

# AN EFFICIENT ADAPTIVE CONTROL STRATEGY USING BACKSTEPPING CONTROL TECHNIQUE AND FUZZY LOGIC FOR WHEELED MOBILE ROBOT

MỘT CHIẾN LƯỢC ĐIỀU KHIỂN THÍCH NGHI HIỆU QUẢ DỰA TRÊN KỸ THUẬT ĐIỀU KHIỂN BACKSTEPPING VÀ HỆ LOGIC MỜ CHO XE TỰ HÀNH

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## ABSTRACT

This paper presents a novel adaptive controller for wheeled mobile robots (WMR) based on backstepping control technology (BCT) and fuzzy logic system (FLS). The controller is designed using BCT, and then its uncertainty parameters are adjusted using FLS. In addition, the stability of the closed-loop control system is proven based on the Lyapunov principle. The simulation results show that the WMR can precisely move on its own according to its intended trajectory. Moreover, with the integration of the fuzzy logic system, the proposed controller's quality is significantly improved compared to the conventional backstepping controller.

**Keywords:** 3-wheeler, Backstepping control, fuzzy logic system.

## TÓM TẮT

Bài báo trình bày về một bộ điều khiển thích nghi mới cho xe tự hành bằng việc sử dụng kỹ thuật điều khiển backstepping kết hợp với hệ logic mờ. Đầu tiên bộ điều khiển được thiết kế dựa trên kỹ thuật backstepping, sau đó các tham số bất định của bộ điều khiển được chỉnh định dựa trên hệ logic mờ. Bên cạnh đó, tính ổn định của hệ thống điều khiển vòng kín được chứng minh dựa theo nguyên lý Lyapunov. Những kết quả mô phỏng cho thấy xe tự hành có khả năng di chuyển chính xác theo quỹ đạo mong muốn. Hơn nữa, với việc tích hợp thêm hệ logic mờ, chất lượng của bộ điều khiển đề xuất được cải thiện đáng kể so với bộ điều khiển backstepping thông thường.

**Từ khóa:** Xe 3 bánh, điều khiển Backstepping, hệ Logic mờ.

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## 1. INTRODUCTION

Wheeled mobile robots (WMRs) are nonlinear systems popularly studied in the control field because of their

flexible mobility. For example, WMR has been used in industrial, military and space exploration [1]. In practice, it is difficult to obtain an exact mathematical model of WMRs due to uncertain parameters. Depending on the number of input signals, it is possible to classify WMRs as full actuators [1, 2] or under-actuated systems [3, 4, 5]. Designing a controller for a nonlinear, underactuated system is significantly more challenging than one for full actuators.

In the literature, many studies have been invested in designing controllers for WMRs. The most straightforward approach is based on the PID control law. Although this approach achieves the stability and tracking of the WMR, its control quality is low. In [6], a fuzzy logic system (FLS) was incorporated into the PID controller to adjust the parameters based on bias. This has improved the accuracy of orbital tracking and responsiveness of the system, but there are still many errors. In addition, there are other advanced approaches, such as backstepping and sliding mode control techniques [7]. In [8], the authors proposed an adaptive fuzzy controller (AFSMC) for WMRs, where the control law is given by bias and the derivative of that bias so that the WMRs follow the correct trajectory. Simulation results showed that the system has a good response but stabilizes slowly. A backstepping controller is presented in [9] for WMRs in which the control law is derived from linear velocity and angular velocity signals. However, BCT controllers still have some disadvantages, such as a poor ability to adapt to changes in the environment and many uncertain parameters in the control law that directly affect the system's performance. Therefore, this paper has developed a new adaptive control law based on BCT [10] and FLS to improve the response quality of WMRs.

The rest of the paper is organized as follows: The mathematical model of WMRs is presented in Section 2. Section 3 offers the steps to design a Backstepping controller for WMRs, and the adaptive control law based on BCT and the FLS is described in Section 4. The simulation

results are presented in section 5. Finally, the conclusions and development directions are summarized in section 6.

**2. MATHEMATICAL MODEL**

The model of the WMR is shown in Figure 1. This robot is controlled by differential, so the rear wheel is controlled, and the robot's steering is done independently. The front wheel is added to maintain the balance of the vehicle. The parameters of the robot are as follows:  $m$  and  $J$  are the respective mass of the robot and the moment of inertia of the vehicle,  $T_1$  and  $T_2$  are the torques of the right and left motors linked to the motion of the driving wheels. Also,  $r$  is the radius of the rudder,  $L$  represents half the distance between the rudders, and  $\lambda$  is the Lagrange coefficient. The position and orientation of the robot in the Cartesian coordinate system is determined by the vector  $q = [x, y, \theta]^T$ .

