = \omega R$, χ is a new parameter, and $\mu = \bar{v}$, then

$$\frac{\mu}{\zeta} = \frac{\chi}{\bar{\lambda}} = \eta \quad (13a)$$

$$\frac{v}{\zeta} = \frac{\chi}{\lambda} = y \quad (13b)$$

where $\lambda = v_t/v$, $\bar{\lambda} = v_t/\bar{v}$.

Thus,

$$C_p = e^{\eta}\left[\frac{6}{y^3} - e^{-y}\left(1 + \frac{3}{y} + \frac{6}{y^2} + \frac{6}{y^3}\right)\right] \quad (14)$$

Comparing with (6) and (7), we arrive at

$$y = \alpha\left(\frac{1}{\lambda+\gamma\theta} - \frac{\beta}{1+\theta^3}\right) \quad (15)$$

Thus, instead of 11 parameters in (6) and (7), our model only needs 4 parameters: η, α, β, and γ. Below we will examine the meaning of these four parameters. First, we consider the case $\theta = 0$ and choose a set of four parameters suitable for the above eight semi-empirical models. The results are shown in Fig. 4.

Keeping the three parameters unchanged and changing the remaining parameters, we obtain Figs. 5, 6, 7 (for $\theta = 0$).

Three parameters η, α, β clearly show the physical properties of C_p as well as the wind turbine power P_t. η is related to the magnitude of the maximum value of C_p, that is, related to the maximum power value of the wind turbine. α is related to the increase in C_p as well as the wind turbine power. And β is related to the cutoff point of turbine power.

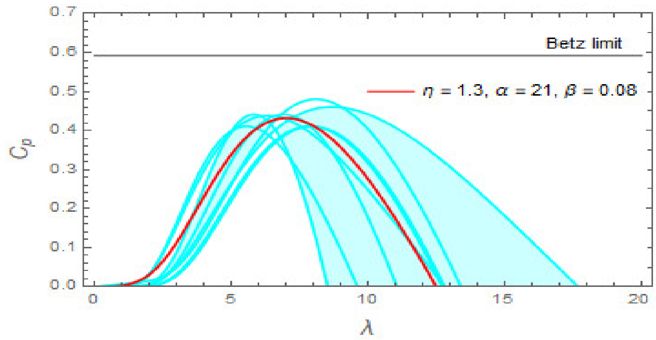

Fig. 4. The power coefficients for $\theta = 0$ based on our physical model. The cyan lines and shading area are exponential functions of Refs. [19-26]

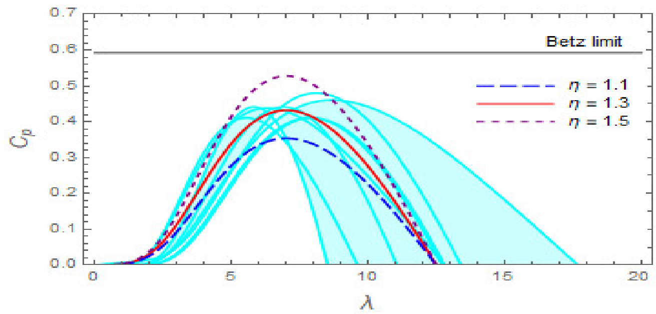

Fig. 5. The power coefficients for $\theta = 0$ based on our physical model as η changes. As η increases, the maximum value of C_p also increases

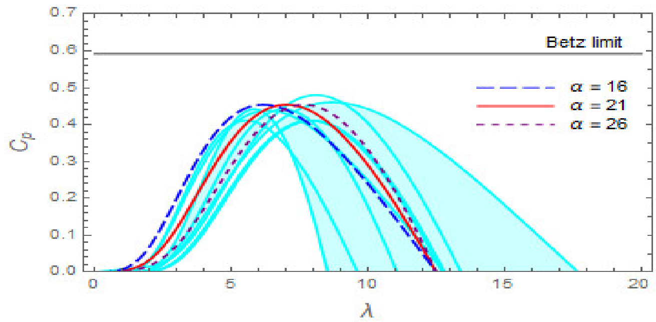

Fig. 6. The power coefficients for $\theta = 0$ based on our physical model as α changes. As α increases, the slope in the small λ region of C_p decreases

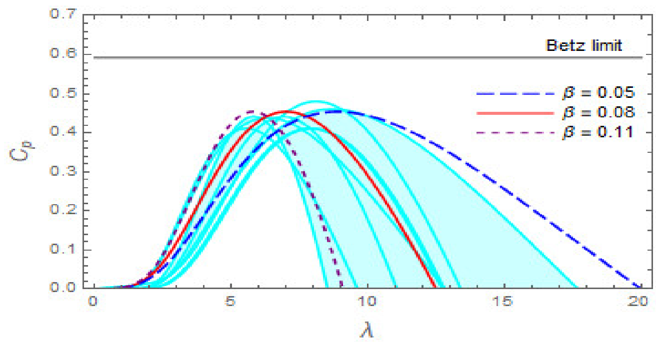

Fig. 7. The power coefficients for $\theta = 0$ based on our physical model as β changes. As β increases, the breakpoint of C_p decreases

Comparing with (6) and (7), we see that

$$\alpha = c_7,\ \ \beta = c_{11},\ \ \gamma = c_9,\ \ e^{\eta} = c_0 c_6 \quad (16)$$

Hence, all the semi-empirical curves obtained by [19-26] can also be obtained from (14) and (15) corresponding to suitable parameter sets of η, α, β, and γ.

By adjusting these four parameters, we can obtain any functional form of C_p. For example, the exponential function of [22] can be obtained from our analytical function corresponding to the parameter sets shown in Fig. 8.

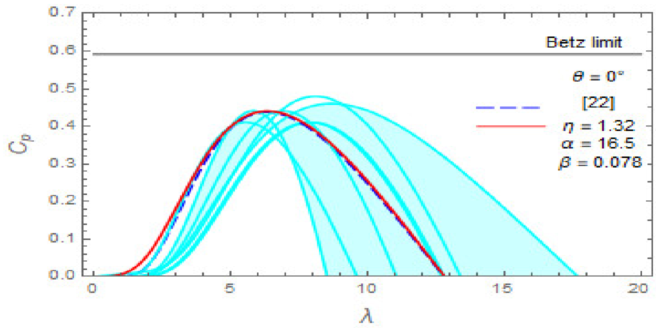

Fig. 8. The power coefficient C_p vs. λ of [22] compared to the our model

4. CONCLUSION

In summary, by using the exponential cumulative distribution function a physical model is derived for the power of the wind turbine, in particular the power coefficient function is found by analytical calculation results. Instead of 11 parameters that have no physical meaning, this analytic function has only 5 parameters that characterize the properties of real wind turbine power, such as maximum value, slope, breakpoint, and pitch angle.

In the article we used the exponential cumulative distribution, to expand it we can use a more general distribution function, such as the Weibull distribution.

$$f(u) = \frac{\kappa}{\zeta}\left(\frac{u-\mu}{\zeta}\right)^{\kappa-1} e^{-\left(\frac{u-\mu}{\zeta}\right)^{\kappa}}$$

With the Weibull distribution function, there is an additional configuration parameter κ. The significance of this parameter is possible for fine adjustments of wind turbine power. The power coefficient function with additional configuration parameters may be a prospective study for us in the future.

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THÔNG TIN TÁC GIẢ

Nguyễn Tuấn Anh, Nguyễn Hữu Đức

Khoa Công nghệ năng lượng, Trường Đại học Điện lực