DETERMINATION OF GAS FLOW PARAMETERS IN THE NOZZLE OF A SOLID FUEL ROCKET ENGINE BY ANALYTICAL METHOD

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ABSTRACT

In this paper, a mathematical model is built to determine the working parameters on the combustion chamber of a solid fuel rocket engine. Based on the theoretical basis established for this model, the parameters of the gas flow in the nozzle of the solid fuel rocket engine such as velocity, pressure, and temperature are determined by Matlab software. Selected C5K solid-fuel rocket engine for model verification. The results of the article are the theoretical basis for the process of calculating the nozzle design to ensure high accuracy.

Keywords: *Rocket engine; solid fuel; nozzle; gas flow.*

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1. INTRODUCTION

Solid fuel rocket (SFR) engines working on chemical fuels are characterized by complex physicochemical processes. This process takes place inside the combustion chamber and nozzle of the engine, see Figure 1. The nozzle is a very important part in a solid-fuel rocket engine, its role is to generate thrust through the process of expansion, accelerating the flow of combustion products. This is where the process of converting the heat of combustion of the fuel into kinetic energy of the gas takes place. The combustion product flow of the propellant is accelerated from subsonic to supersonic. At the critical cross-section of nozzle, the speed of the gas flow is Mach. The thrust of a solid propellant rocket engine depends on the speed of the combustion product at the nozzle outlet [1, 2]. The characteristics of the combustion product flow in the nozzle are pressure, temperature, and velocity. These parameters are important factors in determining the reliability of the rocket engine.

In previous studies, the parameters of gas flow through the nozzle have been studied relatively fully, however, these studies are mainly calculated through simulation method or approximate. In the framework of the article, the authors focused on researching and building a mathematical model to determine the aerodynamic parameters of the air flow in the nozzle of the solid fuel rocket engine such as velocity, pressure, and temperature at any position by analytical method. The equation for determining the parameters of the gas flow in the nozzle is solved numerical methods.

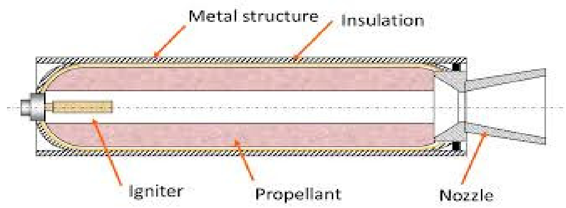

Fig. 1. General structure of rocket engine

2. THEORETICAL FOUNDATIONS OF BUILDING MATHEMATICAL MODELS

In order to determine the aerodynamic parameters of the gas flow in the propeller by analytical method, the interior ballistic problem of a solid fuel rocket engine is solved to determine the input parameters for the gas flow problem throught nozzle.

2.1. System of equations for interior ballistic

a) Some assumptions

The introduction of assumptions is intended to simplify the complex processes that occur in the combustion chamber of a rocket engine. To solve the basic problem of interior ballistic by theoretical methods, the influence of other factors is taken into account by the experiment coefficient. The following assumptions are used to solve the interior ballistic problem of a solid-fuel rocket engine, see in [1, 2, 3, 5]:

- The igniter ignites instantaneously, then at t = 0, p = p_{moi};

- The propellant burns according to the laws of geometry;

- Static pressure does not change along the combustion chamber and is equal to the braking pressure at the nozzle outlet;

- The temperature of the combustion gas in the combustion chamber T_0 does not change and is equal to:

$$T_0 = \frac{T_1}{k}$$

- Heat loss in the combustion chamber is given by the factor χ_n:

$$\chi_n = 1 - \frac{a}{1 + b\psi}$$

- Neglecting the volume of the gas particles themselves: $\alpha = 0$.

