STUDYING THE INFLUENCE OF THE NOZZLE PROFILE ON THE WORKING PERFORMANCE OF A SOLID-FUEL ROCKET ENGINE

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ABSTRACT

In this paper, a method to determine the typical working parameters of a solid-fuel rocket engine taking into account some types of nozzle profiles are established. A mathematical model describing the thermodynamic process of a solid-fuel rocket engine is established based on thermodynamic and aerodynamic relationships. The model has been calculated for the cruise engine of the 9K38 anti-aircraft guided missile (Russian). Based on the results, the paper evaluated the influence of the nozzle profile on the performance of solid-fuel rocket engines. The research results are an important theoretical basis for the process of designing and selecting the optimal nozzle profile to minimize energy losses and improve the working efficiency of the engine.

Keywords: *Solid fuel rocket engine; nozzle; engine structure; 9K38 anti-aircraft missile.*

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1. INTRODUCTION

The nozzle is a part used to convert the heat energy of the combustion product into the kinetic energy of the gas stream. At the outlet of the nozzle, the gas flow has supersonic speed. Therefore, creating a high-value thrust that is used to propel the rocket in motion. In addition, the nozzle also has the effect of regulating the pressure in the engine combustion chamber and adjusting the mass of the rocket. The nozzle works in very harsh conditions: the combustion product (in the form of gas phase and solid phase) has a high temperature, and moves at a high speed (about 1000m/s), so the nozzle is heated intensely about 550ºC) and the inner surface of the nozzle is easily worn, causing the size parameters of the nozzle to change, especially the throat [1].

Thrust loss in the nozzle is a combination of many types of losses, of which there are two main types, which are the loss due to current diffusion and the loss due to friction between the combustion product jet stream and the nozzle shell wall. This loss takes place mainly in the divergent section. By fixing the throat section diameter and the nozzle outlet cross-sectional diameter, the nozzle aperture opening angle decreases with increasing the nozzle overpass length, thereby reducing diffusion loss. However, it increases the loss due to friction with the nozzle's wall. In contrast, in the case of reducing the divergent section length, the diffusion loss increases, while the friction loss decreases. Since then, it is required to determine the appropriate opening angle of the nozzle so that the total loss in the nozzle is the smallest and that means getting the maximum thrust coefficient - the engine thrust reaches the maximum value. The research results of the article can be applied to the calculation and design of the rocket engine nozzle block.

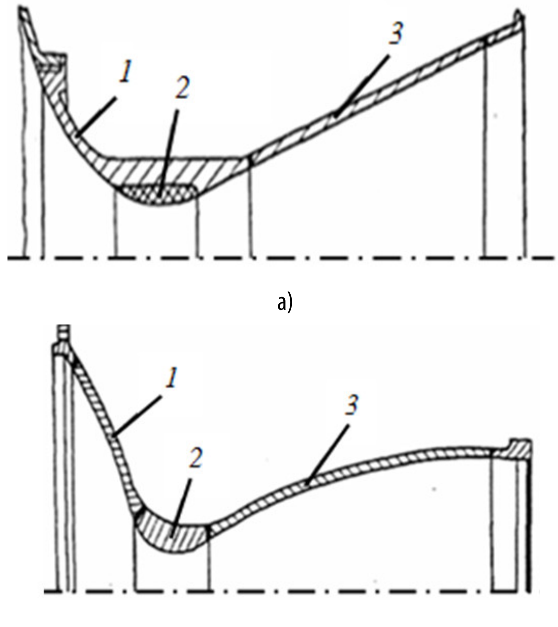

Fig. 1. Structure of cone nozzle (a) and shaped nozzle (b)

