NEARLY FREE- END TIME OPTIMAL CONTROL BASED ON OUTPUT FEEDBACK AND MAXIMUM PRINCIPLE FOR THE SINGLE-PENDULUM CRANE SYSTEM

ĐIỀU KHIỂN CẬN TỐI ƯU THỜI GIAN DỰA TRÊN PHẢN HỒI ĐẦU RA VÀ NGUYÊN LÝ CỰC ĐẠI CHO HỆ THỐNG CẨU TREO CON LẮC ĐƠN

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ABSTRACT

This paper aims to introduce a new approach to design a controller to ensure the single-pendulum system (2D overhead crane) to be stable at the desired position based on output feedback information and maximum principle. Due to the necessity of the information of system's swing angle which is inaccessible, an observer is developed to reconstruct the important value. The estimation from the observer and the information about the trolley's positions are utilized to implement the output feedback controller. In this paper, the controller is constructed based on Maximum principle, to ensure the stability of the system within a fixed-time interval and the minimal cost function of time being. Therefore, the high-frequency phenomenon is eliminated, and the amplitude of input signal also be reduced significantly. Thereby, the lifetime of the actuated system and the closed system's durability can be enhanced. The advantages of the prosed control method are validated through various case studies and scenarios of simulation.

Keywords: *The single-pendulum system, state observer, free-end time-optimal control, output feedback principle.*

TÓM TẮT

Bài báo này nhằm mục địch giới thiệu một cách tiếp cận mới để thiết kế bộ điều khiển cho hệ thống cẩu treo con lắc đơn (hệ thống cẩu treo 2 bậc tự do), để giúp hệ thống ổn định tại vị trí mong muốn dựa trên nguyên lý phản hồi đầu ra và nguyên lý cực đại. Với việc không có thông tin về góc lắc, một bộ quan sát góc lắc đã được phát triển để phục hồi lại giá trị của nó. Các dữ liệu từ bộ quan sát, kết hợp với thông tin về vị trí của xe hàng được sử dụng để thiết kế bộ điều khiển. Bộ điều khiển được xây dựng dựa trên nguyên lý cực đại đảm bảo tính ổn định của toàn hệ kín trong một khoảng thời gian cố định với hàm chi phí thời gian gần như nhỏ nhất. Những cải tiến không những loại bỏ hiện tượng rung tần số cao, mà còn giúp giảm đáng kể biên độ của tín hiệu điều khiển. Từ đó bảo vệ tuổi thọ của hệ thống truyền động và nâng cao độ bền vững của hệ kín. Tất cả các kết quả mô phỏng được chứng minh thông qua 3 trường hợp và được thực hiện trên nền tảng Malab/Simulink.

Từ khóa: *Hệ thống cẩu treo con lắc đơn, bộ quan sát trạng thái, điều khiển tối ưu thời gian, nguyên lý phản hồi đầu ra.*

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1. INTRODUCTION

Overhead Cranes [1, 2] are widely deployed in factories and habours in order to move heavy objects and dangerous cargoes from a position to another position. With the rapidly increasing productivity, the claim for the operation of the overhead crane system is more strictly that it can be deployed with an incredibly high speech, robustness, and accuracy. In addition to these mentioned tasks, because the mathematical model of the overhead crane has various nonlinearities and uncertainty parameters, the problem of smooth and efficient control that to make the system can be handled all tasks is a challenge for engineers and research groups around the world. Therefore, the research topic involving the overhead crane has attracted much attention from scientists, and its potentials are the motivation for many papers to improve the system's performance.

With great potential in industrial sectors, the developed control methods for applying to the single-pendulum system are diverse with many incredible results. The paper [3] presented preliminarily methods that could be applied to this system. In general, the single-pendulum system's issue is separated into two missions. The first mission is to design a controller to help the trolley track the desired trajectory while minimizing the swing angle is the main task of the second mission. These epitomizing papers which design a controller based on this perspective are [4, 5] where the swing angle is about 5-7 degrees. Input shaping studied in [6], as well as its enhancements [7], is widely used to minimize the swing angle. Nevertheless, the diagram of the input shaping method was developed based on a feedforward scheme, some disadvantages happened such as restlessness, the convolution function chosen, etc. Particularly, if the frequency of the input impulse series must not be calculated carefully, the oscillation is amplified, and the amplitude of the oscillation becomes maximum due to the vibration resonance. In addition to the classic methods, there are many advance controllers studied such as the partly linearization [8], fuzzy logic controller [9, 10], applying neural network [11], high order sliding mode control [12, 13], hierarchical sliding mode controller [14], etc. All of them have own disadvantages listed in [15].