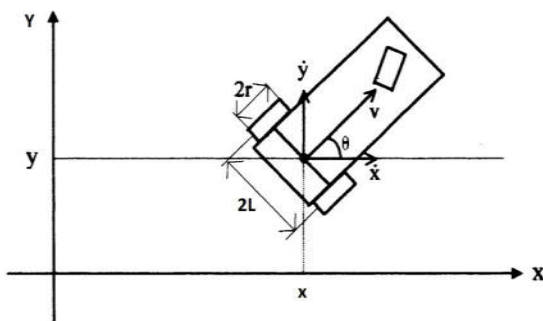


Figure 1. The dynamic model of the robot

The following Euler-Lagrange equation is applied to describe the kinematics and dynamics of the above WMRs as follows [7]:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} = E(q)T - A^T\lambda \tag{1}$$

In there:

$$M(q) = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J \end{bmatrix} \text{ is the calculation matrix.}$$

$$C(q, \dot{q}) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ is the Coriolis matrix and the centripetal}$$

force.

$$E(q) = \frac{1}{r} \begin{bmatrix} \cos\theta & \cos\theta \\ \sin\theta & \sin\theta \\ L & -L \end{bmatrix}, T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}, A^T(q) = \begin{bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{bmatrix}$$

The given Lagrange coefficient:

$$\lambda = -m\dot{\theta}(\dot{x}\cos\theta + \dot{y}\sin\theta)$$

Combined with the kinematic nonholonomic linkage and the non-slip rolling condition, we have:

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0$$

The robot model is rewritten as follows:

$$\begin{cases} \ddot{\theta} = b_1 u_1 \\ \ddot{x} = \frac{\lambda}{m} \sin\theta + b_2 u_2 \cos\theta \\ \ddot{y} = \frac{\lambda}{m} \cos\theta + b_2 u_2 \sin\theta \end{cases} \tag{2}$$

In there:  $b_1 = L/(rJ)$ ,  $b_2 = 1/(rm)$  is a constant and  $u_1 = T_1 - T_2$ ,  $u_2 = T_1 + T_2$  are the control inputs. So  $T_1 = 0.5(u_1 + u_2)$ ,  $T_2 = 0.5(u_2 - u_1)$ .

**3. BACKSTEPPING CONTROL**

Model (2) is split into two subsystems. The first system depends on the control signal, and the second subsystem depends on the control signal  $u_2$ . The traction controller for WMR uses a Backstepping controller to synthesize control signals  $u_1$  and  $u_2$ . The backstepping control system architecture is shown in Figure 2.

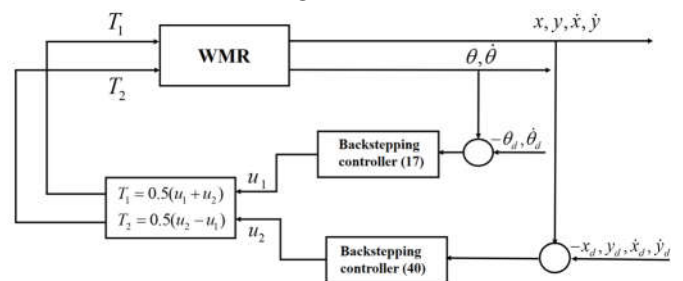


Figure 2. Backstepping control structure diagram

The following content will present the steps of controller synthesis.

**3.1. Backstepping controller for the car to follow the direction angle**

Consider the first subsystem from model (2):

$$\ddot{\theta} = b_1 u_1 \tag{3}$$

The task is to design the control signal  $u_1$  so that the output signal angle  $\theta$  follows the set direction angle  $\theta_d$ . We use BCT to design an angle controller.

