PROPOSED FAST CONVERGENCE TECHNIQUE FOR SINR MAXMIN PROBLEM IN MIMO WIRELESS RELAY NETWORK

ĐỀ XUẤT KỸ THUẬT HỘI TỤ NHANH CHO BÀI TOÁN CỰC ĐẠI TỐI THIỂU SINR TRONG MẠNG TRUYỀN DẪN VÔ TUYẾN MIMO

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ABSTRACT

Nonconvex SINR constraints in amplify-and-forward (AF) multiple-input multiple-output (MIMO) relay beamforming problems make the design mathematically intractable. A more practical optimization is to maximize the minimum SINR achievable data rate among all the users. The previously existing methods invoke the vectorization of pre-coding matrix X and then introduce the auxiliary variable matrix of lager dimension which results in an increased computational complexity. In order to deal with large size problems, the number of additional variables should be reduced. In this article, we propose to use the optimization technique to improve the optimal speed through the minmax SINR problem in which the dimension of the optimization problem is significantly smaller than that of the mentioned DCI approach.

Keywords: *Nonsmooth optimization, maxmin SINR, exact penalty function optimization, total transmit power.*

TÓM TẮT

Bài toán thiết kế điều hướng (beamforming) với các điều kiện ràng buộc SINR không lồi trong mạng truyền dẫn vô tuyến chuyển tiếp MIMO phương thức AF là những bài toán khó giải. Trong đó, vấn đề cực đại tối thiểu SINR giữa các người dùng phía thu có nhiều ý nghĩa trong thực tế. Các kỹ thuật tối ưu đề xuất trước đây sử dụng việc véc-tơ hóa ma trận tiền giải mã và tạo ra ma trận phụ với chiều không gian lớn dẫn tới sự gia tăng độ phức tạp tính toán. Để giải quyết vấn đề giảm thiểu tính toán thông qua việc thực hiện giảm số các biến phụ. Bài báo đã đề xuất sử dụng kỹ thuật tối ưu nhằm tăng tốc độ hội tụ đối với bài toán cực đại tối thiểu SINR. Trong đó, kích thước của bài toán tối ưu hóa sẽ giảm đáng kể so với kỹ thuật DCI đã được đề cập trước đây.

Từ khóa: *Tối ưu hóa không trơn, cực đại tối thiểu SINR, tối ưu hóa hàm phạt chính xác, tổng công suất phát.*

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1. INTRODUCTION

Relay communication is a subject that has received a phenomenal attention from both industry and research community in the last ten years. "Conventional" relay communication has focused on the use of multiple SISO (single-input singleoutput) relays to assist the communication between users. Recently, much attention has been paid to the studies of using MIMO (multiple-input multiple-output) relays to further improve the data rate of relay communications, especially in the context of multi-user wireless networks. In fact, using a single MIMO relay node can improve transmission quality of a multi-user system more efficiently than using multiple SISO relays [1-2]. The natural problems are (i) to design a precoding matrix to minimize the relaying power subject to threshold constraints on signal-to-interference-and-noise ratios (SINR) at destinations, or (ii) to maximize the SINRs subject to constraints on relaying power budget. While relaying powers are convex quadratic functions, the SINRs are quadratic fractional functions, which are not concave in the precoding matrix variables. This implies that all concerned precoding matrix optimization problems are nonconvex quadratic programs.

The minimum weighted SINR, at first glance, appears to characterize only the worst performing user relative to a set of user priorities. However, maximizing the minimum weighted SINR actually amounts to equalizing the weighted SINR performance of all users (where the weights reflect the user priorities); and in the case when all users are assigned the same priority, amounts to equalizing the SINR performance. Hence, maximizing the minimum weighted SINR is a strategy for enforcing the desired level of fairness in the network. Thus, two optimization problems of interest are the max-min weighted SINR problem subject to a given total power constraint and the total power minimization problem subject to given minimum SINR constraints. Whichever optimization goal is chosen depends on the priorities of the system designer. Moreover, both optimization goals are closely related. It was proved that for multiple-input-single-output (MISO) downlink systems, these two optimization problems are inverse problems. This means that, suppose we know the optimal minimum weighted SINR in the max-min weighted SINR problem, then by plugging in this optimal minimum weighted SINR as the weighted SINR constraint in the total power minimization problem, the optimal total transmit power will be equal to the total transmit power constraint in the max-min weighted SINR problem.