Optimal control is an effective tool to deal with many optimization problems in control engineering. In addition to the energy optimization of the closed-loop system, the free-end time-optimal problem is an interesting problem but has not received much attraction from researcher due to its complexity in designing. Due to the high frequency of the switching in optimal time controller, inspite of the stability of system's state at the origin, the control input, as well as the state oscillates with a high frequency around the equilibrium value in case of using method in [16]. Therefore, in this paper, a nearly free-end time-optimal controller is proposed. Although the optimal property has not just been rigorous, it not only makes the control input smoother, but the input's amplitude also reduces significantly whereas the trolley's position is very close to the optimal response. Additionally, the end time only depend upon the initial point and the bounded of the control signal. Thereby, depending upon each task, the time to stability is changed suitably. For the single-pendulum system, with the weak stability at the origin of the swing angle, the equation for control the trolley's position, by a linearization transform, has a form of the second-order system. This controller is significant when applying to this system.

To implement the proposed controller effectively, a compulsory requirement is to determine the exact value of the swing angle or the swing angle is measurable. This requirement, normally, is handled by sensors. Nevertheless, in many cases, by the damage of the sensor systems or technical restrictions, this task is not fulfilled. Thus, a swing-angle observer is developed to estimate the values of the swing angle. The convergence of the proposed observer is proven rigorously. The data from the novel observer is utilized to implement the nearly free-end time-optimal controller. This scheme which is constructed by the connection between the optimally proposed controller and the swing-angle observer is called the output feedback-based nearly free-end time-optimal controller. The stability of the closed-loop system is also proven mathematically.

The contributions are listed as below:

- Developing a new state observer to resuscitate the values of the swing angle of the singular-pendulum system. In term of science, this observer helps the estimated value to be asymptotically stable at the real angle. In term of application, this contribution helps to save the cost of installing the sensors.

- A free-end time-optimal controller is improved to steer the trolley's position to be stable at the desired value with minimizing the cost function of time. This enhancement not only makes the control input smoother, but the input's amplitude also reduces significantly whereas the trolley's position is close to the optimal response. Thereby, it protects the lifetime of the actuated system.

In this paper, $\underline{x}$ is symbolised for the column of n real numbers $x_1, x_2, ..., x_n$. The absolute value of a real number x is $|x|$. The notation $\|\underline{x}\|$ represents the 2-norm of a vector $\underline{x}$ and $\|\underline{x}\|_\infty$ is the infinite-norm of a vector $\underline{x}$. The scalar product between 2 vectors $\underline{x}$ and $\underline{y}$ is notated by $\langle \underline{x}, \underline{y} \rangle$ and calculated by $\langle \underline{x}, \underline{y} \rangle = \underline{x}^T \underline{y} = \underline{y}^T \underline{x}$.

2. PROBLEM PRELIMINARY

Consider the single-pendulum system describes in [15] and the its physical model is shown in Figure 1. As can be seen from this figure, a single-pendulum system includes a trolley moving on the rails, a long cable without elasticity, and a cargo.

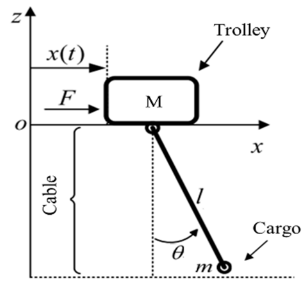

Figure 1. The physical model of a single-pendulum system

The masses of the trolley and the cargo are symbolized by M and m, respectively. l is presented for the length of the system's cable. All of them are assumed as to be known. The single-pendulum system has a unique input force symbolized by F(t). The force F(t) makes the trolley move and the cargo swing simultaneously. Therefore, the vector of states for this system consists of the trolley's position x(t) and the swing angle θ(t). For convenience, this paper will using these notations F, x, θ instead of F(t), x(t), θ(t), respectively.