Set a new state variable:

$$z_1 = \theta, z_2 = \dot{\theta} \tag{4}$$

Now, a subsystem (3) can be rewritten as:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = b_1 u_1 \end{cases} \tag{5}$$

Set angle deviation variable:

$$r_1 = \theta - \theta_d \tag{6}$$

Derivative  $r_1$  with respect to time we have:

$$\dot{r}_1 = \dot{\theta} - \dot{\theta}_d = z_2 - \dot{\theta}_d \tag{7}$$

To falsify  $r_1 \rightarrow 0$ , we choose Liapunov function:

$$V_{11} = \frac{1}{2} r_1^2 \tag{8}$$

Derivative  $V_{11}$  with respect to time we have:

$$\dot{V}_{11} = r_1 \dot{r}_1 = r_1 (z_2 - \dot{\theta}_d) \tag{9}$$

Set:

$$r_2 = z_2 - \alpha_1 \tag{10}$$

So:

$$z_2 = r_2 + \alpha_1 \tag{11}$$

where  $\alpha_1$  is the virtual control signal for the first subsystem of (5). We select virtual control signal for the first subsystem:

$$\alpha_1 = -a_1 r_1 + \dot{\theta}_d \tag{12}$$

Replace (11), (12) into (9) then:

$$\begin{aligned} \dot{V}_{11} &= r_1 \dot{r}_1 = r_1 (z_2 - \dot{\theta}_d) = r_1 (r_2 + \alpha_1 - \dot{\theta}_d) \\ &= r_1 (r_2 - a_1 r_1) = -a_1 r_1^2 + r_1 r_2 \end{aligned} \tag{13}$$

In there, operand  $-a_1 r_1^2$  makes the system stable, operand  $r_1 r_2$  will be discarded in the next step.

From (10), the derivative  $r_2$  with respect to time we have:

$$\dot{r}_2 = \dot{z}_2 - \dot{\alpha}_1 = \ddot{\theta} - \dot{\alpha}_1 = b_1 u_1 - \dot{\alpha}_1 \tag{14}$$

Choose the Lyapunov function for system (14) as follows:

$$V_1 = V_{11} + \frac{1}{2} r_2^2 \tag{15}$$

Derivative  $V_1$  with respect to time:

$$\begin{aligned} \dot{V}_1 &= \dot{V}_{11} + r_2 \dot{r}_2 \\ &= (r_1 \dot{r}_1 + r_2 \dot{r}_2) \\ &= r_1 (z_2 - \dot{\theta}_d) + r_2 \dot{r}_2 \\ &= r_1 (z_2 - \dot{\theta}_d) + r_2 (b_1 u_1 - \dot{\alpha}_1) \\ &= -a_1 r_1^2 + r_1 r_2 + r_2 (b_1 u_1 - \dot{\alpha}_1) \end{aligned} \tag{16}$$

We choose the control signal:

$$\begin{aligned} r_1 + b_1 u_1 - \dot{\alpha}_1 &= -a_2 r_2 \\ u_1 &= -\frac{1}{b_1} (r_1 + a_2 r_2 - \dot{\alpha}_1) \end{aligned} \tag{17}$$

In which constants:  $a_1 > 0, a_2 > 0$ , substitute (17) into (16) we have:

$$\dot{V}_1 = -a_1 r_1^2 - a_2 r_2^2 < 0, \forall r_1, r_2 \neq 0 \tag{18}$$

It will ensure that system (3) is asymptotically stable, and the output  $\theta$  angle follows the set direction angle  $\theta_d$ .

### 3.2. Backstepping controller for tracking vehicle

From equation (2), we have the model of the second subsystem:

$$\begin{cases} \ddot{x} = \frac{\lambda}{m} \sin\theta + b_2 u_2 \cos\theta \\ \ddot{y} = \frac{\lambda}{m} \cos\theta + b_2 u_2 \sin\theta \end{cases} \tag{19}$$

We rewrite equation (19) as follows:

$$\begin{cases} \dot{x}_1 = J_{11} x_2 \\ \dot{x}_2 = f_1(\mathbf{X}) + g_1(\mathbf{X}) \tau_1 \\ \dot{x}_3 = J_{22} x_4 \\ \dot{x}_4 = f_2(\mathbf{X}) + g_2(\mathbf{X}) \tau_2 \end{cases} \tag{20}$$

With  $\tau_1, \tau_2$  are two control signals. In which,

$$\begin{aligned} \mathbf{X} &= [x_1 \ x_2 \ x_3 \ x_4]^T = [x \ \dot{x} \ y \ \dot{y}]^T \\ f_1(\mathbf{X}) &= \frac{\lambda}{m} \sin\theta \ ; \ f_2(\mathbf{X}) = \frac{\lambda}{m} \cos\theta \\ g_1(\mathbf{X}) &= b_2 \cos\theta \ ; \ g_2(\mathbf{X}) = b_2 \sin\theta \end{aligned} \tag{21}$$

