IMPROVE PATH FOLLOWING EFFICIENCY FOR 4WD4WS ROBOT BY USING EXPONENTIAL FUNCTIONS IN VIRTUAL TARGET GUIDANCE ALGORITHM

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ABSTRACT

Most of the path following research for 4WD4WS mobile robots is conducted with two control loops, the kinematic loop and the dynamic loop. This article proposes a method using exponential functions in virtual target guidance algorithm to decrease the convergence time in the kinematic control loop. The process of synthesizing the control law is strictly mathematically guaranteed. Simulations in Matlab visually represent the research results.

Keywords: *mobile robot, path following, four-wheel-drive/four-wheel-steer, virtual target guidance algorithm, kinematic control.*

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1. INTRODUCTİON

The four-wheel-independent-drive and four-wheel-independent-steer robot or 4WD4WS is a redundant drive system with high flexibility and mobility with 8 motors (4 drive motors and 4 steering motors) is being used widely. In addition to its high road holding ability and performance, 4WD4WS is also used in harsh conditions such as military tasks.

Path following is a common problem in mobile robot control and many researches have been done for many years. Most of the cases, authors built two control loops: kinematic and dynamic.

With 4WD4WS, some authors built the controller on a dynamic model with unrealistic assumptions about the system's response [6], when the robot, in fact, is a non-holonomic system despite the residual drive. Research in [7] is one of the very detailed studies with the introduction of two separate control loops, in which the steering loop also provides independent forward and rear steering angles. However, by imposing the same steering angle on the front and rear wheels [10, 11], the author has created a side slip phenomenon at the drive wheels in all operating cases.

Geometric steering methods are also widely used [1-5, 12] for path following problems. One of those is virtual target guidance method [1, 2, 12] which was applied to the 4WD4WS robot, however, when results in [1, 2] have large convergence time, the method in [12] can not assure the robustness against disturbances and the robot would be swayed along the desired path.

Due to the use of two control loops with the residual driving nature of the system, the control methods of both control loops must ensure fast convergent speed and robustness against disturbances. Therefore, the authors propose to use an exponential function in the virtual target guidance algorithm in the kinematic control loop to enhance efficiency of the path following problem for the 4WD4WS robot.

2. METHODOLOGY

2.1. Kinematic model of a 4WD4WS mobile robot

A popular kinematic model for 4WD4WS robots used by many researchers is a simplified model of a two-wheeled bicycle. In which the steering angles of each real wheels as well as the virtual wheels F and R (as shown in Fig.1) are determined through the Ackermann steering method with the instantaneous center of rotation I_{CR}.

$$\delta_c = \tan^{-1}\frac{\tan(\delta_f)+\tan(\delta_r)}{2} \quad (1)$$

$$R = \frac{l}{\cos(\delta_c)*(\tan(\delta_f)-\tan(\delta_r))} \quad (2)$$

where: $\delta_f, \delta_r, \delta_c$ are the virtual steering angles at points F, R, C; wheel base $l = 2*l_f = 2*l_r$, d is the vehicle width. R is the trajectory radius, as in Fig. 1: $R = I_{CR}C$; $\delta_i, i = 1, 2, 3, 4$ are the real steering angles at real wheels – angles between the longitudinal car axis and wheel planes.

The real steering angles of each wheel can also be calculated from δ_c, R:

$$\delta_1 = \tan^{-1}\frac{\frac{l}{2}+R*\sin(\delta_c)}{R*\cos(\delta_c)-\frac{d}{2}} \tag{3}$$

$$\delta_2 = \tan^{-1}\frac{-\frac{l}{2}+R*\sin(\delta_c)}{R*\cos(\delta_c)-\frac{d}{2}} \tag{4}$$

$$\delta_3 = \tan^{-1}\frac{-\frac{l}{2}+R*\sin(\delta_c)}{R*\cos(\delta_c)+\frac{d}{2}} \tag{5}$$

$$\delta_4 = \tan^{-1}\frac{\frac{l}{2}+R*\sin(\delta_c)}{R*\cos(\delta_c)+\frac{d}{2}} \tag{6}$$