The mathematical model of the single-pendulum system is given by [15]:

$$\begin{cases} (M+m)\ddot{x} + B\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F \\ ml^2\ddot{\theta} + b\dot{\theta} + ml\ddot{x}\cos\theta + mgl\sin\theta = 0 \end{cases} \quad (1)$$

Where g is the acceleration of gravity, B, b are the static coefficients of the friction.

Because of no information about the exact value of the swing angle θ, as well as its unavailable measurements, the control task here is to determine the input signal F which can make the position of trolley x, starting at an arbitrarily initial point x_0, achieve the desired position x_d such as the under cost function of time is minimum:

$$\Pi(x,t) = \int_0^T dt \to \min \quad (2)$$

Remark 1: The valuable T is arbitrary. Therefore, the optimal control problem here is called free-end time problem or minimum-time problem [17].

To solve the above objective, this paper focuses on:

• Designing a swing-angle observer to estimate the real value of θ by only utilizing the information about the acceleration of the trolley.

• The data from the output of the swing-angle observer is used for developing the controller. The estimated value of swing angle θ, the achieved controller is a nearly free-time optimal controller.

By denote $e_x = x - x_d$ is the tracking error of position, the first equation of equation (1) can be rewritten as the form:

$$\begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = F_m + f_\theta\end{cases} \qquad (3)$$

Where $x_1 = e_x$, $x_2 = \dot{e}_x$, $F_m = \frac{F}{M+m}$ and $f_\theta = -\frac{\left(ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta - Bx_2\right)}{M+m} - \ddot{x}_d$.

Assumption 1: The states of the single-pendulum system are bounded, and the trolley moves slowly enough.

3. MAIN RESULTS

3.1. Design swing-angle observer for single-pendulum system

The symbol $\hat{\theta}$ is the output of the proposed observer and it presents for the estimate avlue of the swing angle θ.

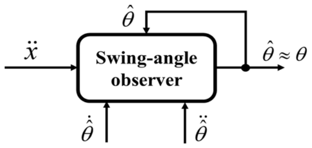

Figure 2. Model of the swing-angle observer

The model of the swing-angle observer is designed as the following:

$$\begin{cases}\dot{\hat{\varphi}}_1 = \hat{\varphi}_2\\ \dot{\hat{\varphi}}_2 = -\alpha^{-1}\left(b\hat{\varphi}_2 + mgl\sin\hat{\varphi}_1\right) - \left(l^{-1}\cos\hat{\varphi}_1\right)\ddot{x}\\ \hat{\theta} = \underline{c}^T\left[\hat{\varphi}_1 \quad \hat{\varphi}_2\right]^T\end{cases} \qquad (4)$$

Where $\alpha > 0$ is a positive number, $\underline{c} = \begin{pmatrix}1\\0\end{pmatrix}$ and $\hat{\varphi}_1, \hat{\varphi}_2$ are two states of the proposed observer.

3.2. Design nearly free-time optimal controller

The design procedure of the nearly free-end time-optimal controller consists of 2 steps. Firstly, the nonlinear mathematical model in (3) is converted to the second-order LTI system with model error using an output feedback regime. And the following step is to design the free-end time- optimal controller based on the HJB equations. This controller will ensure the closed-loop system to be stable at the origin in a finite time.

Step 1: Transmitting the nonlinear single-pendulum system to the second-order LTI system with model error by output feedback principle.

Consider the nonlinear system which is depicted in equation (3). Proposing the output feedback controller

$$F_m = v + \frac{\left(ml\ddot{\hat{\theta}}\cos\hat{\theta} - ml\dot{\hat{\theta}}^2\sin\hat{\theta} - B\dot{x}\right)}{M+m} \qquad (5)$$

Where v is an auxilary input signal. The equation (3) becomes:

$$\Leftrightarrow \begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = v + \tilde{f}\end{cases} = \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}v + \begin{pmatrix}0\\ \tilde{f}\end{pmatrix} = A\underline{x} + \underline{b}^T v + \underline{\tilde{d}} \qquad (6)$$

Where

$\tilde{f} = \frac{\left(ml\ddot{\hat{\theta}}\cos\hat{\theta} - ml\dot{\hat{\theta}}^2\sin\hat{\theta} - B\dot{x}\right)}{M+m} - \frac{\left(ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta - B\dot{x}\right)}{M+m}$,

$\underline{x} = \begin{pmatrix}x_1\\ x_2\end{pmatrix}$, the matrix $A = \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$, $\underline{b} = \begin{pmatrix}0 & 1\end{pmatrix}$ and $\underline{\tilde{d}} = \begin{pmatrix}0\\ \tilde{f}\end{pmatrix}$.