The definition of the error vector between the output signal and the reference signal is as follows:

$$\mathbf{e}(t) = \begin{bmatrix} e_1 \\ e_3 \end{bmatrix} = \begin{bmatrix} x_1 - x_{1d} \\ x_3 - x_{3d} \end{bmatrix} \tag{22}$$

Considering system (20) as two subsystems (23), (24) with control signals for each system, we have:

$$\begin{cases} \dot{x}_1 = J_{11} x_2 \\ \dot{x}_2 = f_1(\mathbf{X}) + g_1(\mathbf{X}) \tau_1 \end{cases} \tag{23}$$

$$\begin{cases} \dot{x}_3 = J_{22} x_4 \\ \dot{x}_4 = f_2(\mathbf{X}) + g_2(\mathbf{X}) \tau_2 \end{cases} \tag{24}$$

The common control signal for system (19), (20) is selected according to the following law:

$$\mathbf{u}_2 = \alpha \tau_1 + \beta \tau_2 \tag{25}$$

With  $\alpha, \beta$  are positive constants.

System (23), (24) is the reverse transport system 2. According to the Backstepping technique, to determine the control signal  $\tau_1, \tau_2$  we must proceed in two steps:

Synthesize the control law  $\tau_1$  consider the system (23):

**Step 1:** Determine the position error in the x-axis  $e_1$  is as follows:

$$e_1 = x_1 - x_{1d} \tag{26}$$

Derivative of  $e_1$  with respect to time, we have:

$$\dot{e}_1 = \dot{x}_1 - \dot{x}_{1d} = J_{11} x_2 - \dot{x}_{1d} \tag{27}$$

Set:  $\mathbf{e}_2 = \mathbf{x}_2 - \alpha_1$  with  $\alpha_1$  is the virtual control signal.

Instead (27):

$$\dot{e}_1 = J_{11} (\mathbf{e}_2 + \alpha_1) - \dot{x}_{1d} \tag{28}$$

To determine the guaranteed virtual control signal  $\mathbf{e}_1 \rightarrow 0$ , we choose the function Lyapunov:

$$V_{22} = \frac{1}{2} \mathbf{e}_1^T \mathbf{e}_1 \tag{29}$$

Derivative  $V_1$  with respect to time we have:

$$\begin{aligned} \dot{V}_{22} &= \mathbf{e}_1^T \dot{\mathbf{e}}_1 = \mathbf{e}_1^T (J_{11} (\mathbf{e}_2 + \alpha_1) - \dot{x}_{1d}) \\ &= -c_1 \mathbf{e}_1^T \mathbf{e}_1 + \mathbf{e}_1^T J_{11} \mathbf{e}_2 \end{aligned} \tag{30}$$

To get (30) virtual control signals of the following form:

$$\alpha_1 = J_{11}^{-1}(-c_1 e_1 + \dot{x}_{1d}) \tag{31}$$

With  $c_1$  is a positive constant. Let  $e_1 \rightarrow 0$  then  $e_2 \rightarrow 0$

**Step 2:** We have:

$$e_2 = x_2 - \alpha_1 \tag{32}$$

Derivative  $e_2$  with respect to time we have:

$$\dot{e}_2 = \dot{x}_2 - \dot{\alpha}_1 = f_1(\mathbf{X}) + g_1(\mathbf{X})\tau_1 - \dot{\alpha}_1 \tag{33}$$

To determine the control signal  $\tau_1$  ensures  $e_2 \rightarrow 0$ , we choose the function Lyapunov:

$$V_2 = V_{22} + \frac{1}{2} e_2^T e_2 \tag{34}$$

The derivative  $V_2$  with respect to time we have:

$$\begin{aligned} \dot{V}_2 &= \dot{V}_{22} + e_2^T \dot{e}_2 \\ &= -c_1 e_1^T e_1 + e_1^T J_{11} e_2 + e_2^T (f_1(\mathbf{X}) + g_1(\mathbf{X})\tau_1 - \dot{\alpha}_1) \end{aligned} \tag{35}$$

Select the control signal derived from the equation (35):

$$\tau_1 = g_1^{-1}(\mathbf{X})(-c_2 e_2 - J_{11}^T e_1 - f_1(\mathbf{X}) + \dot{\alpha}_1) \tag{36}$$

With  $c_2$  is a positive constant ( $c_2 > 0$ ).