Specifically, the recast programs are shown to be optimizations of d.c. (difference of convex) functions subject to convex constraints. Accordingly, a d.c iterative procedure (DCI) is developed to locate their optimal solutions. Such a DCI approach has been successfully applied in our previous works [3-4] for optimized beamforming. In contrast to [5-6], our approach is practical for the average and even high dimensions of the MIMO relay antennas, for which the SDR method cannot work. Moreover, the proposed Zero-Forcing(ZF) precoding approach is able to solve the problem of SINR maximin optimization directly. In fact, the use of d.c. structure for SINR functions makes their maximin optimization even more computationally efficient than the relaying power optimizations. Performance of the Zero-Forcing precoding method is confirmed by numerical results in computational complexity.

The rest of this article is organized as follows: Section 1 presents an overview of the maxmin SINR problem in the MIMO radio transmission system on the beamforming designs. Section 2 shows the system models and optimization problems. Next, section 3 provides extensive numerical examples to verify performance of the proposed method. And finally, section 4 presents our conclusion.

Notations: Matrices and column vectors are denoted by boldfaced uppercase and lowercase characters, respectively. For a Hermitian matrix **A**, $\lambda_{max}(\mathbf{A})$ is its maximal eigenvalue, while $\rho(\mathbf{A})$ is its spectral radius defined by $\rho(\mathbf{A}) = \max_i |\lambda_i(\mathbf{A})|$ with $\lambda_1(\mathbf{A})$, i = 1, 2,... being its eigenvalues. Furthermore, $\mathbf{A} \geq 0$ means **A** is positive semidefinite. We denote $\langle \mathbf{A} \rangle = \text{trace}(\mathbf{A})$ for a square matrix **A** and $\langle \mathbf{A},\mathbf{B} \rangle = \text{trace}(\mathbf{A}^H\mathbf{B})$ for matrices **A** and **B** of appropriate dimension, where $\mathbf{A}^H$ is the conjugate transpose of **A**. Accordingly, for two complex vectors **x** and **y** of the same dimension, $\langle x,y \rangle := x^H y$ and accordingly, $\|x\| = \sqrt{\langle x,x \rangle}$ and $|\langle x,y \rangle|^2 = \langle xx^H, yy^H \rangle$. Also, $\mathbf{E}_x$ denotes the expectation operator in respect to random variable x.

2. SYSTEM MODEL AND OPTIMIZATION PROBLEMS

Let $s = [s_1, s_2, ..., s_M]^T \in \mathbb{C}^M$ be the vector of signals sent by M sources, which are assumed to be zero mean and component-wise independent with variance $\sigma_s^2 = E[|s_i|^2]$. Let $u_i = [u_{i1}, u_{i2}, ..., u_{iN}]^T \in \mathbb{C}^N$, $i = 1,2,...,M$ be the uplink channel vector between source i and the relay while $v_j = [v_{j1}, v_{j2}, ..., v_{jN}]^T \in \mathbb{C}^N$, $i = 1,2,...,M$ denotes the downlink channel vector between the relay and destination j.

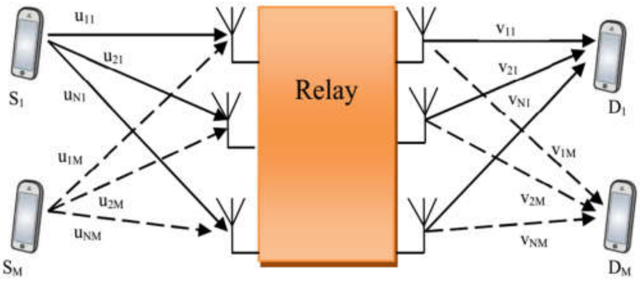

Figure 1. MIMO wireless relay model with AF processing method

2.1. Optimization problem

The signal is processed from the sources to the destination as follows:

+ On the first stage, all sources simultaneously transmit the signals to the relay. The received signal at the relay is given by

$$\mathbf{y}_{up} = \mathbf{Us} + \mathbf{n}_r \quad (1)$$

where **U** is the uplink channel matrix that contains all the channel vectors as its columns and $n_r = [n_{r1}, n_{r2}, ..., n_{rN}]^T \in \mathbb{C}^N$, $i = 1,2,...,N$ represents the additive noise at the relay receivers, which is assumed to be white Gaussian noise with zero mean and variance $\sigma_r^2 = E[|n_{rn}|^2]$.