Remark 2: To overcome the difficulty of the inaccessibility of the swing angle θ and its derivations $\dot{\theta}, \ddot{\theta}$, an observer therefore is proposed to estimate them. However, it is difficult to achieve $\hat{\theta} = \theta$ in a finite time interval, it directly leads to $\underline{\tilde{d}} \neq \underline{0}$. The linear system guaranteed in (6) is a nearly LTI system.

Assumption 2: $\underline{\tilde{d}}$ is finite, that means $\left\|\underline{\tilde{d}}\right\|_\infty < +\infty$.

Step 2: Design the nearly free-end time-optimal controller

To construct the free-end time-optimal controller for system (6), firstly we consider the LTI system being similar to system (6) without model error and with respect to (w.r.t) the saturation condition of auxilary input signal:

$$\dot{\underline{x}} = A\underline{x} + \underline{b}^T v \ \text{ w.r.t } |v| \leq k \qquad (7)$$

Denote that $\underline{\varphi}$ is the costate vector for system (7) and $\underline{x}^*, v^*, \underline{\varphi}^*$ are the optimal value of $\underline{x}, v, \underline{\varphi}$, respectively. According to the Halminton-Jacobi-Bellman principle, these values $\underline{x}^*, v^*$ and $\underline{\varphi}^*$ need to satisfy [17]:

$$H\left(\underline{x}^*, v^*, \underline{\varphi}^*\right) = 1 + \left\langle A^T\underline{x}^*(t), \underline{\varphi}^*(t)\right\rangle + \left\langle v^*(t), \underline{b}\underline{\varphi}^*(t)\right\rangle = 0, \quad \forall t \in [0, T] \qquad (8)$$

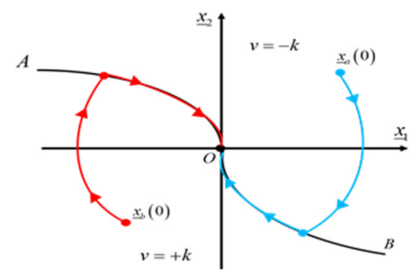

Figure 3. The switching-curve

And $\underline{x}^*, v^*, \underline{\varphi}^*$ is the unique optimal solution [17]. Because the cost function (2) is linear and the constraint $|v| \le k$ defines an isochrone $S_T = \{v \in \mathbb{R} | -k \le v \le k\}$. Therefore, according to the HJB principle and the optimazation theory, the optimal control signal v^*, which makes the system's state $\underline{x}(t)$ moving from an arbitrarily initial point $\underline{x}(0)$ to $\underline{x}(T) = \underline{0}$ in a finite time T, is determined by:

$$v^* = \underset{v \in \mathbb{R}}{\mathrm{argmin}} H(\underline{x}, v, \underline{\varphi}) = -k\,\mathrm{sign}(\underline{b}\underline{\varphi}^*) = \begin{cases} -k, \text{if} & \underline{b}\underline{\varphi}^* \ge 0 \\ k, \text{if} & \underline{b}\underline{\varphi}^* < 0 \end{cases} \quad (9)$$

Where $\underline{\varphi}^*$ satisfies $\dot{\underline{\varphi}}^* = -\dfrac{\partial H(\underline{x}^*, v^*, \underline{\varphi}^*)}{\partial \underline{x}^*} = -A^T \underline{\varphi}^*$. Figure (3) depicted the state trajectory of system (7) in phase space in cases $v = \pm k$ with two diffirently initial point $\underline{x}_a(0)$ and $\underline{x}_b(0)$ (blue line and red line, respectively). As can be seen that, when the state trajectory is in the switching-curve line, it goes to the origin in a finite time. Moreover, whenever the state trajectory is not in AOB, the optimal control signal v^* is opposite in sign to $L_{AOB}(\underline{x})$. But when $\underline{x}(t) \in L_{AOB}(\underline{x})$, the optimal control signal v^* is opposite in sign to x_1. Thus, the equation of the swiching-curve line AOB, and the state feedback optimal controller v^* are achieved by substittuting (9) into (7) and Figure (3):