- The geometrical parameters of the motor do not change during engine operation.

b) System of differential equations for interior ballistic of SFR in the burning propellant phase

The system of basic equations for internal projection of SFR is established on the basis of thermodynamic and aerodynamic relationships. It describes the state of the medium in the combustion chamber and the nozzle, the size and shape characteristics of the propellant. Using the system of differential equations of interior ballistic of ĐTR presented in the document [6] with the following equations: the law of change of pressure with time p(t), the mass fraction of burnt powder ψ(t), heat loss in the combustion chamber $\chi_n(t)$.

$$\begin{cases} \dfrac{d\psi}{dt} = \dfrac{\rho_T . S.u}{\omega} \\ \dfrac{d\chi_n}{dt} = \dfrac{ab}{(1+b\psi)^2} . \dfrac{d\psi}{dt} \\ \dfrac{dp}{dt} = -\dfrac{1}{V}\left[\begin{aligned} &\left(\varphi_2 . K_0(k) . F_{th} . \sqrt{\chi_n R T_0} + S.u - V.\chi_1\right).p \\ &- S.u.\chi_n . f_0 . \rho_T \end{aligned} \right] \end{cases} \tag{1}$$

Where:

$$\chi_1 = \frac{1}{\chi_n} . \frac{d\chi_n}{dt}$$

ω, ρ_T, S, u respectively, the mass, density, burning surface area and burning rate of the propellant to determine the relative amount of propellant ψ at time t; a, b are the empirical coefficients used to determine the heat loss in the combustion chamber χ_n at time t; p, T_0, R are pressure, combustion temperature and gas constant in the combustion chamber, respectively; f_0 is the powder force; F_{th}, V, φ_2 are the critical cross-sectional area, the volume of gas occupied in the combustion chamber and the flow loss coefficient, respectively. The aerodynamic function $K_0(k)$ is determined through the adiabatic exponent k of the propellant.

$$K_0(k) = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} . \sqrt{\frac{2k}{k+1}}$$

Initial conditions: t = t_0; $\psi = \psi(t_0)$; $p_0 = p(t) = p_{m«i}$; $\chi_n = \chi_n(t_0)$.

To solve this system of equations, we used Runge-Kutta method of order 4, time step h = 0.0001s. Algorithm diagram used to solve system of equations (1) as shown in Fig. 2.

The result after the system of equations (1) is solved is: Law of time change of pressure p(t), the mass fraction of burnt powder ψ(t), factor taking into account heat loss in the combustion chamber $\chi_n(t)$.

c) Calculation of gas parameters in the free gas injection stage

Suppose by the time t_k the propellant burns out, The pressure of the gas in the combustion chamber is p_k. The mass of gas in the combustion chamber m_k is determined directly from the equation of state, [6]:

$$m_k = \frac{p_k . V_k}{f_0} \tag{2}$$

The specific volume of gas v_k in the combustion chamber at time t_k is determined from the expression:

$$v_k = v(t_k) = \frac{V_k}{m_k} = \frac{f_0}{p_k} \tag{3}$$

Gas pressure in the combustion chamber at this stage:

$$p = p_k . (1 + B.t)^{-\frac{2k}{k-1}} \tag{4}$$

Where, $B = \frac{k-1}{2.k} . b$, with $b = \frac{\phi_2 . K_0(k) . k . F_{th}}{m_k} \sqrt{\frac{p_k}{v_k}}$.

The algorithm diagram determines the working parameters of the combustion chamber as shown in Fig. 3.

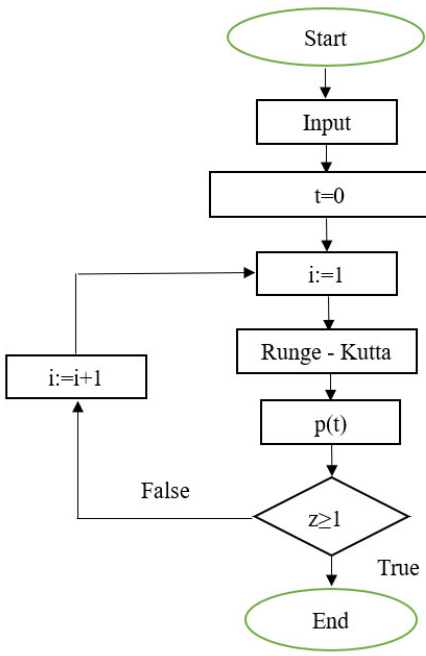

Fig. 2. Algorithm diagram for determining the working parameters of the combustion chamber in the burning propellant phase

