

TOPOGRAPHY OPTIMIZATION IN VIBRATION ACOUSTIC RADIATION OF VEHICLE BODY STRUCTURE

TỐI ƯU HÓA TOPOGRAPHY TRONG BỨC XẠ DAO ĐỘNG ÂM CỦA CẤU TRÚC THÂN XE

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ABSTRACT

The structural vibration acoustic radiation is commonly applied in controlling noise of structure. Basing on that, author controls acoustic radiation through the structure topography optimization. The structure topography optimization plays an important role in processing stiffener, decreasing in vibration and coherence in processing as well as assembling. Among the control noise of car structure that the author has been researching, the folded plate is one of the most major one of structural mechanics. The application of the finite element method and boundary element method together with the software calculation has contributed effectively to analyze both vibration characteristics and calculation of the acoustic radiation power of the structure, as well as sound pressure in the closed environment. Besides these, it also builds up finite radiation power with optimized, complex structure; all the calculations show clear effectiveness of optimization and contributes to control noise of structure influenced by structural sound vibration.

Keywords: *Finite element method (FEM); boundary element method (BEM); structural acoustic radiation; topography optimization.*

TÓM TẮT

Bức xạ âm rung động cấu trúc thường được áp dụng trong việc kiểm soát tiếng ồn của cấu trúc. Trên cơ sở đó, tác giả kiểm soát bức xạ âm thông qua việc tối ưu hóa Topography cấu trúc. Việc tối ưu hóa Topography cấu trúc đóng một vai trò quan trọng trong quá trình gia công, giảm độ rung và tính liên kết trong quá trình gia công cũng như lắp ráp. Trong số các tiếng ồn tác động lên kết cấu ô tô mà tác giả đã nghiên cứu, tấm gấp là một trong những loại kết cấu có độ rung lớn nhất về cơ học kết cấu. Việc áp dụng phương pháp phần tử hữu hạn và phương pháp phần tử biên cùng với phần mềm tính toán đã góp phần hiệu quả vào việc phân tích đặc tính rung, tính toán công suất bức xạ âm của kết cấu và áp suất âm trong môi trường kín. Bên cạnh đó, nó cũng tạo ra năng lượng bức xạ hữu hạn với cấu trúc phức tạp, tối ưu hóa...; tất cả các tính toán cho thấy hiệu quả rõ ràng của việc tối ưu hóa kết cấu và góp phần hạn chế tiếng ồn của kết cấu chịu ảnh hưởng của dao động âm.

Từ khóa: *Phương pháp phần tử hữu hạn (FEM); phương pháp phần tử biên (BEM); bức xạ âm cấu trúc; tối ưu hóa Topography.*

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1. INTRODUCTION

Automotive technology is one of engineering multi-disciplinary demanding combination of almost all the small fields. Vibration is an essential field to calculate design of the automotive technology. The automotive technology takes significant steps and gains high achievements in the world. However, control noise inside and outside the car is still one of the scientific aspects needing to be paid attention to and to be researched. The noise inside car affects sound pollution as well as quality and comfort of the passengers of all the structures, plate is one of the structures causing noise mostly. Until now, there have been a lot of researches controlling noise above the plate structure. The plate, in formation of folded and angular ones, has not been researched much, but it is the most important plate structure inside the car. Along with technology and advance in computer science, and applied software have been made contribution to calculate design and simulate engineering. Author has been doing the research between theory of sound, vibration and mechanical engineering. And the application of all the engineering software such as HypeWorld, Nastran, Sysnoi, Matlab to do the calculating and simulating with the title: Topography optimization in vibration acoustic radiation of vehicle body structure

Topography optimization is one of the most effective one of the shape optimization. Unlike topology optimization, topography optimization only makes shape and structure change to meet the demand of design without making material disappear. The change in structure and shape makes not only the power of acoustic radiation but also plate vibration decrease. All the results of sound control are clearly indicated in the analysis by charts.

By now, there have been a lot of researches controlling noise with the following various ways; Topology optimization of the plate by researching interaction between plate and Acoustic cavity is a research that not only a group of the authors such as; W. Akl, A. El-Sabbagh, K. Al-Mitani, A. Baz [1] but also others deeply paid attention to because it indicates relationship among fluid-structures, one of the relationship which is necessary to do the research in acoustic - structure. Also the authors like: T. Yamamoto, S.

Maruyama [2] did research in thickness optimization of multilayered structure by the coupling surface between a structure and an acoustic cavity to make out relationship among fluid-structures. Besides these, the author focuses on transferring matrix representation of multilayered structures, which plays an important part in optimization.