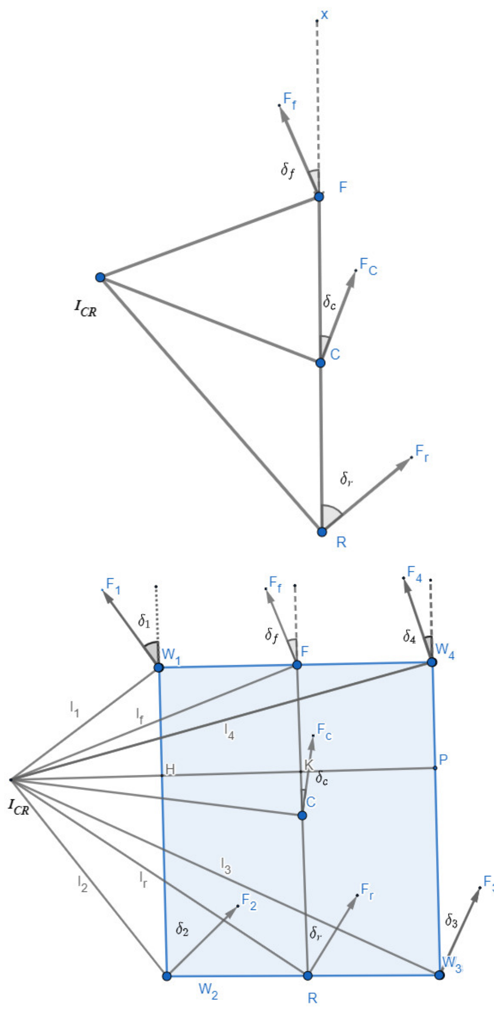

Fig. 1. The simplified two-wheeled bicycle kinematic model

Meanwhile, with the assumption of driving according to the Ackermann method, friction forces will eliminate lateral movements of the vehicle body and slipping will not occur, and the vehicle will move at a constant speed v. Then the angular velocity of the vehicle body is calculated by:

$$\dot{\psi} = \omega = \frac{v}{R} \tag{7}$$

Project the velocity vector v onto the two axes of the fixed ground coordinate system:

$$\begin{cases} \dot{x} = v\cos\psi \\ \dot{y} = v\sin\psi \end{cases} \tag{8}$$

Combining (1), (2), (7), (8) one can get the kinematic equations of 4WD4WS:

$$\begin{cases} \dot{x} = v\cos\psi \\ \dot{y} = v\sin\psi \\ \dot{\psi} = \frac{v}{R} \\ R = \frac{l}{\cos(\delta_c)*(\tan(\delta_f)-\tan(\delta_r))} \\ \delta_c = \tan^{-1}\frac{\tan(\delta_f)+\tan(\delta_r)}{2} \end{cases} \tag{9}$$

Using symmetrical steering method, then:

$$\begin{cases} \delta_c = 0 \\ \delta_f = -\delta_r = \delta \end{cases} \tag{10}$$

Then, (9) can be written as:

$$\begin{cases} \dot{x} = v\cos\psi \\ \dot{y} = v\sin\psi \\ \dot{\psi} = \frac{2v}{l}\tan(\delta) \end{cases} \tag{11}$$

With δ as control input.

2.2. Path following for 4WD4WS mobile robot by using virtual target guidance method

Fig. 2 shows the virtual target guidance method applying for the path following problem of a 4WD4WS robot, where D is the virtual target to determine the angles:

$$e_\psi = \psi - \psi_r \tag{12}$$

$$\psi_d - \psi_r = \operatorname{atan}\left(\frac{e_y}{\beta}\right) \tag{13}$$

$$\Delta\psi = \psi_d - \psi_r - e_\psi = \operatorname{atan}\left(\frac{e_y}{\beta}\right) - e_\psi \tag{14}$$