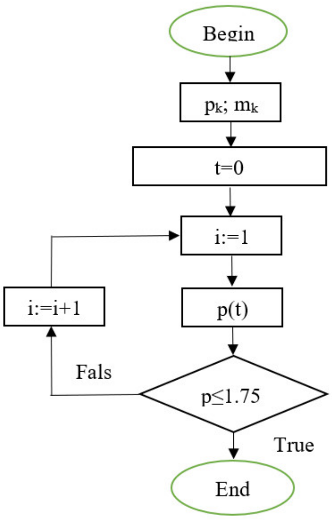

Fig. 3. Algorithm diagram for determining the working parameters of the combustion chamber in the free gas injection phase

2.2. Determine the parameters of the air flow in the nozzle

To investigate the movement of air flow, Laval overpass nozzle is used as a survey model, see Fig. 4 [1, 2, 6].

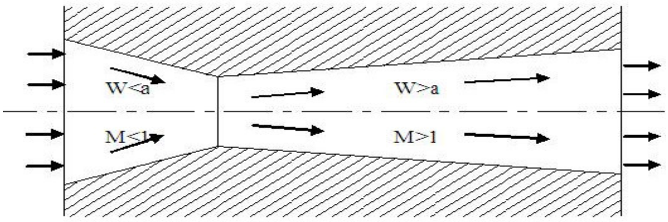

Fig. 4. Laval nozzle diagram

W- the speed of gas movement; a- speed of sound; M- Mach number

The movement of air through the nozzle is a very complex thermodynamic process. It is very difficult to describe it theoretically. Therefore, to study computation by analytical method, we need to use the following assumptions, refer to [5]:

- The flow is considered to be continuous, a gas phase, unidirectional and steady, the flow parameters are a function of the variable X, having the same value at all points on the same cross-section of the flow;

- The flow is isoentropic, there is no heat loss to heat the pipe wall and there is no internal friction;

- The isobaric and isothermal heat capacity, the adiabatic exponent are constants;

- The speed of movement of the gas flow in the combustion chamber and at the nozzle inlet is very small (w_0 = 0).

With these assumptions, the expressions determining the distribution rules of the parameters of pressure, temperature and gas flow velocity through the Laval booster nozzle were determined according to the following formulas, see in [1, 2, 6]:

- Air flow speed:

$$w = \varphi_1 \sqrt{\frac{2k}{k-1} R T_0 \left(1 - X^{\frac{k-1}{k}}\right)} \quad (5)$$

- Airflow temperature:

$$T = T_0 \cdot X^{\frac{k-1}{k}} \quad (6)$$

- Pressure of the gas stream:

$$p = p_0 \cdot X \quad (7)$$

Where: $X = X(k, F/F_{th})$ is a function of the adiabatic exponential k and the area ratio F/F_{th}; F is the cross-sectional area of the nozzle at the calculated cross-section; F_{th} is the critical cross-sectional area of the nozzle; φ_1 is the coefficient of motion loss of the gas flow, determined experimentally.

Thus, we have built the rule of changing parameters of pressure p, temperature T and gas flow velocity w along the length of the nozzle. The analytical method has the advantage of being able to solve the general problem, the nozzle has any shape and expansion coefficient. However, to solve the problem, many assumptions must be used and the results of the gas flow parameters obtained on the same cross-section are considered to be the same.

3. RESULTS AND DISCUSSION

3.1. Input data

The parameters of the gas flow in the nozzle of the solid fuel rocket engine C5K are determined with the input parameters shown in Table 1, refer to [7].

