A STUDY ON THE DESIGN OF A NOVEL CLOSED-CHAIN ROBOT USING THE THEORY OF FINITE SYMMETRY GROUPS

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ABSTRACT

The field of robotics has been a remarkable human invention in the past century. With advancements in science and technology, industries worldwide are striving to enhance and produce cutting-edge robot designs capable of operating at high frequencies, while optimizing cost and efficiency to replace human labor across various domains. This article proposes a novel robot structure based on the theory of finite symmetry groups, aiming to systematically and comprehensively enumerate all closed planar kinematic chains with n links and F degrees of freedom. Initially, a configuration with one degree of freedom and eight links is selected. Then, the study proceeds to establish a set of link parameters by considering the maximum number of links, binary link properties, and the combination of odd numbers of links within the closed kinematic chain. The outcomes demonstrate the development of 16 new robot structures that successfully fulfill the criteria of being both planar and closed chains. These research findings hold practical significance in the realm of designing innovative robot structures that meet production demands, while simultaneously facilitating automation and enhancing overall production efficiency.

Keywords: *robot, kinematic chain, degrees of freedom.*

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1. INTRODUCTION

Since their initial introduction, robots have become increasingly vital, providing numerous benefits in both daily life and production. They have the ability to replace human labor in order to handle large workloads with high accuracy and fast response times. This is especially true in the modern era of industry 4.0, where industrial manufacturing industries are widely adopting robotic technology. As science and technology continue to advance, there is a growing desire to create simplified robot designs that can still perform complex practical tasks. Researchers have shown interest in exploring mechanisms with flat, closed kinematic chain structures as a way to develop new robot designs that are suitable for production purposes. In 2016, Dinh et al. [1] proposed a systematic approach for synthesizing planar non-fractionated kinematic chains with up to six independent loops. This method involves creating contracted graphs, classifying them based on planar and non-planar characteristics, and deriving constraint equations. Yan et al. [2] presented an improved method for constructing generalized kinematic chains using cut-links checking and Kuratowski graph checking algorithms. They provided examples and listed atlases of kinematic chains with up to sixteen links. Venkata et al. [3] introduced a novel algorithm based on graph theory for detecting isomorphism in planar and geared kinematic chains (GKCs). Their algorithm eliminates isomorphic chains without the need for complex calculations or comparisons. Mo-ha Shadab Alam et al. [4] introduced a new method for automatically sketching planar kinematic chains using line graphs. Their goal was to eliminate unnecessary edges in the line graph and correct defective multiple links in the kinematic chain. They defined the concept of the inner angle of a joint and proposed a correction algorithm to eliminate unnatural link crossings and concave angles. In 2018, Sun et al. [5] proposed a novel method for determining isomorphism in planar kinematic chains with multiple joints. Their method utilized a joint-joint matrix to describe the kinematic chain structure, extracted information about links and joints, introduced link and joint codes, and established standardization rules. The efficiency of the method was demonstrated through comparisons of links, joints, and matrices to identify isomorphisms. Rai et al. [6] introduced an algorithm to address the issue of cumbersome links labelling in kinematic chains (KCs), particularly for large KCs. The algorithm was tested on KCs with different types of links, including simple joints, multiple joints, and Epicyclic gear trains (EGTs), demonstrating its efficiency and reliability. In [7], the authors further tested the algorithm on KCs with 8 links, 15 links, and 28 links, proving its feasibility and efficiency. The method introduces improved high-order adjacency link values, calculates them repeatedly, and compares the high-order adjacency link strings from two different KCs. In 2021, Chen [8] presented a simple and efficient method for the structural synthesis of plane kinematic chain inversions without the need for isomorphism detection. The method utilizes the fifth power of the adjacency matrix to identify similar vertices and directly derive non-isomorphic kinematic chain inversions based on non-similar vertices. Huang and Liu [9] proposed a systematic approach to synthesize all possible non-isomorphic fractionated kinematic chains with up to 16 links and up to 6 degrees of freedom (DOFs), combining two or three non-fractionated kinematic chains. They introduced a new isomorphism identification algorithm and provided structural constraint equations to obtain all possible combinations. Helal et al. [10] employed a graphical technique to enumerate available N-bar chains with prismatic joints. They developed a new topological Loop Code to detect isomorphic and rejected KCs, resulting in 21, 16, and 1350 KCs with P-joints. Cui et al. [11] presented an algorithm for synthesizing planar kinematic chains with prismatic pairs (P-pairs), which reduces the number of isomorphic KCs and enhances synthesis efficiency. The method was applied to synthesize KCs with 9 links and 2 DOFs, 10 links and 1 DOF, and 11 links and 2 DOFs, incorporating one or two P-pairs, aligning with existing literature and potentially extending to other kinematic pairs. In [12], the authors analyzed constrained joints in closed chains and employed the recursive Gibbs-Appell methodology to extract motion equations. The recursive formulation allows for the symbolic derivation of motion equations for each chain, regardless of the number of links. Additionally, in [13], a method was proposed to solve the problem of isomorphism identification among kinematic chains based on the distance between non-binary vertices. To ensure the uniqueness and comprehensiveness of the topological structure of kinematic chains, especially with a large number of joints and degrees of freedom, it is necessary to solve this problem regardless of the method used to automatically generate the outline of planar kinematic chains.