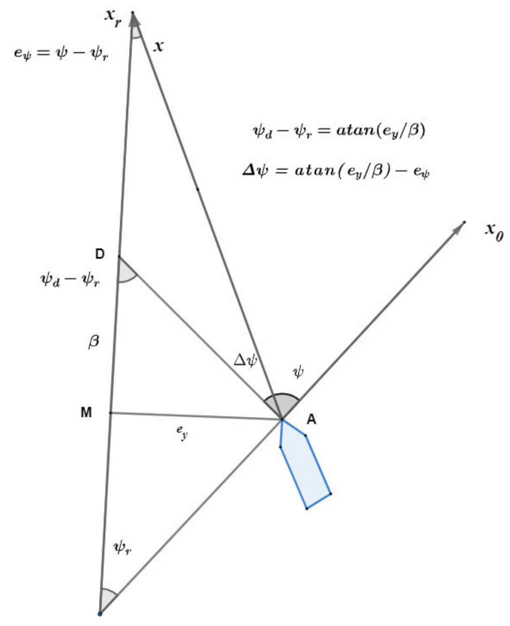

Fig. 2. Virtual target guidance method

Thus, with β being a positive parameter (according to the requirement for convergence speed as well as system response (maximum steering angle)), it is possible to determine the necessary deviation angle Δψ for the vehicle to drive towards point D with direction angle ψ_d and then gradually follows the required trajectory, in which the values e_ψ and e_y are obtained from measuring devices. Usually, the

e_y value is the only response value so (14) is often replaced by:

$$\Delta\psi = \operatorname{atan}\left(\frac{e_y}{\beta}\right) \tag{15}$$

To steer the robot car at an angle of Δψ using the virtual target guidance method, the steering angle of the virtual steering wheel is required to be proportional to this angle:

$$\delta = K\Delta\psi = K \operatorname{atan}\left(\frac{e_y}{\beta}\right) \tag{16}$$

Similar to (8), it is possible to describe the distance from the vehicle center to the required trajectory e_y in the lateral direction of the vehicle body determined by the following differential equation:

$$\dot{e}_y = v\sin(\psi - \psi_d) = -v\sin(\Delta\psi) = -v\sin(\operatorname{atan}\frac{e_y}{\beta}) \tag{17}$$

With a sufficiently small value of e_y relative to β, especially when the vehicle is moving just around the required trajectory, (17) can be approximated as:

$$\dot{e}_y = -\frac{v}{\beta}e_y \tag{18}$$

Since (19) is a basic differential equation with a solution of the form:

$$e_y = e^{-\frac{v}{\beta}t} \tag{19}$$

And this solution will approach without actually reaching zero value. With this control method, the convergence time of the system is not able to determine. That is why the authors proposed using an exponential function to bring the system's convergence time to a finite value.

2.3. Improve trajectory tracking efficiency using an exponential function

To improve the quality of path following task, the author proposes to use the exponential function in formula (16) to calculate the steering angle of the virtual rudder, then (16) becomes:

$$\delta = K\Delta\psi = K \operatorname{atan}[\left(\frac{e_y}{\beta}\right)^n] \tag{20}$$

Where n is a positive value and:

$$n = \frac{p}{q} \tag{21}$$

Where p, q are odd positive integers such that:

$$q = 2p - 1 \tag{22}$$

Similar to (17), it is possible to describe the distance from the vehicle center to the required trajectory e_y in the lateral direction of the vehicle body determined by the following differential equation:

$$\dot{e}_y = -v\sin(\Delta\psi) = -v\sin(\operatorname{atan}[\left(\frac{e_y}{\beta}\right)^n]) \tag{23}$$