+ The second stage, relay transmits signal after processing to the users in destination. Then, **X** is the optimal weight matrix is multiplied by the receiver signal at the relay. Therefore, the relay sends the following signals to the destinations:

$$\mathbf{y}_{relay} = \mathbf{X}\mathbf{y}_{up} = \mathbf{XUs} + \mathbf{Xn}_r \quad (2)$$

Accordingly, the received signal vector at the destinations is

$$y_d = \mathbf{VXUs} + \mathbf{VXn}_r + \mathbf{n}_d \quad (3)$$

where $\mathbf{n}_d$ is additive white Gaussian noise at the destination with element variance σ_d^2 and **V** is the downlink channel matrix. Specifically, the received signal at destination i can be presented by

$$\mathbf{y}_{di} = \mathbf{v}_i^T \mathbf{X}\mathbf{u}_i \mathbf{s}_i + \sum_{j \neq i}^{M} \mathbf{v}_i^T \mathbf{X}\mathbf{u}_j \mathbf{s}_j + \mathbf{v}_i^T \mathbf{X}\mathbf{n}_r + \mathbf{n}_{di} \quad (4)$$

The total transmit power at the relay is determined as follows:

$$P_T(\mathbf{X}) = E\left\{ |\mathbf{y}_{relay}|^2 \right\} = \text{trace}((\sigma_s^2 \mathbf{UU}^H + \sigma_r^2 \mathbf{I}_N)\mathbf{X}^H\mathbf{X}) \quad (5)$$

Signal to interference and noise ratio SINR at i[th] user at destination:

$$\text{SINR}_i(\mathbf{X}) = \frac{\sigma_s^2 \text{trace}(\mathbf{v}_i \mathbf{v}_i^H \mathbf{X} \mathbf{u}_i \mathbf{u}_i^H \mathbf{X}^H)}{\sigma_s^2 \sum_{j \neq i} \text{trace}(\mathbf{v}_i \mathbf{v}_i^H \mathbf{X} \mathbf{u}_i \mathbf{u}_i^H \mathbf{X}^H) + \sigma_r^2 \text{trace}(\mathbf{v}_i \mathbf{v}_i^H \mathbf{X}\mathbf{X}^H) + \sigma_d^2} \quad (6)$$

with $v_i = (v_i)^*$ is is the complex conjugate of the downlink channel information on user i. In relay communications, the key objective of the relay is to assist the communications between sources and destinations obtaining the predetermined quality of service (QoS) while minimizing the consuming power at the relay. The problem of minimizing the total beamforming power under the SINR constraints is mathematically posed as

$$\min_{\mathbf{X}\in C^{N\times N}} \operatorname{trace}((\sigma_s^2\mathbf{U}\mathbf{U}^H+\sigma_r^2\mathbf{I}_N)\mathbf{X}^H\mathbf{X}) \quad (7)$$

$$\text{s.t} \quad \mathrm{SINR}_i(\mathbf{X}) \geq \alpha_i, i = 1, 2, ..., M. \quad (8)$$

A more practical optimization is to maximize the minimum achievable data rate among all the users. The optimization problem is formulated as,

$$\max_{X} \quad \min_{i=1,\dots,M} \quad \frac{1}{2}\log_2(1+\mathrm{SINR}_i(\mathbf{X})) \quad (9)$$
$$\text{s.t} \quad P_T(\mathbf{X}) \leq P_0$$

where P_0 is a prescribed relaying power budget and the factor $\frac{1}{2}$ is due to that the transmission is conducted in two time slots. The optimization problem is also equivalent to maximizing the minimum SINR among all users,

$$\max_{X} \quad \min_{i=1,\dots,M} \quad \mathrm{SINR}_i(\mathbf{X}) \quad (10)$$
$$\text{s.t} \quad P_T(\mathbf{X}) \leq P_0$$

2.2. D.C. approach for MAX-MIN SINR problem

The maximin program (10) in decision variable **X** is equivalent to the following program in decision variable (**X, y**):

$$\max_{X} \quad \min_{i=1,\dots,M} \lambda(X,y_i) := \frac{\left\langle v_i v_i^H X u_i u_i^H X^H \right\rangle}{y_i + \sigma_{de}^2/\sigma_s^2} \quad (11.a)$$

$$\text{s.t} \quad P_T(\mathbf{X}) \leq P_0 \quad (11.b)$$

$$\sum_{j\neq i}^{M}\left\langle v_i v_i^H X u_i u_i^H X^H \right\rangle + \frac{\sigma_{de}^2}{\sigma_s^2}\left\langle v_i v_i^H X X^H \right\rangle \leq y_i \quad (11.c)$$
$$i = 1,2,\dots,M$$

where $y = (y_1,\dots,y_M)^T \in R^M$ and each function $\varphi_i(\mathbf{X},y_i)$ is smooth and convex while all constraints (11.b) and (11.c) are convex.