Actually, topology optimization, based on the iterative steps, is the most effective method to build up structural optimization, especially continuum structures. Evolutionary topological optimization of vibrating continuum structures for natural frequencies is one of all the researches of X. Huang, Z.H. Zuo, Y.M. Xie [3] which has pointed out one of the best characteristics of topology optimization. That is material interpolation scheme.

Jun Dong, Kyung K. Choi, Nam H. Kim [4] has also showed acoustic – structure. In addition to this, they did researches on design optimization for acoustic - structural problems using finite element and boundary element with adjoint variable method. The finite element and boundary element methods are popular with researching acoustics with the fields of noise - vibration - harshness (NVH). Software for vibration analysis such as: NASTRAN, COMET/ACOUSTICS were also applied to calculate numerical examples.

Author Zang Xianguo, YU dejia [9] based on not only mode optimization to control acoustic radiation above plate structure but principle of superposition from response surface method, as well as applied professional software like: Nastran, Sysnoi, Isight to analyzing acoustic radiation to carry out shape optimization for plate with a view to control acoustic radiation. However, that there is one limitation in that method is that it was mainly applied to in complex structure and a little element due to the limitation in optimized software.

S. Kilian [6] focused on analyzing finite element, frequency response function FRF and topology optimization method to do the structure optimization for a coherent structure. The result of the calculation and simulation shows us that change in structure has contributed to increase torsion capability and stable operation.

For scientific researches on control noise, author based on basic concepts about vibration and acoustics [15] to optimize structural acoustics vibration with topography structure and control structural acoustic radiation.

2. THE CALCULATION OF STRUCTURAL VIBRATION ACOUSTIC RADIATION

Structural radiation under the interaction of exciting force $f(x, t)$ with circular frequency ω can be written as:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = f(x, t) \quad x \in \Omega^S \quad t > 0 \quad (1)$$

Where: Ω^S is the structural domain; $\{u\}$ is the node displacement vectors of structure; $[M]$ is the mass matrix of structure; $[C]$ viscous damping matrix of structure; $[K]$ is the stiffness matrix of structure; $f(x, t)$ is the exciting force vector from outside. Fourier transform both of equations (1);

outside force is the harmonic force, and the following numbers can be defined:

$$F(x, t) = f(\omega)e^{i\omega t} \quad (2)$$

Where $f(\omega)$ is the amplitude of the exciting and harmonic force; ω is assumed frequency of number, i is imaginary unit demanding $i^2 = -1$. Also, applying a complex form of solution, vector of node displacement can be changed as the following:

$$u(x, t) = u(\omega)e^{i\omega t} \quad (3)$$

Where: $[u(\omega)]$ is the vector of node displacement

Substituting formula of the exciting and harmonic force (2) and formula of vector of node displacement (3) for the first equation (1) without time variables, the equation can be obtained as:

$$\{-\omega^2[M] + i\omega[C] + [K]\}u(\omega) = f(\omega) \quad (4)$$

Like (4), It can be applied the simple change as;

$$[A(\omega)]u(\omega) = f(\omega)$$

Where: $[A(\omega)] = -\omega^2[M] + i\omega[C] + [K]$

The matrix of the node displacement is

$$[u(\omega)] = [A(\omega)]^{-1}f(\omega)$$

If velocity vector of the node is $v(\omega)$, it can be obtained as:

$$v(\omega) = i\omega u(\omega)$$

At the structures of the vibration and fluid and above the surface of the structure, the normal velocity of the node can be written as:

$$v_n = i\omega T A^{-1} = \frac{i\omega T f(\omega)}{(-\omega^2[M] + j\omega[C] + [K])} \quad (5)$$

Where: v_n is the normal velocity above the surface of the structure; T is the transfer matrix which says the relationship to shape of structure's surface.

For the plate structure in material environment with infinite obstacles on the surface (in the acoustic environment). From above the structure's surface under the pressure of exciting and harmonic force and the acoustic pressure P above plate surface, it can be obtained as integral of Raleigh:

$$p(P) = i\omega\rho \int_S G(P, Q)v_n(Q)dS \quad (6)$$

Where:

$p(P)$ is the pressure of point P

ρ is the fluid medium density

S is the structure's surface

Q is the accidental point on the surface of structure

$v_n(Q)$ is the normal velocity of the point Q on the structure's surface

$G(P, Q)$ is the Green function of the semi - infinite free space the Green function of the semi - infinite free space can be written as:

$$G(P,Q) = \exp(-ikr)/2\pi r \tag{7}$$

Where: k is the wave number, $k = \frac{\omega}{c}$

c is the acoustic velocity

r is the distance between two points

$$r = |Q - P|$$

From the formula (6), digitizing the structure's surface, element becomes small, It can be clearly shown that acoustic pressure and the vibration velocity on the surface of the element have constrained condition, so the acoustic pressure of element's surface can be written as:

$$p = Zv_n \tag{8}$$

Where, p is the acoustic pressure of the structure's surface

Z is the matrix of the acoustic impedance

For accidental structure, the formula of power for structural vibration acoustic radiation can be obtained as

$$W = \frac{1}{2} \int_S R_e(pv_n^*) dS = \frac{1}{2} v_n^H \left(\int_S R_e(Z) dS \right) v_n = \frac{1}{2} v_n^H R v_n \tag{9}$$

Where, W is the power of the structural vibration acoustic radiation

v_n^H is the complex conjugate transpose of v_n

R is the acoustic impedance matrix under influence of shape of the structure's surface and the relation between analysis of frequency and fluid medium

$$R = \int_S R_e(Z) dS \tag{10}$$

From formula (9), it can be shown that the normal vibration velocity control of the structure's surface v_n as well as the acoustic impedance matrix of the structure's surface R is two parameters probably strolling acoustic radiation of the structure. Here, we can make out both parameters are under influence of the shape of structural topography.

3. MODEL OPTIMIZATION ANALYSIS

3.1. The mathematical model of structural optimization

Structural optimization consists of 3 factors: design variable, objective function and constrained condition. The design variable is the change coming from the period like parameters improving functions. Whereas, objective function is performance requiring optimization in design as well as the most important of all design variables. The constrained condition is the limit to design; it is also design variable and its performance requirement.

Mathematical model of design optimization can be written as

Minimize:

$$f(X) = f(x_1, x_2, \dots, x_n) \tag{11}$$

Constrained condition:

$$g_j(X) \leq 0 \quad j = 1, 2, \dots, m \tag{12}$$

$$h_k(X) = 0 \quad k = 1, 2, \dots, m_n$$

$$x_i^L \leq x_i \leq x_i^U \quad i = 1, 2, \dots, n$$

Where: $X = x_1, x_2, \dots, x_n$ is the design variable of the structure; $f(X)$ is the objective function; $g(X)$, $h(X)$ is the inequality constraints parameter; L and U is the lower limit and upper limit, $f(X)$ is objective function, $g(X)$ and $h(X)$ is the constrained condition obtained from the structure response in analyzing finite element. Design variable X is the vector, and its choice depends on optimization level.

The author has been carrying out shape optimization of the outside structure; the design variable is linear combinatorial factors of disturbance shape structure.

3.2. Sensibility analysis

The sensibility design is the structure response to the partial derivatives of the design variable - Gradient of the structure response finite element equation can be written as

$$KU = P \tag{13}$$

Where: K is the stiffness matrix; U is the node displacement vector of the element; P is the node load vector of the element the partial derivatives of both sides with the design variable X

$$\frac{\partial K}{\partial X} U + K \frac{\partial U}{\partial X} = \frac{\partial P}{\partial X}$$

It can be rewritten as:

$$\frac{\partial U}{\partial X} = K^{-1} \left(\frac{\partial P}{\partial X} - \frac{\partial K}{\partial X} U \right)$$

According to the structure response with constrained function g , it can be point out function of the displacement vector U as:

$$g = Q^T U$$

Sensibility of the structural radiation can be obtained as:

$$\frac{\partial g}{\partial X} = \frac{\partial Q^T}{\partial X} U + Q^T \frac{\partial U}{\partial X} \tag{14}$$

Adding adjoint variable E to calculating sensibility of the structural radiation. The adjoint variable E demands the following condition:

$$KE = Q$$

Substituting into the equation (14), it can be obtained as

$$\frac{\partial g}{\partial X} = \frac{\partial Q^T}{\partial X} U + E^T \left(\frac{\partial P}{\partial X} - \frac{\partial K}{\partial X} U \right) \tag{15}$$

The above method is called adjoint variable method.

3.3. The approximate expressions model

For the finite element model, the optimization at each iterative step needs to calculate the finite element many times. Therefore, there are a lot of things to do. Also, the finite element implicit needs doing. Approximate model was designed to make the next optimization easy to do (the next model is calculated based on that model).