1. Convergent section; 2. Throat; 3. Divergent section

According to the profile of the supersonic part, the nozzles are divided into cone-shaped and profile nozzles (variable profile), Fig. 1 [2, 3]. The cone-shaped nozzle has the advantage of high technology. However, because the velocity vector of the gas flow at the outlet section of the nozzle is deviated from the axis of the nozzle (flow scattering), it causes a loss of thrust value (the thrust loss due to the scattering phenomenon). Profile nozzles have a more complex structure and are more difficult in manufacturing technology than Laval nozzles, but they have a compact size, which limits more losses due to friction of the gas stream coming out into the nozzle wall, the nozzle longitudinal expansion loss (the nozzle forms shorter than the Laval nozzle when used for the same engine family), thus minimizing the total loss through the nozzle. At present, the technology of manufacturing nozzles in our country for scientific research projects almost exclusively uses cone-shaped nozzles and has not appreciated the problem of limiting energy loss through the nozzle. The research method given below allows a more accurate re-evaluation of the problem mentioned above.

2. PROBLEM FORMULATION

2.1. The system of equations describes the laws of processes occurring in the combustion chamber of a rocket engine

The system of differential equations describes the processes occurring in the combustion chamber of the engine, including combustion processes, gas generation; the process of injecting air through the nozzle; the process of changing the gas state [1, 2].

+ Equation of the law for the change of the burning surface area (S) of the propellant:

$$S = S(t) \tag{1}$$

+ Equation of gas generation:

$$\frac{d\psi}{dt} = \frac{Su\rho_T}{\omega_T} \tag{2}$$

Where, ω_T, ρ_T are the mass and density of the propellant, respectively; ψ is the relative amount of propellant; u is the burning rate, determined by the following formula [2]:

$$u = u_1 p^{\nu} f(T_{bd})\varphi(v_{kh})$$

+ Equation for ejecting gas through the nozzle:

$$\frac{d\eta}{dt} = \frac{\varphi_2 K_0(k) F_{th}}{\omega_T \sqrt{RT}} p \tag{3}$$

Where, η is the relative amount of gas ejected through the nozzle; p, T is the pressure and temperature of the gas, respectively; R is the gas constant; k is the adiabatic exponent of the combustible gas; φ_2 is the flow loss of the gas flow; F_{th} is the throat cross-sectional area of the nozzle.

$$K_0(k) = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \sqrt{\frac{2k}{k+1}}$$

+ Equation of state for gas:

After time t, the mass of the propellant that can be ignited is $\omega_c = \omega_T\psi$, The mass of the ejected gas is $\omega_T\eta$. The mass of gas remaining in the combustion chamber at time t is $\omega_T(\psi - \eta)$.

The equation of state for the gas in the combustion chamber is expressed as:

$$p = \frac{\omega_T(\psi - \eta)RT}{V_K - \frac{\omega_T}{\rho_T}(1-\psi) - \alpha_T\omega_T(\psi - \eta)} \tag{4}$$

Where: V_K is the volume of the combustion chamber; α_T is the co-volume of powder gas.

After synthesizing equations (1), (2), (3) and (4), the system of differential equations describing the processes occurring in the combustion chamber is established (5).

$$\begin{cases} S = S(t) \\ \dfrac{d\psi}{dt} = \dfrac{Su\rho_T}{\omega_T} \\ \dfrac{d\eta}{dt} = \dfrac{\varphi_2 K_0(k) F_{th}}{\omega_T \sqrt{RT}} p \\ p = \dfrac{\omega_T(\psi - \eta)RT}{V_K - \frac{\omega_T}{\rho_T}(1-\psi) - \alpha_T\omega_T(\psi - \eta)} \end{cases} \tag{5}$$

2.2. Determining the thrust of the rocket engine

Thrust is the most important specification of a solid-fuel rocket engine. The value of thrust, as well as its transformation law, is decisive to the law of motion, flight distance, and control system of the rocket. The general formula for determining the thrust of a rocket engine is as follows [4]:

$$P = \varphi_c \cdot C_P \cdot p \cdot F_{th} \tag{6}$$

Where P is the thrust of the engine; φ_c is the nozzle loss factor; C_P is the thrust coefficient; p is the working pressure in the engine combustion chamber;

When the engine is working stably, the pressure p in the combustion chamber of the engine is always maintained at a certain value, this value depends mainly on the throat cross-sectional diameter of the nozzle F_{th}. Therefore, in order to increase the thrust of the engine, the nozzle needs to be designed with a reasonable profile to increase the thrust coefficient C_p, which depends on the diameter of the nozzle outlet and the nozzle factor φ_c. On the other hand, to ensure the kinetic dimensions of the rocket in orbit as well as in the movement phase in the launch tube, the outlet diameter of the nozzle d_a is limited to a specific value range. Then the value of thrust is proportional to the value of the nozzle coeffiction φ_c.