Substitute equation (36) into (35) we have:

$$\dot{V}_2 = -c_1 e_1^T e_1 - c_2 e_2^T e_2 < 0, \forall e_1, e_2 \neq 0 \tag{37}$$

Synthetic control  $\tau_2$ . Consider system (24):

Backstepping design is similar to  $\tau_1$  design, we get the control signal for the second subsystem:

$$\tau_2 = g_2^{-1}(\mathbf{X})(-c_4 e_4 - J_{22}^T e_3 - f_2(\mathbf{X}) + \dot{\alpha}_2) \tag{38}$$

With:  $c_3, c_4$  are positive constants. In there:

$$\begin{aligned} e_3 &= x_3 - x_{3d} \\ e_4 &= x_4 - \alpha_2 \\ \alpha_2 &= J_{22}^{-1}(-c_3 e_3 + \dot{x}_{3d}) \end{aligned}$$

According to (25) the control signal for the position of the tricycle is:

$$u_2 = \alpha\tau_1 + \beta\tau_2 \tag{39}$$

With  $\tau_1, \tau_2$  calculated according to (36), (38) instead of equation (39) we have:

$$\begin{aligned} u_2 &= \alpha g_1^{-1}(\mathbf{X})(-c_2 e_2 - J_{11}^T e_1 - f_1(\mathbf{X}) + \dot{\alpha}_1) \\ &\quad + \beta g_2^{-1}(\mathbf{X})(-c_4 e_4 - J_{22}^T e_3 - f_2(\mathbf{X}) + \dot{\alpha}_2) \end{aligned} \tag{40}$$

### 3.3. Theorem statement and proof of the stability of the closed system

To analyze the stability of the system, the control signal for WMR is synthesized as follows:

$$T = [T_1 \quad T_2]^T = \begin{bmatrix} \frac{u_1 + u_2}{2} & \frac{u_2 - u_1}{2} \end{bmatrix}^T \tag{41}$$

**Theorem:** If the control law is designed as in (41), where  $u_1, u_2$  are described as in (17) and (40) respectively, then the closed system will be asymptotically stable.

**Proof:**

Choose a Lyapunov function for a closed system:

$$V = \frac{1}{2} r_1^2 + \frac{1}{2} r_2^2 + \frac{1}{2} e_1^T e_1 + \frac{1}{2} e_2^T e_2 = V_1 + V_2 \tag{42}$$

$$\text{In there: } V_1 = \frac{1}{2} r_1^2 + \frac{1}{2} r_2^2 \text{ và } V_2 = \frac{1}{2} e_1^T e_1 + \frac{1}{2} e_2^T e_2$$

Derivative  $V$  with respect to time:

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \tag{43}$$

From (18), we have:

$$\dot{V}_1 = -a_1 r_1^2 - a_2 r_2^2 < 0, \forall r_1, r_2 \neq 0 \tag{44}$$

And from (37):

$$\dot{V}_2 = -c_1 e_1^T e_1 - c_2 e_2^T e_2 < 0, \forall e_1, e_2 \neq 0 \tag{45}$$

From equation (44) see  $\dot{V}_1 < 0$ , and equation (45) there  $\dot{V}_2 < 0$ . Therefore:

$$\dot{V} = \dot{V}_1 + \dot{V}_2 < 0 \tag{46}$$

This proves that the closed system is stable Lyapunov.

### 4. ADAPTIVE BACKSTEPPING CONTROL

In the control law in equation (40), the  $c_1, c_2, c_3, c_4$  parameters must be detected by trial and error method and the quality of the system is also directly affected through the parameter selection. With  $c_1, c_2$  are the two parameters that directly affect the tracing in the x-direction, with  $c_3, c_4$  are the two parameters that affect the tracking in the y-direction. In each state, if a suitable set of parameters is selected, it will give a good response, so in this part, the fuzzy backstepping controller is studied for 3-wheel self-propelled vehicle in order to adjust the best parameters for the Backstepping controller according to the change of error and position deviation, the control structure diagram is shown in Figure 3.