Sensitivity information is in use to carry out the Taylor expansion with structural radiation, and then the approximate model is obtained. The linear approximation method is one of all approximation methods:

$$\tilde{g}_j(X) = g_{j0} - \sum_{i=0}^N \frac{\partial g_j}{\partial X_i} (X_i - X_{i0}) \tag{16}$$

Approximate inverted:

$$\tilde{g}_j(X) = g_{j0} - \sum_{i=0}^N \frac{\partial g_j}{\partial X_i} X_{i0}^2 \left(\frac{1}{X_i} - \frac{1}{X_{i0}} \right)$$

Approximate convex:

$$\tilde{g}_j(X) = g_{j0} + \sum_{i=0}^N \frac{\partial g_j}{\partial X_i} c_{ji} (X_i - X_{i0}) \tag{17}$$

Where:

$$\text{If } \frac{\partial g_j}{\partial X_i} \geq 0, c_{ji} = 1$$

$$\text{If } \frac{\partial g_j}{\partial X_i} < 0, c_{ji} = \frac{X_{i0}}{X_i}$$

The optimized software is used to select approximate method automatically for pointing out approximate optimization model.

4. THE CALCULATION EXAMPLES

4.1. The theory model

In the aspects of engineering, acoustic characteristics of car body structure is estimated by acoustic sensibility. Where, acoustic sensibility of car body is both the unit of force equal to each point above the car body and noise coming from the resonance inside car. It is also the inherent acoustic characteristics of the car body's structure and the significant index to estimate acoustic characteristics of the structure. Therefore, decrease in acoustic sensibility of car body and the control noise radiation inside car are the same. However, the geometric structure of car body is mainly consists of metal plates in form of folded existence: up mounting plate, before mounting plate, before clapboard, before floor channel, floor before, side frame, rear wheel cover.

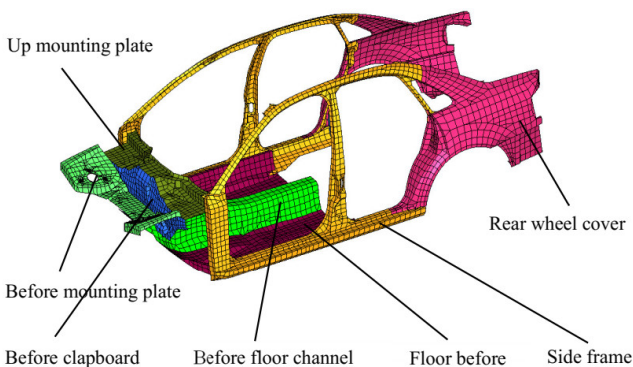


Figure 1. The finite element model car body structure of parts

To carry out the research on structural optimization, author points out theoretical model of structure. Model is a thin plate with folded existence, two beside plates combine with down plate, which makes angle α , plate has measure as: Long 0.3m, wide 0.2m and thickness 1mm. The material parameter function of the structure: The density is 7800kg/m³, air density is 1.21kg/m³, Poisson's ratio is 0.3, acoustic velocity is 343m/s. After having been optimized, the above structural model is changed from plate to shell. Therefore, the calculation is done based on the application of finite element method. The plate is divided into four-node quadrilateral element shell. The above model consists of 2400 elements and 2500 nodes. The node (0, 0.1, 0)m is under pressure of external- exciting force with 10N value, the calculation of acoustic radiation power of the structure in the frequency band is 15 ÷ 300Hz, the step which is acoustic frequency band mainly existing inside the car is 1Hz. The structure is constrained at the outside node (200 nodes) and that is the optimization condition of the structure, and the rest is the topography optimization design area. The Figure 2 is the finite element model - the first theory of the structure

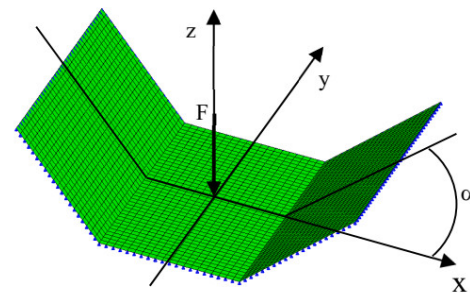


Figure 2. The finite element model the first theory of the structure

4.2. The experimental calculation

To carry out topography optimization of the structure with the decrease condition in acoustic radiation, author has pointed out as the following flow charts as Figure 3.

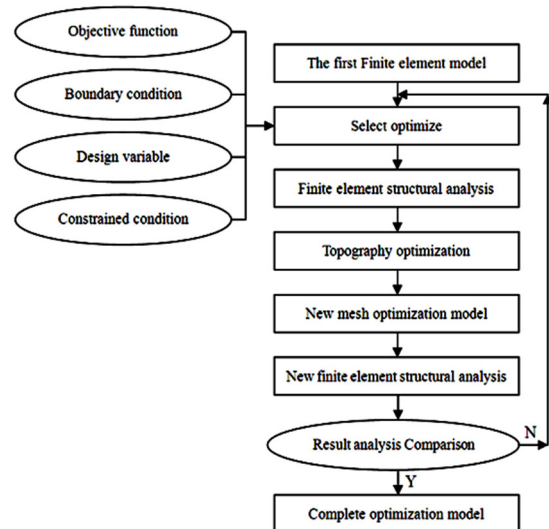


Figure 3. Flowchart of the structural topography optimization

When starting the research, a question is raised? Comparing between the plane plate and the plate with folded existence, apart from required structure with mechanical engineering and assembling, in the respects of acoustics how is the structural vibration acoustic radiation? The following chart shows the result of the structural vibration acoustic radiation power with the plate folded existence ($\alpha = 60^\circ$) and the plane plate ($\alpha = 0^\circ$), Figure 4.