Table 1. Engine parameters and propellant of the C5K engine

Parameters	Value	Unit
Parameters of the rocket engine		
Inner diameter of the combustion chamber	0.054	m
Outer diameter of the combustion chamber	0.056	m
Length of combustion chamber	0.375	m
Diameter of the nozzle outlet	0.035	m
Diameter of throat	0.012	m
Diameter of nozzle inlet	0.054	m
Number of nozzle	1	m
Length of throat	0.017	m
Nozzle wall thickness at the throat	0.0061	m
Angle of inclination of the converging of the nozzle	50	deg
Angle of inclination of the diverging of the nozzle	102	deg
Parameters of the propellant		
Initial length of the propellant	0.349	m
Outer diameter of the propellant	0.045	m
Inner diameter of the propellant	0.0085	m
Number of the propellant bars	1	
Density of propellant	1600	kg/m^3
Fire rate exponent	0.698	

Law coefficient of fire rate	$1.2x10^{-7}$	m^3/Ns
Thermal stability coefficient of pressure	0.011	1/K
Powder force	570000	J/kg
Primer pressure	400000	Pa
Erosion coefficient	0.000006	S^2/m^2
Initial temperature of the propellant	293	^0K

3.2. Calculation results and comments

Solve the system of equations (1) and (2), (3), (4) by Matlab software with input parameters including structural parameters of the combustion chamber and parameters of propellant. The obtained results include the working characteristics of the combustion chamber as follows: Working pressure in the engine combustion chamber over time (Fig.e 5); Air ejector flow through the nozzle over time (Fig. 6); the total amount of air that has been ejected through the nozzle (Fig. 7).

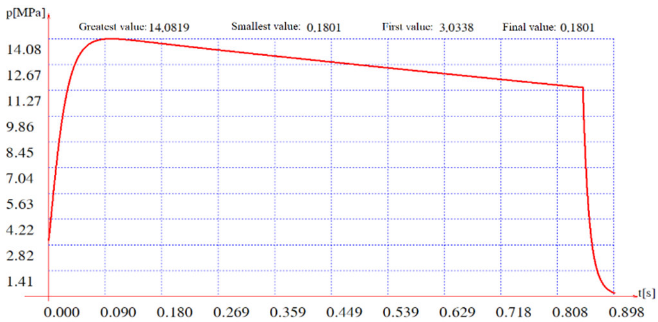

Fig. 5. Working pressure in the engine combustion chamber over time

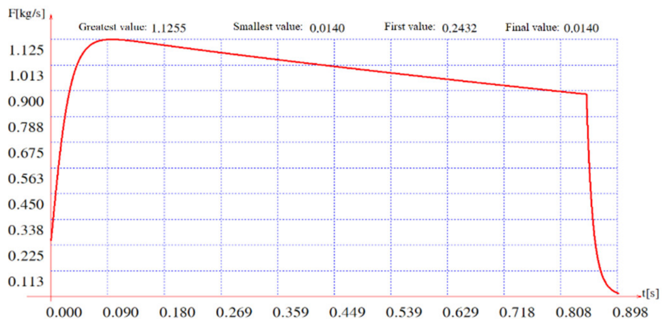

Fig. 6. Air flow through the nozzle over time

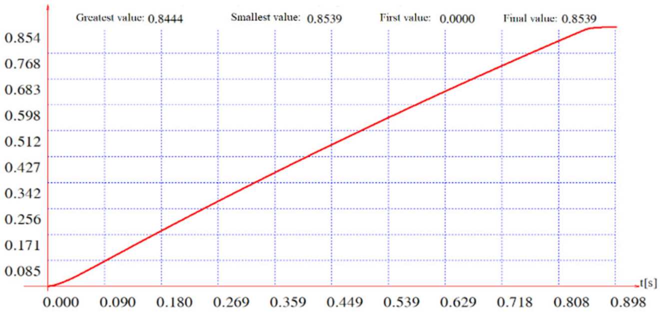

Fig. 7. Total amount of gas ejected through the nozzle over time

After the interior ballistic systems is solved, the input parameters for the problem of determining the air flow parameters in the nozzle are determined. The system of equations (5), (6) and (7) is solved by Matlab software, we get the following results: Velocity of the combustion product flow along the length of the nozzle (Fig. 8); Pressure of combustion product flow along the length of the nozzle (Fig. 9); The temperature of the combustion product stream along the length of the nozzle (Fig. 10).