Next, we use the following d.c. decomposition for the objective function of (11) [7].

where functions

$$f(\mathbf{X},y) := \max_{i=1,2,\dots,M}\sum_{j\neq i}^{M}\varphi_j(X,y_i), \quad g(\mathbf{X},y) := \sum_{j\neq i}^{M}\varphi_i(X,y_i) \quad (12)$$

are convex as f(**X**, y) is a maximization of convex functions, while g(**X**, y) is their sum [7]. The maximin program (11) becomes the following canonical d.c. program:

$$\min_{X,y=(y_1,\dots,y_M)^T\in C^M} [f(\mathbf{X},y) - g(\mathbf{X},y)] \quad (13.a)$$

$$\text{s.t} \quad P_T(\mathbf{X}) \leq P_0 \quad (13.b)$$

$$\sum_{j\neq i}^{M}\left\langle v_i v_i^H X u_i u_i^H X^H \right\rangle + \frac{\sigma_{de}^2}{\sigma_s^2}\left\langle v_i v_i^H X X^H \right\rangle \leq y_i \quad (13.c)$$
$$i = 1,2,\dots,M$$

Since g(**X** , y) is convex and smooth, its affine minorant can be easily obtained by using its gradient [11]:

$$g(X,y) - g(X^{(k)},y^{(k)}) \geq \left\langle \nabla(X^{(k)},y^{(k)},(X-X^{(k)},y-y^{(k)}) \right\rangle =$$

$$\sum_{i=1}^{M}\left[\frac{2\mathrm{Re}\left\{\left\langle v_i v_i^H (X-X^{(k)}) u_i u_i^H X^{(k)H} \right\rangle\right\}}{y_i^{(k)}+\frac{\sigma_{de}^2}{\sigma_s^2}} - \frac{\left\langle \mathbf{v}_i \mathbf{v}_i^H X^{(k)} u_i u_i^H X^{(k)H} \right\rangle (y_i - y_i^{(k)})}{(y_i^{(k)}+\frac{\sigma_{de}^2}{\sigma_s^2})^2}\right] \quad (14)$$

for any given $(X^{(k)},y^{(k)},y^{(k)} = (y_1^{(k)},\dots,y_M^{(k)})^T$.

The corresponding convex majorant for the d.c. objective function in (11.a) is transparent and the following convex program provides an upper bound optimization for d.c. program (11):

$$\min_{X,y=(y_1,\dots,y_M)^T\in C^M}\left\{f(\mathbf{X},y) - g(\mathbf{X}^{(k)},y^{(k)})\right\}$$
$$+\sum_{i=1}^{M}\left[\frac{2\mathrm{Re}\left\{\left\langle v_i v_i^H (X-X^{(k)}) u_i u_i^H X^{(k)H} \right\rangle\right\}}{y_i^{(k)}+\frac{\sigma_{de}^2}{\sigma_s^2}} - \frac{\left\langle \mathbf{v}_i \mathbf{v}_i^H X^{(k)} u_i u_i^H X^{(k)H} \right\rangle (y_i - y_i^{(k)})}{(y_i^{(k)}+\frac{\sigma_{de}^2}{\sigma_s^2})^2}\right] \quad (15.a)$$

$$\text{s.t} \quad P_T(\mathbf{X}) \leq P_0 \quad (15.b)$$

$$\sum_{j\neq i}^{M}\left\langle v_i v_i^H X u_i u_i^H X^H \right\rangle + \frac{\sigma_{de}^2}{\sigma_s^2}\left\langle v_i v_i^H X X^H \right\rangle \leq y_i \quad (15.c)$$
$$i = 1,2,\dots,M$$

The optimal solution $(\mathbf{X}^{(k+1)},y^{(k+1)})$ of (15) is a better solution of (13) than $(\mathbf{X}^{(k)},y^{(k)})$. In summary, the following iterations generate a sequence $\left\{(\mathbf{X}^{(k)},y^{(k)})\right\}$ of improved solutions to (7), which converges to its optimal solution.