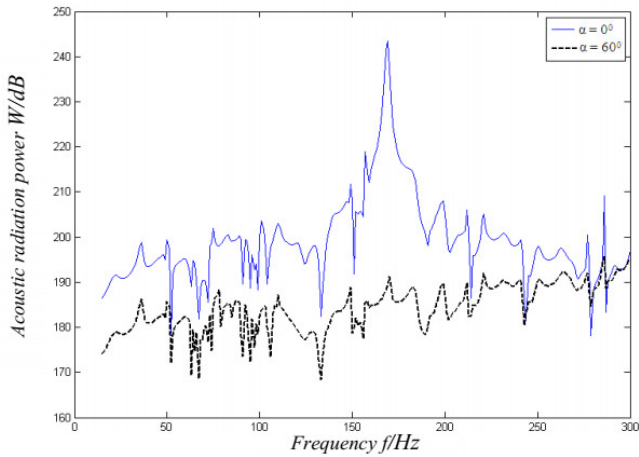


Figure 4. Comparison between results of the plate folded existence and the plane

The chart describes the structure with folded existence, especially for the plate optimization. Nevertheless, we can make out that when α angle changes, the structure will change, too and then change the angle value α to see the change in the structural vibration acoustic radiation, the angle α is changed as: $\alpha = 15^\circ, 45^\circ, 60^\circ, 75^\circ$ and -75° , Figure 5.

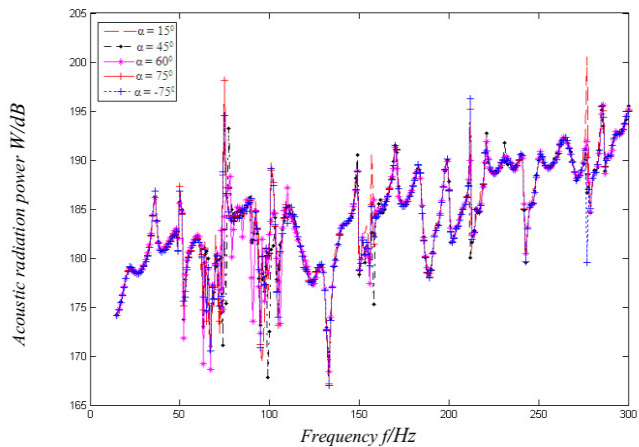


Figure 5. Comparison among acoustic radiation power when changing the value of α angle

Here made some basic comments as: when changing the value of α angle, there is almost no change in the structural vibration acoustic radiation.

By carrying out the theoretical model optimization with the different value of α angle like the above, we can obtain the equal topography optimization and then calculate power of the structural vibration acoustic radiation.

Comparing with the basic theoretical model, we can obtain the optimization value after changing topography shape of structure and the results are shown as Figures 6, 7.

When $\alpha = 75^\circ$ and -75° optimization model and the results of the structural acoustic radiation can be obtained as:

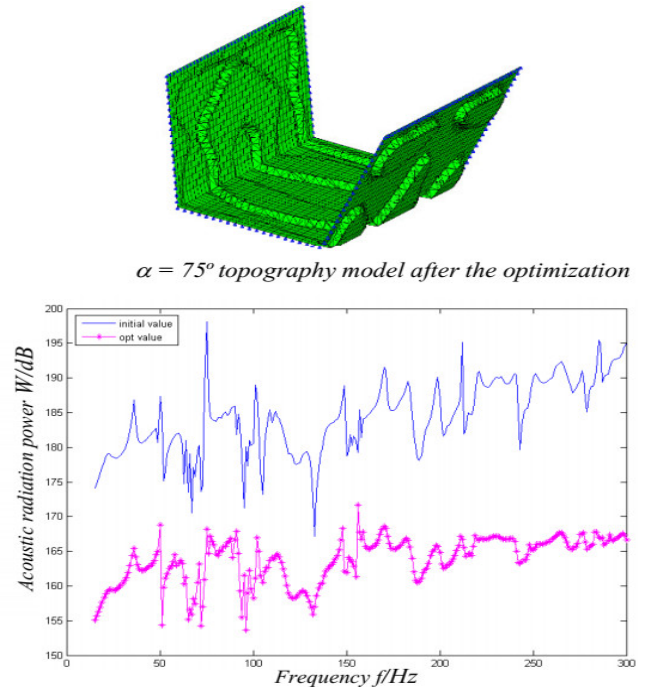


Figure 6. $\alpha = 75^\circ$ Acoustic radiation power model before and after the optimization