$$L_{AOB}(\underline{x}) = x_1 + \frac{1}{2k} x_2 |x_2| \quad (10)$$

$$\Rightarrow v^* = v^*(\underline{x}) = \begin{cases} -k\,\mathrm{sign}\left(x_1 + \dfrac{1}{2k} x_2 |x_2|\right), \text{if} \quad L_{AOB}(\underline{x}) \ne 0 \\ -k\,\mathrm{sign}(x_1), \text{if} \quad L_{AOB}(\underline{x}) = 0, \text{and} \quad x_1 \ne 0 \end{cases} \quad (11)$$

Remark 3: Since $H(\underline{x}^*, v^*, \underline{\varphi}^*) = 0, \forall t \in [0, T]$, the optimal costate vector $\underline{\varphi}^*$ must be a nonzero vector: $\underline{\varphi}^* \ne 0, \forall t \in [0, T]$.

Nevertheless, using the signal function in equation (11) makes the control input is discrete or the vibration frequency of the oscillation in $v^*(\underline{x})$ becomes immensely large. This phenomenon brings a negative effect on system's performance, as well as shortening the system's durability. So, this paper proposes a smoother free-end time-optimal controller:

$$v^*(\underline{x}) = \begin{cases} -k\tanh\left[\alpha\left(x_1 + \dfrac{1}{2k} x_2 |x_2|\right)\right], \text{if} \quad L_{AOB}(\underline{x}) \ne 0 \\ -k\tanh(\alpha x_1), \text{if} \quad L_{AOB}(\underline{x}) = 0, \text{and} \quad x_1 \ne 0 \end{cases} \quad (12)$$

Where α is a positive number and large enough. The controller (12) is called nearly free-end time-optimal controller.

Remark 4: Although the optimal property is broken, replacing the signal function in equation (11) with the tanh hyperbolic function in (12) can reduce the amplitude of control input significantly. Simultaneously, the oscillation's frequency is also smaller. By choosing α to be large enough, the state trajectory of system (7) is extremely close to the optimal trajectory.

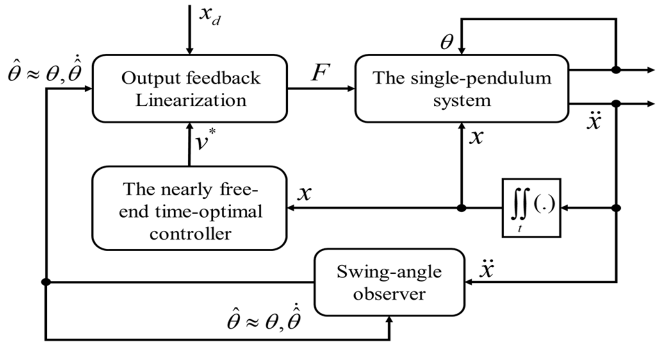

Figure 4. The scheme of the closed-loop system

Theorem 1: The free-end time-optimal controller $v^*(\underline{x})$ will steer the state trajectory of (7) in a fixed time T calculated by:

$$T = 2\sqrt{\frac{|\varepsilon|}{k}} \quad (13)$$

Where ε is the first element of $\underline{x}(0) = (\varepsilon \quad 0)^T$.

Proof: The state trajectory of the LTI system (7) is calculated by [18]:

$$\underline{x}(t) = e^{At}\underline{x}(0) + \int_0^t e^{A(t-\tau)} \underline{b}^T v^*(\tau) d\tau \quad (14)$$

Define t_1 is the time interval that $v^*(\underline{x}) = \delta k$. It is easy to see that the time interval that $v^*(\underline{x}) = -\delta k$ is $(T - t_1)$. From (14) we have:

$$\underline{x}(t) = e^{At}\underline{x}(0) + \delta k \int_0^{t_1} e^{A(t-\tau)} \underline{b}^T d\tau - \delta k \int_{t_1}^{T-t_1} e^{A(t-\tau)} \underline{b}^T d\tau \quad (15)$$