Based on this analysis, this study utilizes the theory of finite symmetry groups to create 16 new robot structures that satisfy the conditions of being flat and closed kinematic chains. These structures are based on the input of 8-link and one degree of freedom.

2. MATERIALS AND METHODS

2.1. Franke's Notation

According to [14], a chain is determined by the arrangement of links and their connections. A link refers to a rigid body with joint elements surrounding it. These links are interconnected by lines or bands, forming either a direct coupling (two elements joined to form a kinematic pair) or a chain of one or more binary links. The structure symbol consists of two elements: (1) A circle represents n-links, with a number inside to indicate whether it is a ternary link, quaternary link, or n-link. This number corresponds to the number of matching elements. (2) A connecting line represents the connection between links. Each line signifies one or more binary links. The number on each line indicates the count of binary links in the chain, or zero if it denotes a direct connection.

In Fig. 1, we observe a 12-link chain with 1 degree of freedom. Fig. 2 showcases the chain depicted using the corresponding symbols.

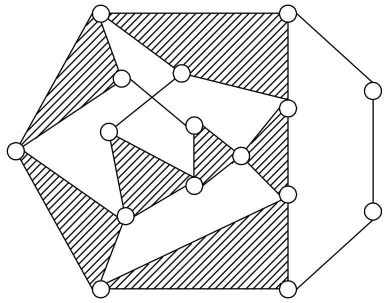


Fig. 1. Chain of 12 links with 1 degree of freedom

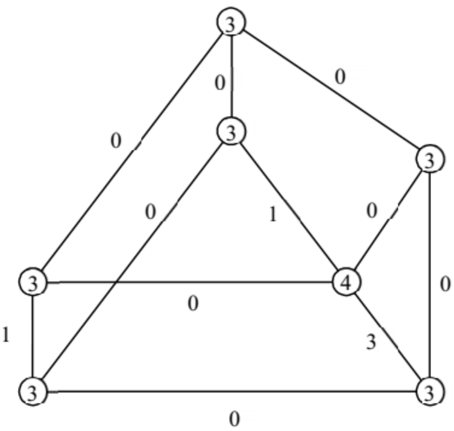


Fig. 2. Chain symbol

Table 1 presents various methods to depict the interconnections between links. These lines are labeled with numbers, indicating the count of binary links.

2.2. Properties of links in the chain

Definition 1: In a chain, the number of joints is always greater than or equal to the number of links.

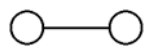
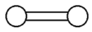
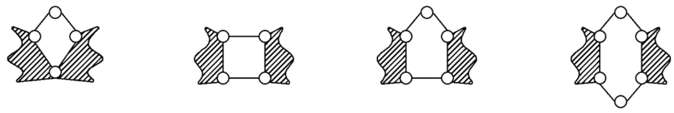
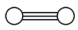
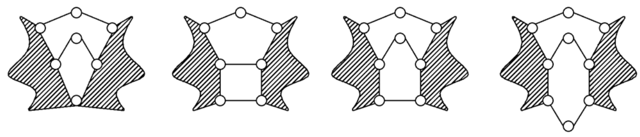
$$n = \sum_{i=2}^{r} q_i \tag{1}$$

$$J = \frac{1}{2}\sum_{i=2}^{r} i q_i \tag{2}$$

Where, n denote the number of links, J represent the number of joints, and q_i indicate the number of link i, satisfying the condition that $i \geq 2$ and r constitute the maximum number of links within the kinematic chain. It is important to note that $i \geq 2$ must $J \geq n$. The equality condition arises when the chain solely consists of binary links.

Definition 2: The minimum requirement for the number of binary links within the chain is equivalent to the sum of the chain's degrees of freedom and 3.

Table 1. Symbols and codes for links between links

Number of connections	Symbolic representation	Connection drawing
1		
2		
3		

Based on Gruebler theory [15] we have:

$$F = 3(n-1) - 2J \quad (3)$$

where F is the degrees of freedom. From equations (1), (2) and (3) we have:

$$F = 3\sum_{i=2}^{r} q_i - 3 - \sum_{i=2}^{r} iq_i \quad (4)$$

Simplifying the expression we get:

$$\sum_{i=2}^{r}(3-i)q_i = 3 + F \quad (5)$$

From formula (5) we can write as follows:

$$q_2 = 3 + F - \sum_{i=3}^{r}(3-i)q_i \quad (6)$$

From (6) we see that $\sum_{i=3}^{r}(3-i)q_i \le 0$. Thus, we have:

$$q_2 \ge 3 + F(= 3n - 2J) \quad (7)$$

Definition 3: In a chain, the minimum number of links is at least 2.