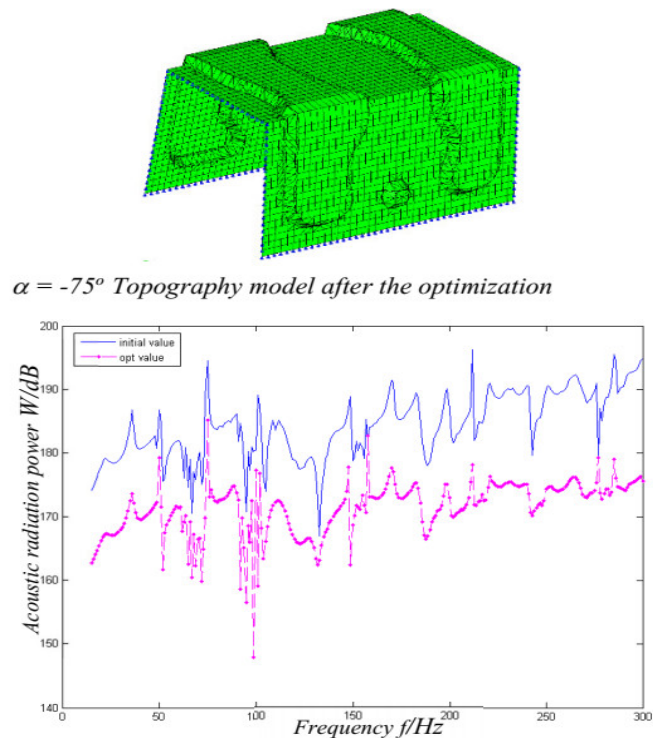


Figure 7. $\alpha = -75^\circ$ Acoustic radiation power model before and after the optimization

And the chart comparing structural acoustic radiation before and after the optimization with $\alpha = -75^\circ$

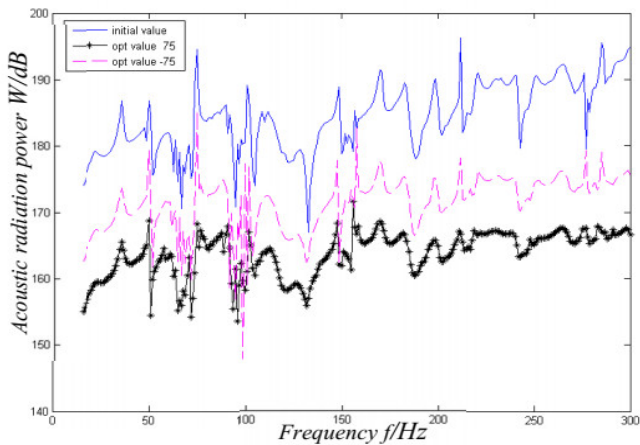


Figure 8. Comparison acoustic radiation power when $\alpha = 75^\circ$ and $\alpha = -75^\circ$

Of all the above optimizations, when $\alpha = 180^\circ$, the theoretical structure shape is symmetric, but the optimization model points out noticeable difference. This is the structural acoustic radiation comparison of two optimization models with $\alpha = 75^\circ$ and $\alpha = -75^\circ$. Figures 8, 9 show the powers of structural acoustic radiation with all the values α given. the comparison helps us to select the coherent value for designing the first shape.

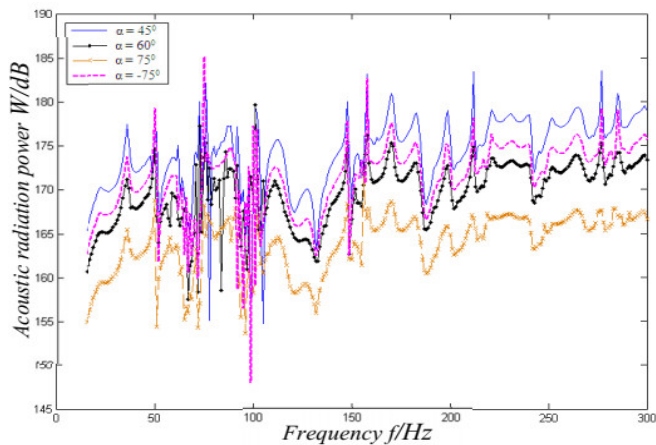


Figure 9. Acoustic radiation power models after the optimization when changing an angle