The nozzle loss factor φ_c is the multiplication of the two components of loss due to friction φ_{ms} and loss due to scattering φ_{tx} [4]:

$$\varphi_c = \varphi_{ms} \cdot \varphi_{tx} \tag{7}$$

Where φ_{ms} is the coefficient of thrust loss due to friction between the combustion product stream and the nozzle wall; φ_{tx} is the coefficient of thrust loss due to current diffusion.

Divide the supersonic part of the nozzle into *n* equal parts along the nozzle axis from the throat cross-section to the nozzle output cross-section (Fig. 2). Then, the thrust loss due to friction of the stream with each area is calculated by the following formula [4]:

$$\Delta P_i^{ms} = c_{fi}\frac{\rho_i w_i^2}{2}\Delta F_i \cos\beta_i \quad (8)$$

Where c_{fi} is the coefficient of friction of the area i-th; β_i is the angle of inclination of the i-th profile relative to the nozzle axis; ρ_i, w_i are the average density and the cross-sectional flow velocity at the i-th part, respectively.

ΔF_i is the area of the i-th curved surface determined by the approximate formula:

$$\Delta F_i = 2\pi R_i^{tb} l_i \quad (9)$$

Where R_i^{tb}, l_i are the average radius of the cross-sections and the length of the i-th part, respectively.

The coefficient of friction is determined by the following expression:

$$c_{fi} = c_{f0}\left(1 + 0.89\frac{k-1}{2}M_i^2\right)^{-0.55} \quad (10)$$

Where c_{fo} is the coefficient of friction for an incompressible fluid, with a nozzle with a statically machined surface: $c_{fo} = 0.002$ [4]; M_i is the Mach value at the i-th part.

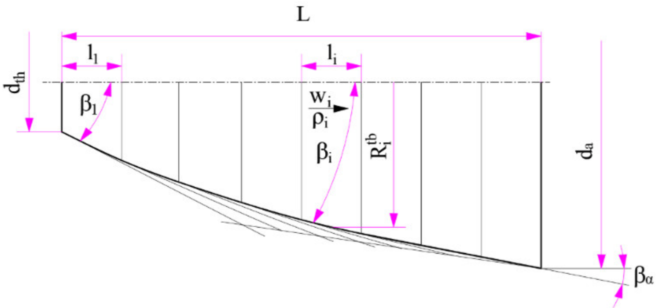

Fig. 2. The supersonic part of the nozzle

The coefficient of thrust loss due to friction is determined as follows:

$$\varphi_{ms} = \frac{P - \sum_i^n \Delta P_i^{ms}}{P} \quad (11)$$

The coefficient of thrust loss due to diffusion is determined as follows:

$$\varphi_{tx} = \frac{1+\cos\beta_\alpha}{2} \quad (12)$$

β_α is the angle of inclination of the nozzle surface relative to the axis at the nozzle outlet position.

3. RESULTS AND DISCUSSION

3.1. Problem solution

To test the theoretical model, the cruise engine of the 9M39 rocket is used for the calculation process, see Fig. 3. To solve the system of equations (5) the input parameters need to be determined. These parameters can be obtained from the technical documentation or measured directly from the sample engine [5]. Due to the large number of input parameters, some basic parameters are presented in Table 1.