2.3. Proposed Zero-Forcing precoding

The Zero-Forcing (ZF) precoding matrix should has the form:

$$X_{zf} = \mathbf{V}^H(\mathbf{V}\mathbf{V}^H)^{-1}\Delta(\mathbf{U}^H\mathbf{U})^{-1}\mathbf{U}^H$$

where Δ is a complex diagonal matrix of the size M x M. Using the DCI method summarized above, the optimal zero-forcing solution can be achieved iteratively. In this case, the DCI is a cheap algorithm as the number of variables for ZF precoding is only M. The ZF precoding matrix can be used as the initial solution for DCI to derive the optimal linear precoding matrix. As ZF precoding matrix is very close to the optimal solution, the number of DCI iterations required is relatively low.

Algorithm for SINR Maxmin Optimization

Initialization: Choose an initial feasible solution $(\mathbf{X}^{(0)}, y^{(0)})$ of (11.b) and (15.a)

κ-th iteration:

Set **κ** = 0 Solve convex program (15) to obtain the optimal solution $(\mathbf{X}^{(k+1)}, y^{(k+1)})$. Given the tolerance level ϵ, stop and output the solution $(\mathbf{X}^{(k)}, y^{(k)})$

if $\left|\min_{i=1,2,\ldots,M} \mathrm{SINR}_i(X^{(k)}) - \min_{i=1,2,\ldots,M} \mathrm{SINR}_i(X^{(k+1)})\right| \prec \varepsilon$ then

Terminate, and output *X*opt **:=** X(κ+1)**.**

else

Reset κ **:=** κ **+** 1 and X(κ) **:=** X(κ+1)**.** Continue to the next iteration.

end if

Output the final solution X opt.

3. NUMERICAL RESULTS

This section presents various numerical results to illustrate the performance of our proposed methods versus the existing ones. All channel coefficients are zeromean, complex Gaussian distributed with unit variance. The transmitted signal power from the sources is set to be $\sigma_s^2 = 20\text{dB}$ while the variances of AWGN at the relays and destinations are $\sigma_{de}^2 = 0\text{dB}$. Recall that M is the number of user pairs, R is the number of MIMO relays and N_R is the number of transmit/receive antennas at each MIMO relay, so the total number N of relaying antennas is N = $RN_{\mathbf{R}}$. All the results are obtained by averaging over 1,000 Monte-Carlo simulation runs. The SeDuMi [8] is used for SDP solvers. The simulation parameters are shown in Table 1.

Table 1. Simulation parameters 1

N°	Parameter	Value
1	The number of user pairs M on the receiver	8
2	the total number N of relaying antennas	6
3	The number of MIMO relays	1
4	The number N_R of transmit/receive antennas	6
5	Number of iterations ITE at each SNR threshold	1000
6	SNR threshold (dB)	2, 3, 4, 5, 6
7	Stopping criterium ε for ZF optimization technique	10^{-6}
8	Stopping criterium ε for DCI optimization technique	10^{-6}

For the comparison purpose, the simulation in Fig. 2, if M < N - 2 optimal ZF precoding matrices can achieve generally optimal precoding solutions. In terms of computational complexity, ZF achieve the minimum computational cost, the proposed DCI initialized by ZF can reduce the computational time remarkably compared with the previously developed DCI.

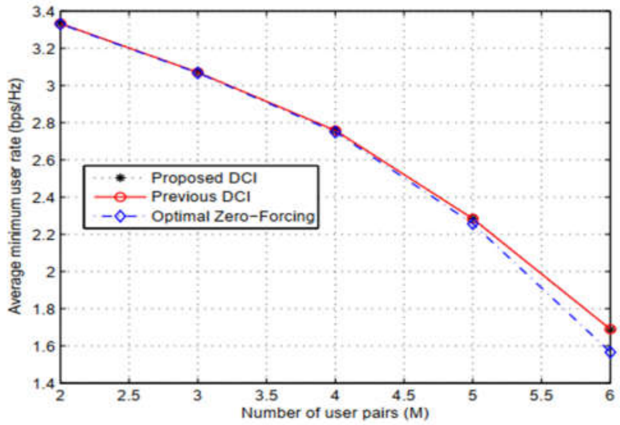

Figure 2. Compare minimum rate by ZF and DCIs at different numbers of user pairs, N = 6

From the data results of Fig. 3 shows that, when the number of user pairs increases from 2 to 6, the average computation time to perform the problem for the proposed method, it is almost maintained at 0.7 seconds. Meanwhile, both DCI methods have rapidly increased average computation time. Specifically, when the number of user pairs M = 6, the average calculation time of the Proposed DCI method is approximately 18 seconds, and for the Previous DCI method, the average time is approximately about 6.8 seconds. All the results are summarized in Fig. 3.