With the results of optimization, we can see the power optimization of the structural vibration acoustic radiation. However, when studying the effect of sound in the closed environment, besides the structural vibration acoustic radiation, the check of the sound pressure radiation is also done. Because of being studied in the closed environment, the power of the acoustic radiation is not high at all but acoustic pressure is high, which has not good effect on activities in that environment. Therefore, the optimization of the acoustic radiation means the structure optimization to reduce acoustic pressure affecting the ears of the people working in the environment. That is called decrease in the noise radiation. To calculate the noise radiation, we need to

build up the model with the boundary element method. The boundary condition of the model can be obtained by calculating the finite element, the structural vibration radiation. Figure 10 points out the building up field point model of hemispherical with centre of hemispherical is equal to center of the base plane model, radial of hemispherical is 320mm. Also, we can make out all the nodes in the hemispherical is the domain point. The radiation pressure of the model can be obtained through the calculation of the boundary elements.

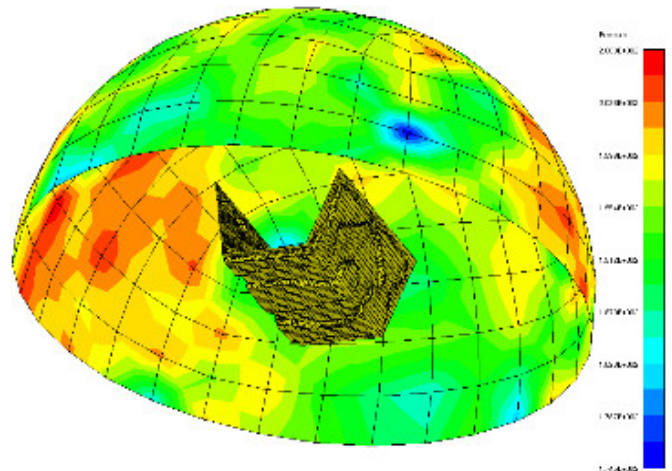


Figure 10. $\alpha = 75^\circ$ The model of field point hemispherical acoustic pressure radiation after the optimization

In the article, author only calculates the acoustic pressure radiation with the model when $\alpha = 75^\circ$ before and after the optimization to estimate the topography optimization result with the frequency of theoretical model. Figure 10 is the model of field point hemispherical acoustic pressure radiation of the model is carried out the optimization with $\alpha = 75^\circ$.

With $\alpha = 75^\circ$, Figure 11 point out comparison of the acoustic pressure domain point before and after the optimization to see the coherence in the topography optimization with structure.

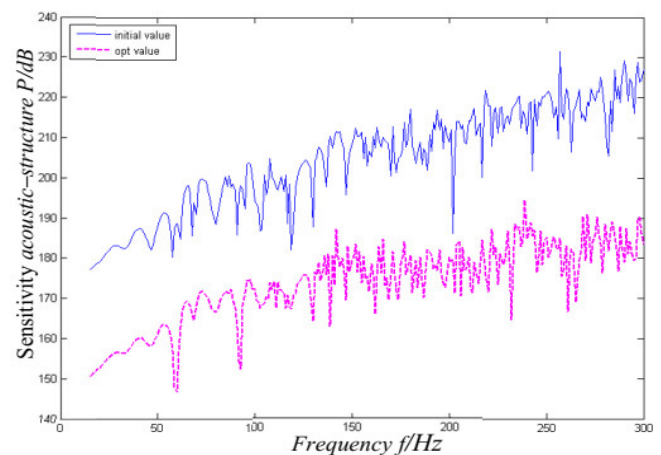


Figure 11. $\alpha = 75^\circ$ Domain point acoustic pressure before and after the optimization

4.3. The application examples

The results of the topography optimization with structural radiation can be seen through the theoretical calculation. However, we have to apply the above results to design model to test the optimization of the method. Here does author do the optimization above the models built up and applied in the the major researches (State key laboratory of advanced design and manufacturing for vehicle body, Hunan University, Changsha 410082). Figure 12 shows the finite element model of the part of car body: Before floor channel before and after the optimization

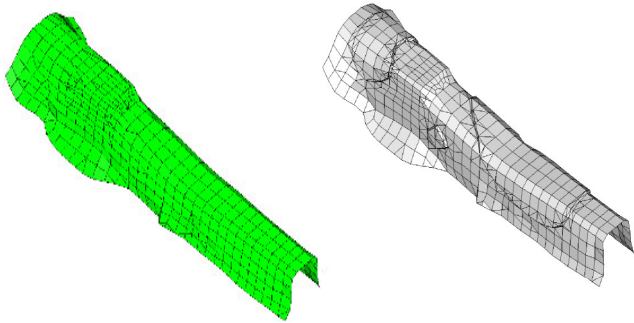


Figure 12. Finite element model of before floor channel before and after the optimization