Utilizing the condition for the end point $\underline{x}(T) = (0 \quad 0)^T$, noting that the initial condition of system is $\underline{x}(0) = (\varepsilon \quad 0)^T$ and $e^{At} = \sum_{n=0}^{+\infty} \dfrac{(At)^n}{n!} = I + At$ (due to $A^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$). From equation (20) we have:

$$\begin{cases} t_1 = \dfrac{T}{2} \\ \delta k\left(2Tt_1 - t_1^2 - \dfrac{T^2}{2}\right) = -\varepsilon \end{cases} \Leftrightarrow \begin{cases} t_1 = \dfrac{T}{2} \\ T^2 = \dfrac{-4\varepsilon}{\delta k} \end{cases} \quad (16)$$

Since $T^2 > 0$, it must be select $\delta = -\text{sign}(\varepsilon)$. From (13), we have $T = 2\sqrt{\frac{|\varepsilon|}{k}}$. The proof is complete.

4. NUMERICAL SIMULATION AND DISCUSSION

To validate the effectiveness of the nearly proposed free-time optimal controller, as well as the estimation performance of the swing-angle observer, the numerical simulations will be handled in Matlab/Simulink platform. The parameters of the single-pendulum system is given as the following: M = 10[kg], m = 1[kg], g = 9.81[m/s^2], B = b = 0.1. Performance of the proposed scheme is verified through two case studies.

Case study 1: Validating the control quality of nearly free-end time-optimal controller

In this case, the paper focus on validating the control quality of the proposed controller with some different values of k. In detail, we simulate with k = 3, k = 6, k = 15, k = 0.5, k = 0.2 and k = 1. To ensure the nearly optimal property, α = 300 is selected. All the simulation results in this case study are implemented within 15 seconds and shown in Figures 5 and 6. The desired position is assumed to be at 2 meters from the initial position. The initial for tracking error $e_x(0)$ is $(2 \;\; 0)^T$.

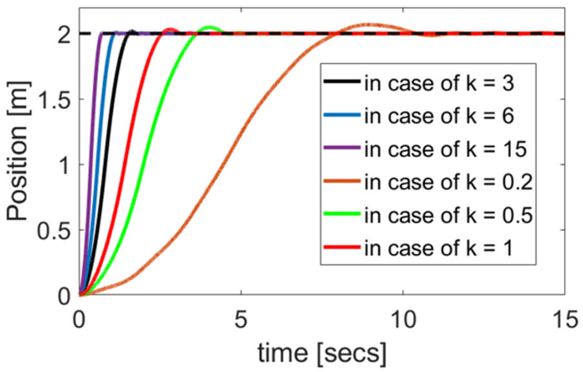

Figure 5. The positional responses of the system with different values of k

As can be seen easily in Figure 5, the proposed method steers the trolley's state at the desired position in a fixed time. With different values of k and using the equation (13) in theorem 1, the data of the time of stability is listed in table 1.

Table 1. The data of the time of stability in accordance with value of k

k = 0.2	12.4 (s)	k = 3	1.633 (s)
k = 0.5	4(s)	k = 6	1.1547 (s)
k = 1	2.8284 (s)	k = 15	0.7393 (s)

The data in table 1 are compared with the data in Figure 5, it can be seen that the proposed controller satisfies the control task. The optimal control input is shown in Figure 6.

The responses in both Figures 5 and 6 show that the greater the value of k is, the greater the control input is also, and the fast the trolley moves to the desired position. By contrast, with the small value of k, the more slowly the trolley moves and the smaller the control input is. Therefore, depending upon the control task and the restrictions of actuated system, the value of k is selected suitably.