With a sufficiently small value of e_y relative to β, especially when the vehicle is moving just around the required trajectory, (23) can be approximated as:

$$\dot{e}_y = -v\left(\frac{e_y}{\beta}\right)^n \tag{24}$$

Thus, with the control input being the steering angle determined according to (20), the differential equation (26) will be obtained. To prove stability, we choose the Lyapunov candidate function:

$$V = \frac{1}{2}e_y^2 \tag{25}$$

Then, its derivative will be:

$$\dot{V} = e_y\dot{e}_y = -\frac{v}{\beta^n}e_y^{n+1} \tag{26}$$

Subtitute (21) to (26):

$$\dot{V} = -\frac{v}{\beta^n}e_y^{\frac{p+q}{q}} \tag{27}$$

Where p, q are odd positive intergers then p+q is an even positive interger. While v and β are also positive then:

$$\dot{V} = -\frac{v}{\beta^n}e_y^{\frac{p+q}{q}} \le 0 \tag{28}$$

Or the system is stable according to Lyapunov.

To determine the convergence time of the system, we use differential equation (24):

$$\frac{de_y}{dt} = -v\frac{e_y^n}{\beta^n} \tag{29}$$

Switching sides:

$$\frac{de_y}{e_y^n} = -v\frac{dt}{\beta^n} \tag{30}$$

Integrating both sides over e_y and t we have:

$$\int_{e_{y0}}^{0}\frac{de_y}{e_y^n} = -\frac{v}{\beta^n}\int_{t_0}^{t_r+t_0}dt \tag{31}$$

In which, t_r is the time to bring e_y from the initial value e_{y0} at the initial time t_0 to zero value.

Then, from (31) convergence time of the system can be determined:

$$t_r = \frac{e_{y0}^{1-n}}{1-n}\frac{\beta^n}{v} \tag{32}$$

When the moving speed of the vehicle body remaining constant, the system's convergence time can be adjusted by changing the value of β accordingly.

Thus, by using the exponential function in the virtual target guidance method, the system has a finite convergence time, improving the quality of the trajectory tracking problem.

3. SIMULATION AND DISCUSSION

The article chooses to simulate straight and circular trajectories on Matlab - Simulink software with the same initial conditions for two control methods: normal virtual target guidance method and virtual target guidance method using an exponential function. In addition, the article also demonstrates the noise resistance of the control method by adding noise to the system. Simulation results of straight trajectory tracking are shown from Fig. 5 to Fig. 8. Noise, that was added to the system, has the form as shown in Fig. 3 and Fig. 4.

With initial conditions :

$$\begin{cases} e_{y0} = -0.8045\text{m} \\ \beta = 20 \\ n = 5/9 \\ v = 30\text{m/s} \end{cases} \tag{33}$$

Then t_r = 0.3596s, this value is consistent with the graph in Fig. 5 with the corresponding lateral error of the vehicle body being 0.002m or 2mm, thus confirming that the convergence time of the system is finite and calculated accurately according to formula (32).