In a closed chain, a link must have at least 2 pin-joints. Therefore, the number of links in the closed chain cannot be less than 2. From equation (1) we have:

$$q_2 + \sum_{i=3}^{r} q_i = n \quad (8)$$

Substituting (4) into (8), we get:

$$3 + F - \sum_{i=2}^{r}(2-i)q_i = n \quad (9)$$

Expanding (9) we have:

$$q_3 + 2q_4 + 3q_5 + ... + (r-2)q_r = n - F - 3 \quad (10)$$

From that we deduce the following formula: $(r-2)h_r \le n - F - 3$. On the other hand, $q_r \ge 1$ due to: $r - 2 \le n - F - 3$. Thus, we have the formula:

$$r \le n - F - 1 \quad (11)$$

2.3. Design of 8-link Chain Robot with 1 Degree of Freedom

Step 1: Utilize formula (2) to determine the number of connections. Employ equation (7) to ascertain the minimum quantity of binary links. Apply formula (11) to identify the maximum number of links within the chain. To illustrate, consider a chain comprising 8 links and 1 degree of freedom. Table 2 displays the corresponding chain parameters.

Table 2. Chain parameters

q_2	q_3	q_4	q_5	q_6
4	4	0	0	0
5	2	1	0	0
6	1	0	1	0
6	0	2	0	0
7	0	0	0	1

Step 2: Employ symbolic notation to arrange circles containing numbers to depict the links. Each category can possess multiple potential structures. For instance, Fig. 3 showcases two configurations of four ternary chains. Please note that cases q_5 and q_6 have been excluded from consideration as they violate the requirement of a closed chain.

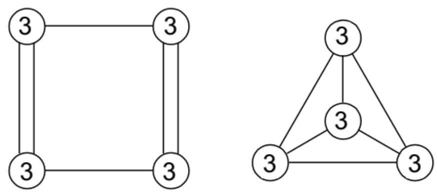
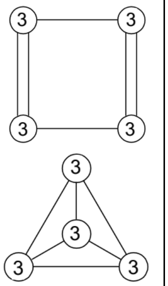
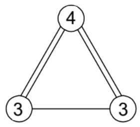
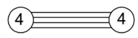


Fig. 3. Linkage structure of 4 ternary links

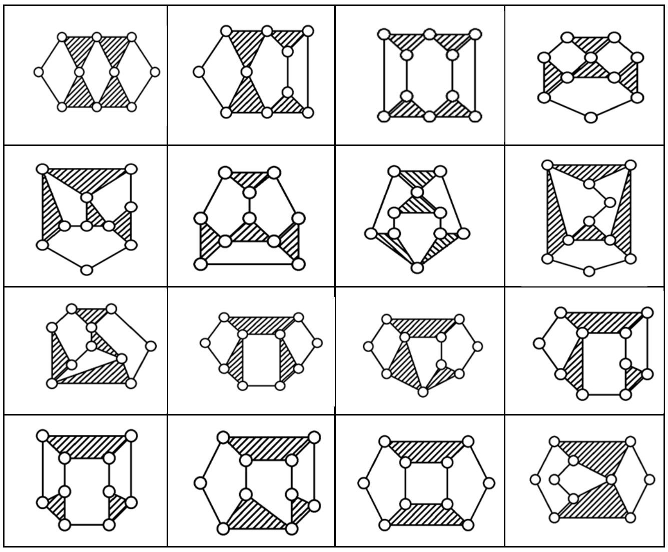
Step 3: Arrange the elements and assign numerical labels to generate Table 3 below. Each link is separated by a dot. Begin by listing stages with a smaller number of links, followed by those with a larger number of links. Permute the numbers to create valid structures.

Table 3. Link group structure

Chain parameters			Molecules	Combinations of connections	Permutations	Total
q_2	q_3	q_4				
4	4	0		0.0.02.02 0.0.02.11 0.0.11.11 0.0.0.1.1.2 1.0.0.1.1.1 2.0.0.0.0.2	1 1 1 3 2 1	9
5	2	1		1.02.02 0.02.12 1.02.11 1.11.11 0.12.11	1 1 1 1 1	5
6	0	2		12.12 22.02	1 1	2
					Total:	16

Step 4: From the results in table 3, we have the robot structures shown in Table 4.

Table 4. Results of 16 robot structures



3. CONCLUSION

Based on the obtained findings, the following conclusions can be drawn:

1. The study successfully applied the theory of finite symmetry groups to construct closed planar chain robot structures. Through this approach, 16 novel robot configurations were developed, leading to improved processes and enhanced efficiency.

2. The design of closed flat structures proves to be effective, particularly when there are no instances of overlapping structures.

3. The outcomes of this research can be readily extended to other structures that follow similar implementation steps.

4. It is worth noting that the automatic classification of structural groups has not been extensively explored in this study. However, the research team plans to address this limitation and publish further advancements in the near future.

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