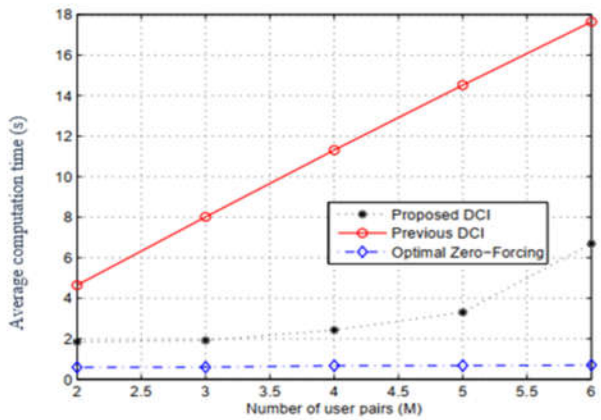

Figure 3. Compare computational time of ZF and DCIs at different numbers of user pairs, N = 6

4. CONCLUSION

In this paper, we propose a method aimed at minimizing the SINR min max of the receiver users based on the problem of optimizing the total transmit power of the antennas at the relay node. The simulation results have proved that the proposed method has achieved fast convergence speed when compared with DCI technique. Numerical results have confirmed the superiority of our proposed algorithms. In future works, we shall investigate the application of the approach devised here to the resolution of huge number of users and other rank constrained problems in multi-input-multi-output (MIMO) communications.

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REFERENCES

[1]. Hanna Pihkola, Mikko Hongisto, Olli Apilo, Mika Lasanen, Saija Vatanen, 2018. *Energy consumption of mobile data transfer Increasing or decreasing*. 5th International Conference on Information and Communication Technology for Sustainability.

[2]. Adeeb Salh, Lukman Audah, Nor Shahida Mohd Shah, Shipun Anuar Hamzah, Hasan Saeed Mir, 2019. *Antenna selection and transmission power for energy efficiency in downlink massive MIMO systems*'. Turkish Journal of Electrical Engineering and Computer Sciences.

[3]. A. H. Phan, H. D. Tuan, H. H. Kha, D. T. Ngo, 2010. *Nonsmooth optimization for beamforming in cognitive multicast transmission*. in Proc. of IEEE Global Telecommunications Conference (GLOBECOM), Miami, FL, USA.

[4]. A. Phan, H. Tuan, H. Kha, H. Nguyen, 2013. *Iterative D.C. optimization of precoding in wireless MIMO relaying.* IEEE Trans. Wireless Communications, vol. 12, pp. 1617–1627.

[5]. B. Chalise, L. Vandendorpe, 2009. *MIMO relay design for multipointto-multipoint communications with imperfect channel state information*. IEEE Trans. Signal Processing, vol. 57, no. 7, pp. 2785–2795.

[6]. B. Chalise, L. Vandendorpe, J. Louveaux, 2007. *MIMO relaying for multipoint to multi-point communication in wireless networks.* in 2nd IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMPSAP).

[7]. H. Tuy, 2000. *Convex analysis and global optimization*. Kluwer Academic.

[8]. J. F. Sturm, 1999. *Using SeDuMi 1.02, a Matlab toolbox for optimization over symmetric cones.* Optim. Methods Softw., vol. 11-12, pp. 625–653.

[9]. H.H. Kha, N.D. Long, 2015. *Optimal precoder designs for sum rate maximization in MIMO multiuser multicells.* Science and Technology Development, vol.18, no. K6.

[10]. W. H. Cai, Tony Q. S. Quek, Chee Wei Tan, 2011. *A Unified Analysis of Max-Min Weighted SINR for MIMO Downlink System.* IEEE Trans. Signal Processing, vol. 59, no. 8, pp. 3850–3862.

[11]. Tran Dinh Thong, Du Dinh Vien, Le Thanh Hai, Phan Huy Anh, 2019. *Improve convergence speed for optimizaton of relay power problem using Spectral optimizaton technique in multi-antenna wireless transmision*. Journal Military Science anh Technology, Special Issue, *No.63A, (11/2019)*, ISSN: 1859-1043, pp.39-51.

THÔNG TIN TÁC GIẢ

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