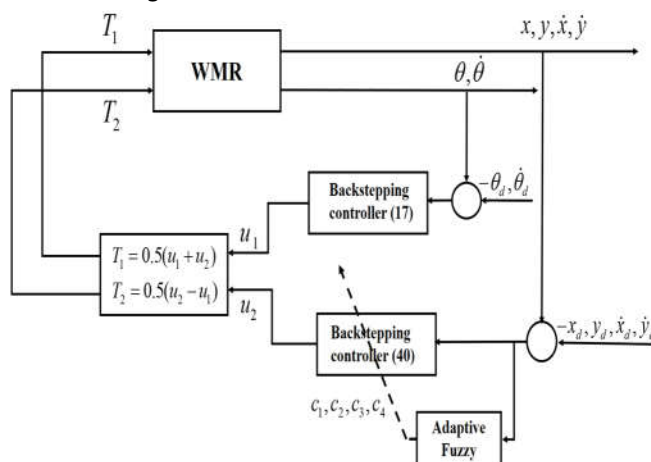
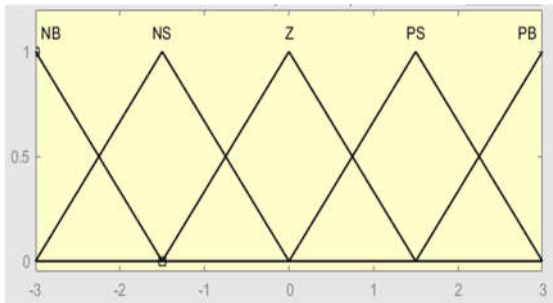
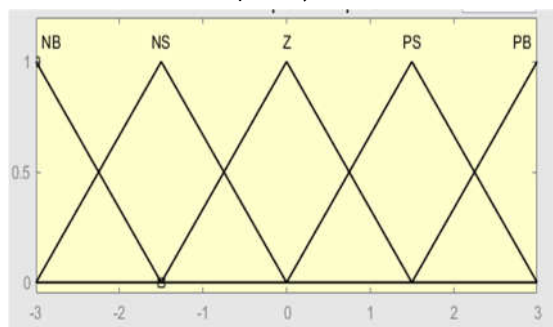


Figure 3. Structural diagram of the fuzzy adjustable Backstepping controller

The input to the fuzzy tuner is the tracking error of the robot's trajectory  $e(t) = [e_1, e_3]^T$  and its time derivative  $\dot{e}(t) = [\dot{e}_1, \dot{e}_3]^T$ . The fuzzy sets for the input language variable are shown in Figures 4 and 5.

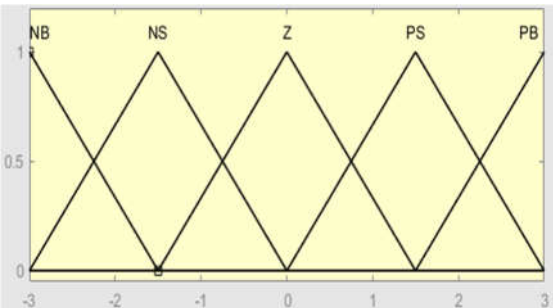


(a) Input fuzzy sets  $e_1$

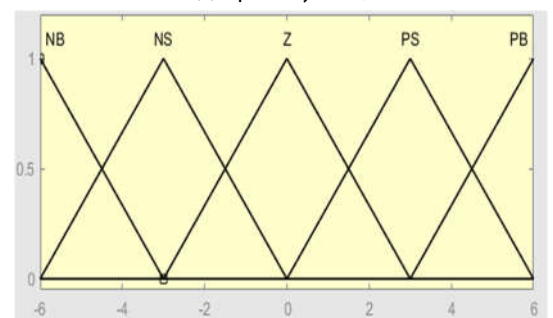


(b) Input fuzzy sets  $\dot{e}_1$

Figure 4. Fuzzy set for error and difference derivative in the x-direction



(a) Input fuzzy sets  $e_3$



(b) Input fuzzy sets  $\dot{e}_3$

Figure 5. Fuzzy set for error and deviation derivative in the y-direction

The fuzzy sets for the input language variable as well as the output values and the composition rules for the fuzzy tuner are built on the Sugeno fuzzy model.