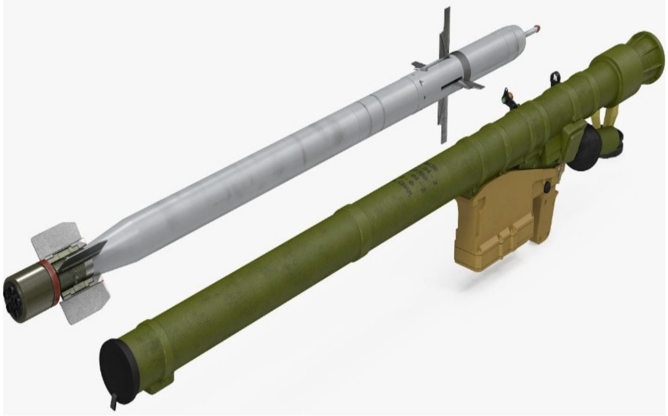
Fig. 3. Anti-aircraft missile complex 9K38

Table 1. Input parameters

Parameters	Value	Unit
The inner diameter of the combustion chamber	64.5×10^{-3}	m
The throat-sectional diameter of the nozzle	14.5×10^{-3}	m
Number of nozzles	1	-
Length of cylindrical part propellant	729.6×10^{-3}	m
Length of cone-shaped propellant	95.3×10^{-3}	m
Half of the cone angle of the propellant	9^o	-
Diameter of cylindrical part propellant	65.4×10^{-3}	m
Minimum diameter of cone-shaped propellant	35.2×10^{-3}	m
Density of propellant	1650	kg/m^3
Powder force	850000	J/kg
Adiabatic exponent	1.25	-
Combustion rate coefficient	0.268×10^{-6}	m/s.Pa
Flow loss coefficient	0.98	-

The system of equations (5) is solved by the numerical integration method by Maple software. Typical results are shown in Figs. 4, 5.

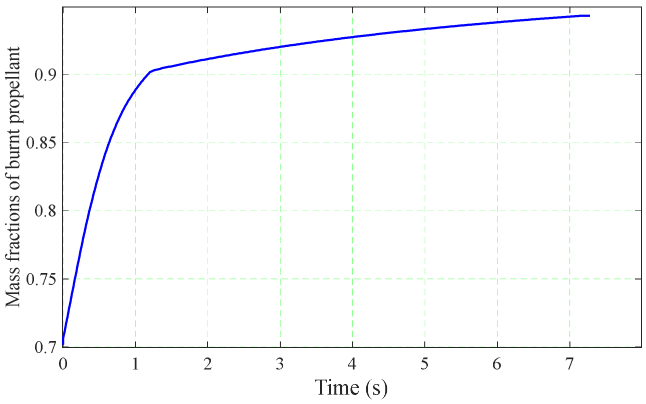

Fig. 4. Mass fractions of propellant

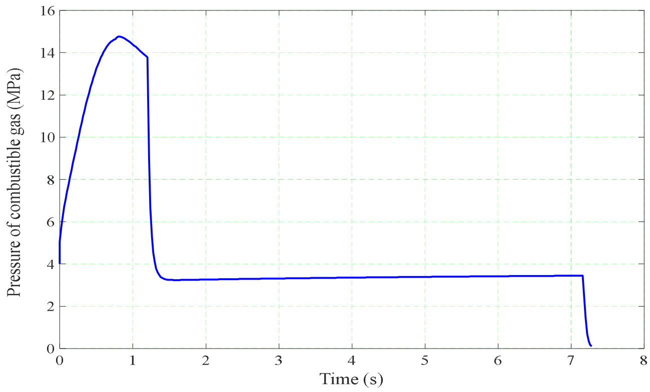

Fig. 5. Law of pressure of combustible gas

Based on the obtained results, some comments are as follows: the law of gas pressure variation and engine thrust over time are consistent with the actual law. The error between the theoretical calculation results and the results announced by the manufacturer [12] is within the allowable limit (< 10%). Thereby, it shows that the theoretical model and the calculation program are completely suitable and reliable.

3.2. Effect of nozzle profile on engine performance

The 9M39 rocket cruise engine is used to accelerate the missile to a certain speed. This engine uses a Laval-type nozzle with the opening angle of the nozzle in the ultrasonic part being constant. Therefore, when the engine is working, it will cause friction and scattering losses, reducing the engine's efficiency, and affecting the rocket's speed on the flight path. With the design of a new nozzle profile that increases the thrust of the engine, the smaller size and weight of the nozzle is also an optimal measure in the design process of rocket engines.