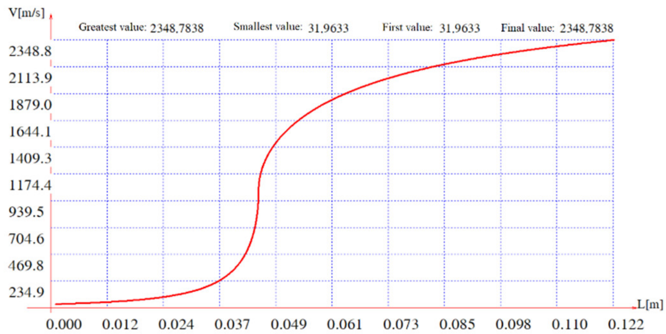

Fig. 8. Graph of the velocity of the combustion gas along the nozzle length

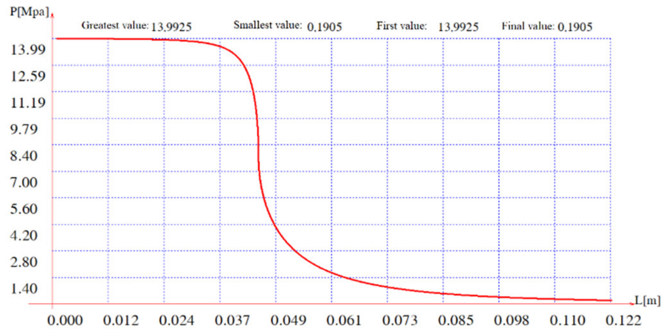

Fig. 9. Graph of the presure of the combustion gas along the nozzle length

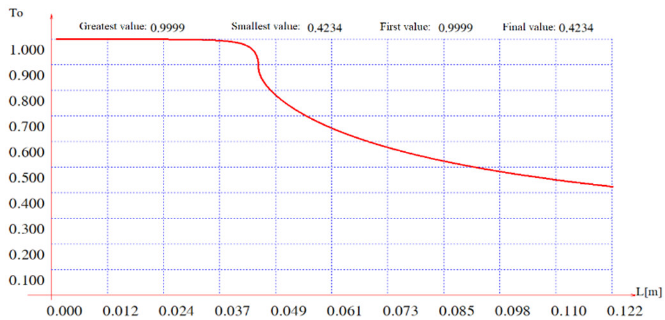

Fig. 10. Graph of the temperature of the combustion gas along the nozzle length

Comment:

- The results of the solution of the interior ballistic problem of SFR give the rule consistent with the actual results. The maximum gas pressure in the engine combustion chamber according to theoretical calculations is 14.0819MPa (see Fig. 5). The maximum pressure in the combustion chamber of the engine experimentally is 14.13MPa [7], error 0.34%.

- Calculation results allow to survey the variation of velocity, pressure and temperature parameters of the flow along the length of the nozzle. From the nozzle inlet to the nozzle outlet, the air flow velocity increases, while the temperature and pressure decrease. This result is completely consistent with reality, refer to [7].

- By analytical method, the optimal size for the nozzle with different designs can be changed such as: change the extension factor of the nozzle, increase or decrease the length of the nozzle's diverging, etc [8]. From there, the design of the nozzle with the highest efficiency is selected.

4. CONCLUSIONS

In this paper, an analytical method is used to determine the gas flow parameters in the nozzle. Through the results of calculating the gas flow parameters in the C5K rocket engine nozzle, the following conclusions are obtained:

- The aerodynamic parameters of the flow in the nozzle calculated by the analytical method have higher accuracy than other calculation methods, so the working parameters of the rocket engine are determined close to more realistic results, refer to [7].

- The results obtained from the analytical method are the aerodynamic parameters of the combustion product flow at any cross section of the nozzle. This is an important theoretical basis in the process of calculating the optimal design of the propeller of a solid fuel rocket engine.

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