Table 1. Fuzzy sets of input language variables

Language variable $e_1, e_3$	Language variable $\dot{e}_1, \dot{e}_3$	Meaning
NB	NB	Large negative interval
NS	NS	Small negative interval
Z	Z	0 interval
PS	PS	Small positive interval
PB.	PB	Large positive interval

The fuzzy sets for the input language variable  $e_1, e_3, \dot{e}_1, \dot{e}_3$  have a triangular shape and the output  $c_1, c_2, c_3, c_4$  is selected experimentally. The names of the fuzzy sets and their meanings are shown in Table 1. The output values of the fuzzy tuner are shown in Table 2.

Table 2. Output values

Output variable	Meaning	Output value for $c_1$	Output value for $c_2$	Output value for $c_3$	Output value for $c_4$
VS	Very small	30	10	35	10
S	Small	40	20	45	20
M	Medium	50	30	55	30
B	Large	60	40	65	40
VB	Very large	70	50	75	50

From formulas (36) and (38), we see that component  $(c_1, c_2)$  is the parameter affecting the grip in the x-direction, the component  $(c_3, c_4)$  is the parameter affecting the grip in the y-direction. To simplify the selection of values for the fuzzy set and reduce unnecessary computation, the outputs  $(c_1, c_2)$  and  $(c_3, c_4)$  are chosen equally and the base inference rule of the fuzzy modifier for the four outputs are shown in Tables 3 and 4.

Table 3. Output-based inference system  $(c_1, c_2)$

$(c_1, c_2)$		$e_1$				
		NB	NS	Z	PS.	PB
$\dot{e}_1$	NB	M	B	VB.	B	M
	NS	S	M	B	M	S
	Z	VB.	S	M	S	VB.
	PS	S	M	B	M	S
	PB	M	B	VB.	M	M

Table 4. Output-based inference system  $(c_3, c_4)$

$(c_3, c_4)$		$e_3$				
		NB	NS	Z	PS.	PB
$\dot{e}_3$	NB	M	B	VB.	B	M
	NS	S	M	B	M	S
	Z	VB.	S	M	S	VB.
	PS	S	M	B	M	S
	PB	M	B	VB.	M	M

5. SIMULATION RESULTS

In this section, some simulations in Matlab are performed to verify the effectiveness of the proposed controller. The model parameters of WMR are selected as follows:

Table 5. Model parameters of the WMR

Parameter	m	J	r	L
Value	1.038kg	0.818kg/m <sup>2</sup>	0.025m	0.075m

The desired trajectories are linear functions for positions along the x-, y-axes with the orientation angle as a constant.

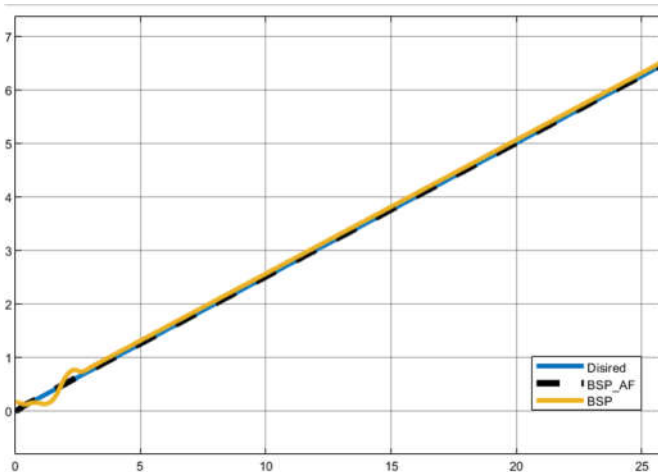
$$\text{Trajectory set: } q = [x, y, \theta]^T = \left[ 0.25t, 0.25t, \frac{\pi}{4} \right]^T$$

The initial controller parameters in (17) and (40) of the Backstepping controller are selected in Table 6.

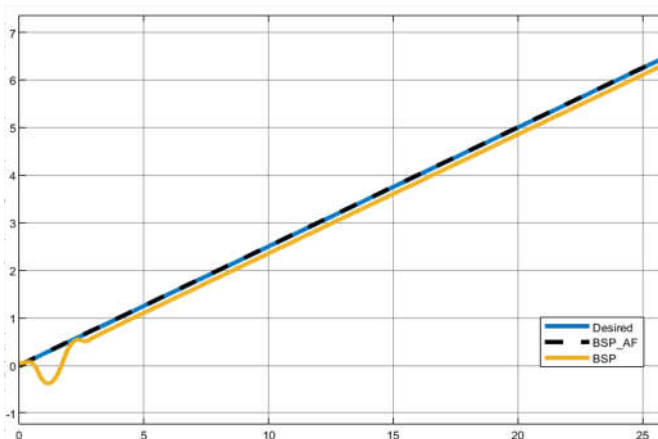
Table 6. Select parameters

Parameter	a <sub>1</sub>	a <sub>2</sub>	c <sub>1</sub>	c <sub>2</sub>	c <sub>3</sub>	c <sub>4</sub>	α	β
Value	2	4	56	9	58	16	125	25