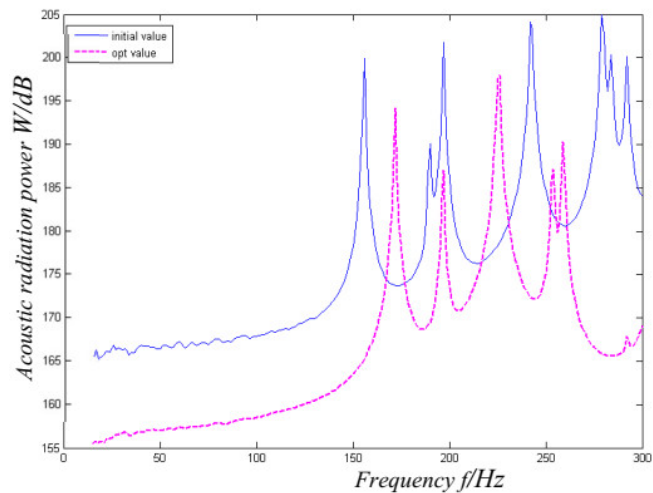


Figure 13. Acoustic radiation power of before floor channel before and after the optimization

Figure 13 is the calculating result chart for the structural acoustic radiation power before and after the optimization with before floor channel model.

As all above, for this calculation, we do not carry out the optimization on the model given. Author builds up the theoretical model from the model given and carry out the topography optimization on the theoretical model, and then compare the result of acoustic radiation power between two models. Figure 14 is the figure of the application model and topography optimization of the theoretical model.

Figure 15 shows the comparative results of the structural vibration acoustic radiation power of the above models.

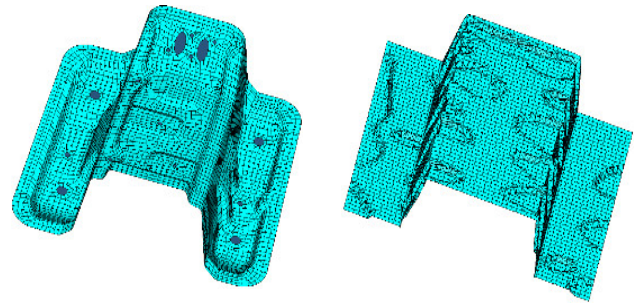


Figure 14. Applied model and topography optimization of the theoretical model

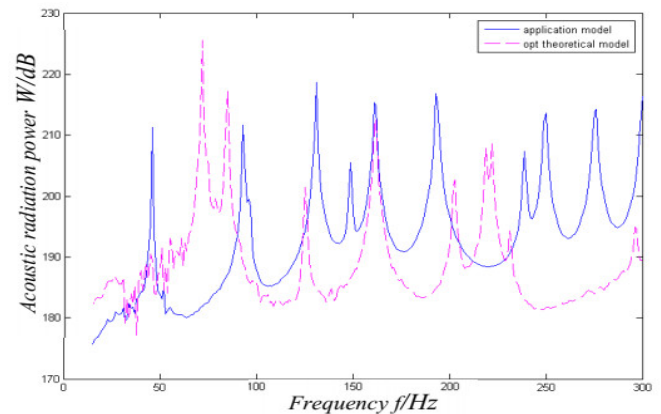


Figure 15. Acoustic radiation power of applied model and optimization of the theoretical model

5. CONCLUSION

With the above results, the following problems can be seen as:

That author points out the structural theoretical model in form of the plate with folded existence is to compare effectiveness with the plane structure (the result of the figure 12 proved it). Although the change in angles of the edge (the change in α angle) does not play a vital role in changing the power value of the structural vibration acoustic radiation of the theoretical model, the change in the α value is important to the optimization shape and suitable for the technology in assembling at the different structures. When α value changes, the optimization shape and the values of the acoustic radiation changes, too. That is one of the problems the designers pay attention to when selecting the best α value for the various objectives.

Figure 11 points out the topography optimization not only demanding acoustic radiation power but also contributing to decrease the value of the acoustic pressure above all the field point given.

The application of topography optimization is not a new method, the main purpose author comments on is the angular structure inside car which is the common structure and effects on making noise in the structure. The structural topography optimization makes contribution to the structural acoustic radiation. It also is one of the steps to design the car body.

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THÔNG TIN TÁC GIẢ

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