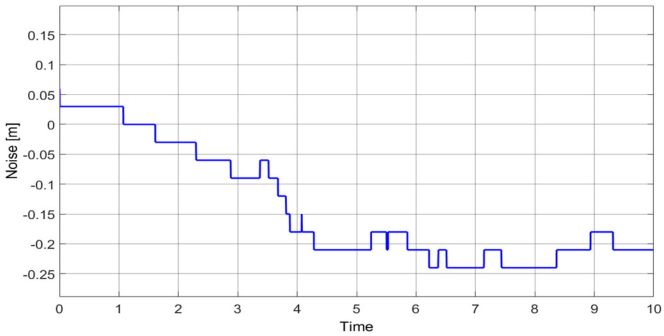

Fig. 3. Noise when follows a straight line

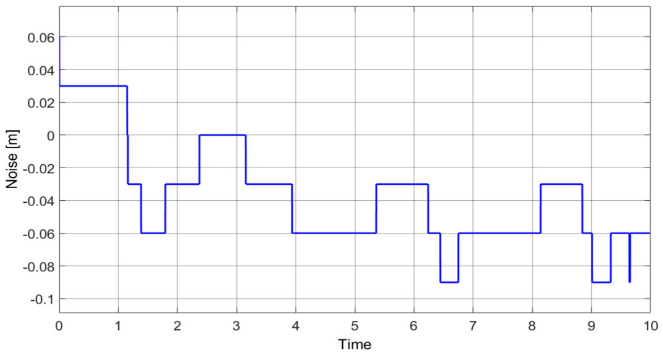

Fig. 4. Noise when follows a circular trajectory

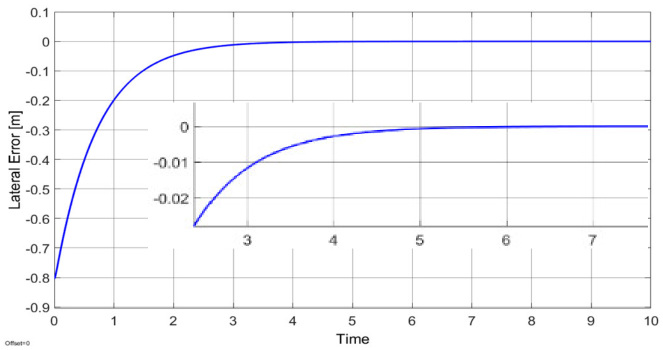

Fig. 5. Lateral error of virtual target guidance method when follows a straight line

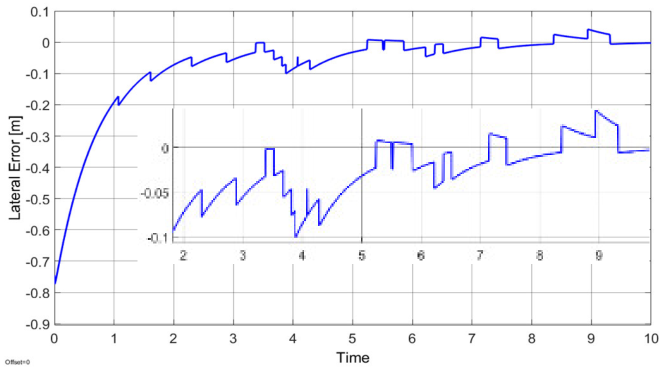

Fig. 6. Lateral error of virtual target guidance method when follows a straight line (disturbance included)

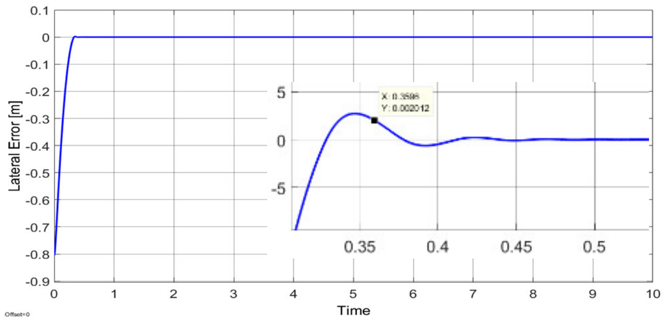

Fig. 7. Lateral error of virtual target guidance method using exponential function when follow a straight line

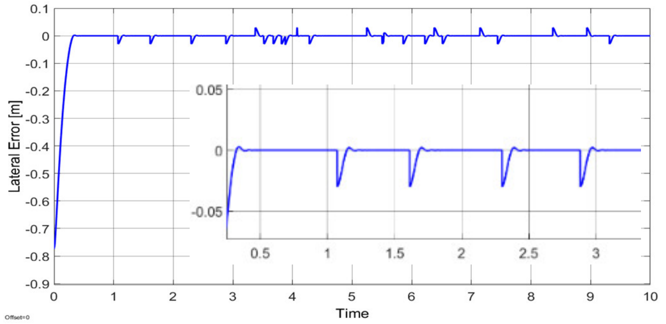

Fig. 8. Lateral error of virtual target guidance method using exponential function when follows a straight line (disturbance included)

Simulation results of circular trajectory tracking are shown from Fig. 9 to Fig. 12 where β is chosen much smaller (β = 1). The results show the differences between the two methods, when normal virtual target guidance in both straight line and circular trajectory need more time to get to a stable lateral error and this error value is also much larger than the virtual target guidance method using exponential function.