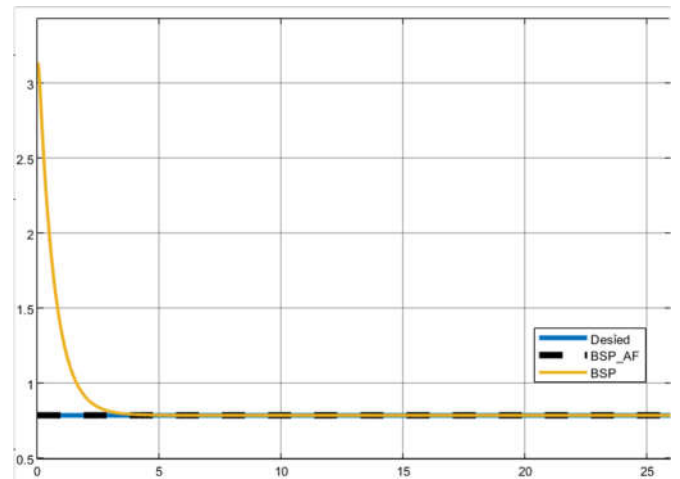
The simulation results using the fuzzy adaptive Backstepping (BSP\_AF) controller and the Backstepping (BSP) controller are shown in Figure 6.



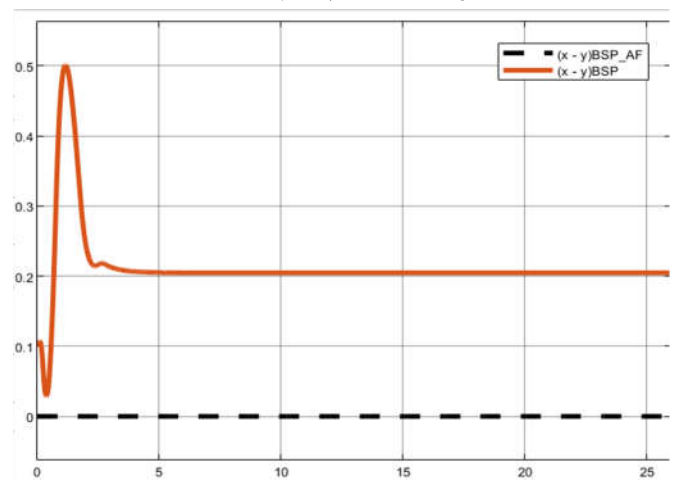
(a) Trajectories follow the x-direction



(b) Trajectories follow the y-direction



(c) The trajectory follows the angle



(d) Vehicle trajectory in terms of x - y

Figure 6. Simulation and verification of two Backstepping algorithms and fuzzy Backstepping

**Comment:** The simulation results in Figure 6 show that the quality of the Backstepping controller with fuzzy tuning is much better than that of the conventional Backstepping controller. With a Backstepping controller with the fuzzy setting, the actual vehicle's trajectory overlaps the desired trajectory entirely, and the x-y difference is always 0. Conventional Backstepping controller for quality control still exists oscillations in the x and y directions for the first 3-3.5 seconds. At the same time, the angular traction is over-adjusted; the initial value is greater than three rads after 3 seconds to stabilize to the set value. In addition, the normal Backstepping controller also has the x-y effect that is always wrong. Even when stable, the x - y value is equal to 0.2. Therefore, the fuzzy Backstepping controller overcomes the overshoot in the direction angle oscillation in the x-, y-directions and optimizes the conventional Backstepping trajectory tracking quality.

6. CONCLUSION

This paper has been proposed an adaptive controller that combines BCT with FLS to adjust the uncertain parameters, improving conventional Backstepping

controllers' performance. Simulation results show that the proposed method is effective, namely, that the response trajectory of the system follows the desired one in a short time. In the future, we plan to install the proposed control algorithm on actual WMRs and develop further more efficient algorithms.

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## THÔNG TIN TÁC GIẢ

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