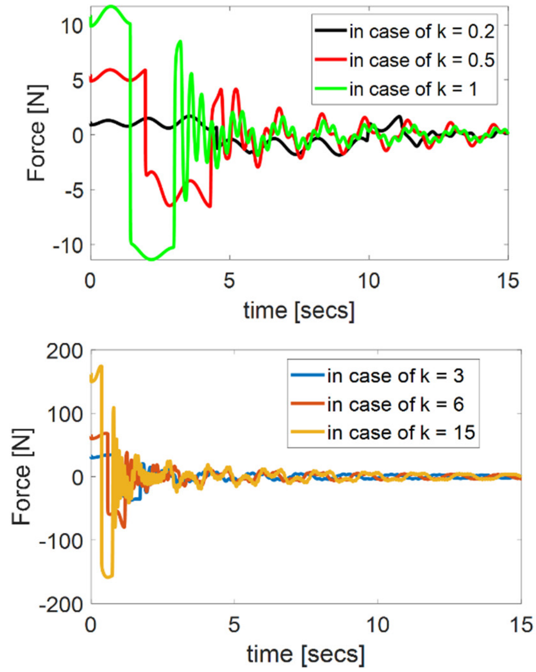

Figure 6. The optimal control input

Case study 2: Comparing to an existing method

The aim of this case study is to compare the proposed method with another existing method and to demonstrate the effectiveness of our proposals. The simulation is implemented within 15 seconds and the existing method chosen to compare is the fixed-time stabilization method for non-autonomous system studied in [16]. All simulations are shown in Figures 7 and 8. In all numerical results in this case, the value of k is fixed at k = 0.8.

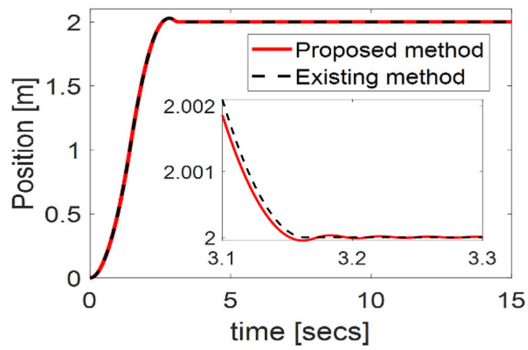

Figure 7. The positional response of the proposed method and existing method

With k = 0.8, using theorem 1, we have T = 3.1623(s). Observing Figure 7 reveals that replacing the signal function in (11) by tanh hyperbolic function in (12), the exactly optimal property is broken. But by selecting α is large enough, the time interval for trolley's stability at the origin deviates non-significantly from the optimal value (about 3.2 (s) - slower 0.04 (s) comparing to the optimal value of time). For single-pendulum system, the truncation error of stable time is immensely small and does not affect the control quality of the closed-loop system.

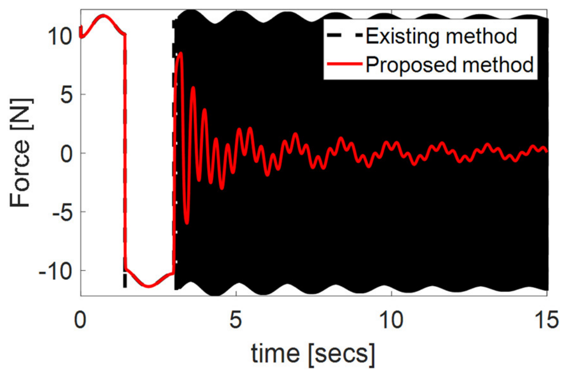

Figure 8. The control input in case of the proposed controller and existing controller

The effectiveness of the proposals can be observed clearly in Figure 8. If utilizing the existing method in [16], the optimal property is reached exactly but it makes the control input signal to be oscillated with many high frequencies. This phenomenon has a negative impact to actuated system. By contrast, the nearly free-end time-optimal control reduces the high frequencies and simultaneously declines the amplitude of input signal. This advantage not only helps the actuated system to prop up the high-frequency phenomenon, but the positional response also is immensely close to the optimal trajectory.

5. CONCLUSION

This paper has presented a new approach for designing a controller to steer the trolley's position to be stable at the desired point in a fixed-time interval without the information about the swing angle. The nearly free-end time-optimal controller has been constructed based on the output feedback principle. The inaccessible information of swing angle has been estimated by a proposed swing-angle observer. This solution is advantages in saving the cost of installing the sensors. The controller has used the estimated data from observer to determine the control input based on the maximum principle. By using the proposed optimal controller, the control input has become smoother, and its amplitude be reduced significantly whereas the trolley's position has been brought back to the desirable optimal value. The effectiveness of our proposals has been validated through two cases studies in Matlab/Simulink platform.