To evaluate the influence of the supersonic boom profile on the rocket engine's performance, a new propeller was designed. This nozzle has a profile determined by the characteristic curve method [3, 9]. The new nozzle has parameters such as the diameter of the combustion chamber, and the throat cross-section is equal to the parameters of the original nozzle. The nozzle with the optimal profile is shown in Table 2 and Fig. 6.

Table 2. Profile parameters of supersonic section

x	y	θ (deg)	M
0	7.25	19.82	1
2.47	8.14	15.83	1.13
9.17	10.41	17.61	1.69
13.86	11.80	15.41	1.82
18.03	12.86	13.21	1.89
22.31	13.78	11.01	1.95
26.92	14.58	8.81	2.02
31.99	15.27	6.60	2.09
37.66	15.82	4.40	2.16
44.04	16.18	2.20	2.23
53.00	16.32	0.78	2.30

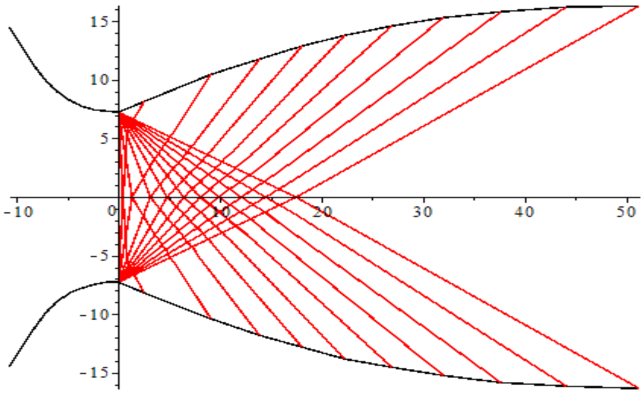

Fig. 6. New nozzle profile (Numerical output for 10 characteristic line)

The thrust force equation (6) is calculated for the original nozzle and the new nozzle. The comparison results are shown in Table 3.

Table 3. Results of comparison of the parameters of the two nozzles

	Unit	Initial nozzle	New nozzle
Nozzle length	mm	53	53
Area ratio F_a/F_{th}		2.76	2.25
The speed at the nozzle outlet w_a	m/s	2.09M	2.30M
Pressure at the outlet of the nozzle p_a	MPa	0.291	0.182
Scattering loss coefficient φ_{tx}	-	0.984	0.998
Frictional loss coefficient φ_{ms}	-	0.980	0.986
Thrust coefficient C_P	-	1.568	1.596
Maximum thrust	N	3461	3523

The obtained results can be interpreted as follows: Based on the results obtained from Table 3, the comparison of the specifications of the new and original nozzles with the same length is as follows:

- The speed at the output of the nozzle w_a increases by 10.05%;
- Thrust coefficient increases by 1.78%;
- The maximum thrust value increases by 1.79%;
- Both scattering and frictional losses are reduced.

The above comparison results show that the parameters are optimized in the calculation process, the new nozzle has basic characteristics that all change in the direction of increasing the efficiency of the rocket engine.

4. CONCLUSIONS

In this paper, a method for determining the thermodynamic characteristics of a solid-fuel rocket engine has been established. Research content has applied modern means with the help of calculation software for high-accuracy results. With the research results obtained, some conclusions have been made as follows:

- The establishment of a system of differential equations of the interior ballistic of rocket engines is done carefully and completely, bringing the theoretical problem closer to the actual model.

- A new nozzle with an optimized profile designed to evaluate the influence of the nozzle profile opening angle on engine performance;

- The results obtained from this study are used to evaluate the performance of solid-fuel rocket engines. This is a reliable scientific basis, which can serve as a reference for designers in the process of designing, manufacturing, and optimizing the overall structure of solid-fuel rocket engines, especially divergent sections of the nozzle.

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