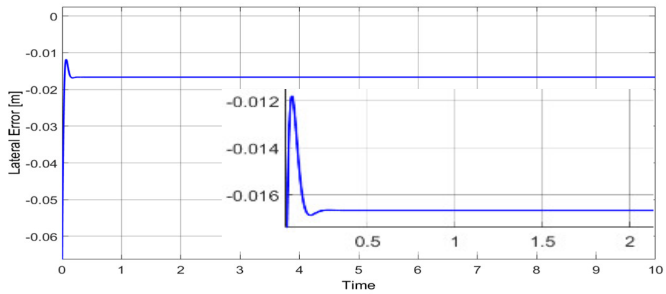

Fig. 9. Lateral error of virtual target guidance method when follows a circular trajectory

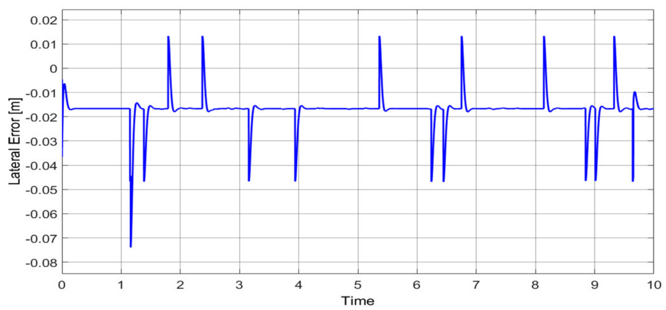

Fig. 10. Lateral error of virtual target guidance method when follows a circular trajectory (disturbance included)

Although both methods well response to disturbances the case when using exponential function is more fluctuated around before getting to the stable value. To reduce this phenomenon one can choose different β but at the cost of the final lateral error.

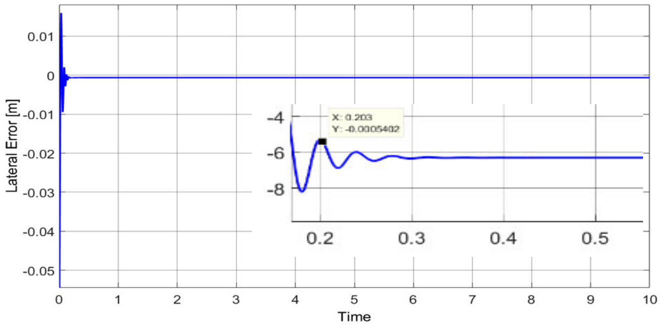

Fig. 11. Lateral error of virtual target guidance method using exponential function when follow a circular trajectory

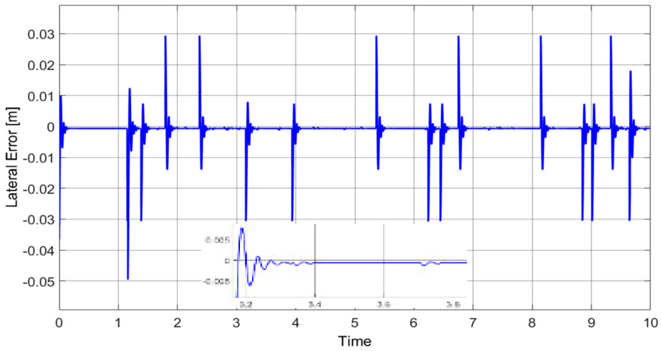

Fig. 12. Lateral error of virtual target guidance method using exponential function when follows a circular trajectory (disturbance included)

4. CONCLUSION

The article has presented a new method of using virtual target guidance algorithm for the problem of trajectory tracking for 4WD4WS mobile robot, whereby the quality of trajectory tracking is improved compared to other available solutions, while ensuring the robustness of the system against disturbances. With that result, this method can be applied to other wheeled mobile robots with similar kinematic configurations, ensuring high efficiency. Research results are rigorously mathematically proven and visualized by simulation.

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