REFERENCES

[1]. Alyazidi N.M., Hassanine A.M., Mahmoud M.S., 2023. *An online adaptive policy iteration-based reinforcement learning for a class of a nonlinear 3d overhead crane*. Applied Mathematics and Computation 447, 127810.

[2]. Huang J., Wang W., Zhou J., 2022. *Adaptive control design for underactuated cranes with guaranteed transient performance: Theoretical design and experimental verification.* IEEE Transactions on Industrial Electronics 69, 2822–2832. doi:10.1109/TIE. 2021.3065835

[3]. Abdel-Rahman E. M., Nayfeh A. H., Masoud Z. N., 2003. *Dynamics and control of cranes: A review*. Journal of Vibration and control, vol. 9, pp. 863-908.

[4]. Park M.S., Chwa D., Hong S.K., 2008. *Antisway tracking control of overhead cranes with system uncertainty and actuator nonlinearity using an adaptive fuzzy sliding-mode control*. IEEE Transactions on Industrial Electronics, vol. 55, pp. 3972-3984

[5]. Dai S., Lv Z., Liu Z., 2010. *Study of precise positioning and antiswing for the varying rope length in 3D crane systems base on the combination of partial decoupling and fuzzy control, in Robotics and Biomimetics (ROBIO)*. 2010 IEEE International Conference on, 2010, pp. 656-661.

[6]. Ahmad M., Ismail R. R., Ramli M., Ghani N. A., Hambali N., 2009. *Investigations of feed-forward techniques for anti-sway control of 3-D gantry crane system, in Industrial Electronics & Applications*. ISIEA 2009. IEEE Symposium on, 2009, pp. 265-270.

[7]. Chwa D., 2009. *Nonlinear tracking control of 3-D overhead cranes against the initial swing angle and the variation of payload weight*. IEEE Transactions on Control Systems Technology, vol. 17, pp. 876-883.

[8]. Park H., Chwa D., Hong K., 2007. *A feedback linearization control of container cranes: Varying rope length*. International Journal of Control Automation and Systems, vol. 5, p. 379.

[9]. Wang Z., Chen Z., Zhang J., 2011. *On PSO based fuzzy neural network sliding mode control for overhead crane*. in Practical Applications of Intelligent Systems, ed: Springer, 2011, pp. 563-572.

[10]. Park M.S., Chwa D., Eom M., 2014. *Adaptive sliding-mode antisway control of uncertain overhead cranes with high-speed hoisting motion*. IEEE Transactions on Fuzzy Systems, vol. 22, pp. 1262-1271.

[11]. Zhan Y., Cheng H.z., Ge N.c., Huang G.b., 2005. *Generalized growing and pruning RBF neural network based harmonic source modeling*. Proceedings-Chinese society of electrical engineering, vol. 25, p. 42.

[12]. Weimin X., Xiang Z., Yuqiang L., Mengjie Z., Yuyang L., 2015. *Adaptive dynamic sliding mode control for overhead cranes*. in Control Conference (CCC), 2015 34th Chinese, 2015, pp. 3287-3292.

[13]. Wang W., Liu X., Yi J., 2007. *Structure design of two types of sliding-mode controllers for a class of under-actuated mechanical systems*. IET Control Theory & Applications, vol. 1, pp. 163-172.

[14]. Qian D., Yi J., Zhao D., 2008. *Hierarchical sliding mode control for a class of SIMO under-actuated systems*. Control and Cybernetics, vol. 37, p. 159

[15]. M. R. Mojallizadeh, B. Brogliato, C. Prieur, 2023. *Modeling and control of overhead cranes: A tutorial overview and perspectives*. Annual Reviews in Control, p. 100877.

[16]. Shen Z., Andersson S. B., 2010. *Minimum time control of a second-order system*. 49th IEEE Conference on Decision and Control (CDC), 4819-4824.

[17]. Michel Athans, Peter L. Falb., 1966. *Optimal Control, An introduction to the theory and its applications*. NewYork McGraw-Hill.

[18]. Isidori A., 1999. *Nonlinear Control Systems II*. Springer Verlag.

THÔNG TIN TÁC GIẢ

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Khoa Các khoa học ứng dụng, Trường Quốc tế, Đại học Quốc gia Hà Nội