SLIDING MODE CONTROL LAW FOR UNCERTAIN NONLINEAR MULTI MOTOR SYSTEMS

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ABSTRACT

Multi-motor drive systems are nonlinear, multi-input multi-output (MIMO) and strong-coupling complicated system, including the effect of friction and elastic, backlash. An emerging proposed technique in the control law is the use of adaptive sliding mode control scheme for the stabilization of closed system. The results of theoretical analysis and simulation show that this controller ensures the quality requirements even when the system is affected by nonlinear factors caused by the mechanical structure.

Keywords: *Multi-motor drive systems, sliding mode control, elastic, backlash.*

TÓM TẮT

Hệ truyền động nhiều động cơ là hệ truyền động phi tuyến, nhiều đầu vào, nhiều đầu ra (MIMO). Đây là hệ động lực học phi tuyến, chứa các liên hệ ma sát, đàn hồi, độ dơ cơ khí giữa các khớp; các mối liên hệ này làm cho mô hình của đối tượng điều khiển trở nên phi tuyến và có thông số bất định. Bộ điều khiển trượt thích nghi được đề xuất để ổn định hệ thống kín. Qua kết quả phân tích lý thuyết và mô phỏng cho thấy bộ điều khiển này đảm bảo được các yêu cầu chất lượng ngay cả khi hệ thống chịu ảnh hưởng của các yếu tố phi tuyến do cấu trúc phần cơ gây ra.

Từ khóa: *Hệ nhiều động cơ, điều khiển trượt, khe hở, đàn hồi.*

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1. INTRODUCTION

Multi-motor drive systems have been employed in systems moving paper, metal, material being quite popular in manufacturing systems and researched by many authors in the recent times. The control method utilized artificial neural network (ANN) technique has been presented by Bouchiba et al. (2017) [2]. However, the disadvantage is to investigate the appropriate networks with associated learning rules in control design. Furthermore, the effectiveness of tracking problem or the stabilization of the cascade system are not still considered under the influences of using neural network approach. Dominique Knittel et al. proposed many linear controllers under the consideration of the approximate model of multi-motor systems without elastic, friction as a linear system to design the controller based on the transfer function technique [3, 4]. The framework of the classical PI controller and H infinity to eliminate disturbance was proposed in the work of [3, 4]. In the elastic multi-motor drive systems, it is necessary to estimate the belt tension to establish the associated state feedback controller. However, the difficulties of the control design lie in the fact that measurement of this belt tension by using sensors. The sliding mode control (SMC) technique based state feedback control enables to eliminate influence of disturbances and unknown parameters was proposed in [2, 5]. The sliding mode based control has been paid much attention in recent years because it is a widely relevant control methodology for uncertain/disturbed systems. Therefore, an adaptive sliding mode controller is proposed to obtain tracking effectiveness. The stability of closed system is obtained and verified by theory analysis, simulations.

2. DYNAMIC MODEL AND PROBLEM STATEMENT

The multi-motor system model considering the components of friction, elastic and tension is as Figure 1.

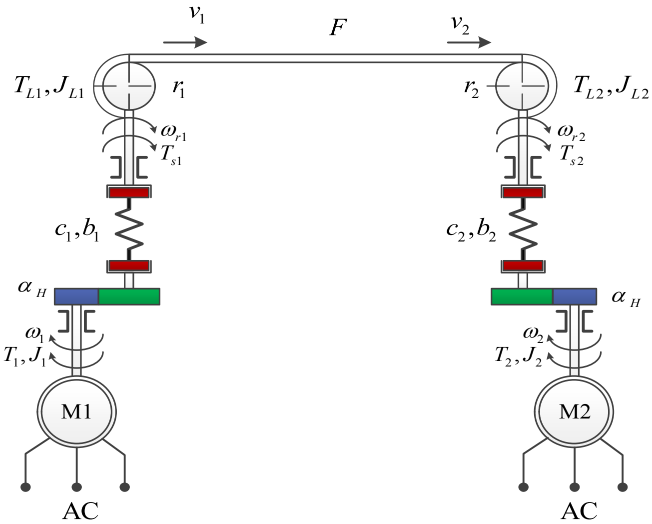


Figure 1. The Two-Motor Drive System

Where ω; and ω_L are the rotor angular speed and the beltpulley angular speed; F is the tension of the belt; T is the moment of Motors; T_L is load torque; v is the expected line speed; r is respectively the radius of beltpulley; c_1, c_2, b_1, b_2 are the stiffness and the friction coefficient; J, J_L are the inertia moment of Motors and Loads; α_H is gear clearance limit angle. In [1, 6], the multi-motor system (in Figure 1) using two induction motor is described by the following dynamic equation (1).

$$\begin{cases} \Delta\dot{\varphi}_1 = \omega_{r1} - \omega_{L1};\ \ \Delta\dot{\varphi}_2 = \omega_{r2} - \omega_{L2} \\ \dot{\omega}_{L1} = \dfrac{1}{K_{L1}}\left[k_{c1}.f_1(\Delta\varphi_1) + k_{b1}g_1(\Delta\omega_1) - (T_{L1} + r_1F_{21})\right] \\ \dot{\omega}_{L2} = \dfrac{1}{K_{L2}}\left[k_{c2}.f_2(\Delta\varphi_2) + k_{b2}.g_2(\Delta\omega_2) - (T_{L2} - r_2F_{12})\right] \\ \dot{F}_{12} = C_{12}\left[r_1\omega_{L1} - r_2\omega_{L2}(1 + \dfrac{1}{C_{12}.l}F_{12})\right] \\ \dot{F}_{21} = C_{12}\left[r_1\omega_{L1} - r_2\omega_{L2}(1 + \dfrac{1}{C_{12}.l}F_{21})\right] \\ y = F_{12} \end{cases} \tag{1}$$

In which, $\Delta\omega_1$, $\Delta\omega_2$, $\Delta\varphi_1$, $\Delta\varphi_2$ are the errors of angle speed and the errors of angle in presence of backlash, elastic; C_{12} is hardness constant of the material when stretched; The functions $f_i(\Delta\varphi)$, $g_i(\Delta\omega)$ are nonlinear functions depending on the gear clearance.

$$f_i(\Delta\varphi) = \begin{cases} 0 \text{ if } |\Delta\varphi| \le \alpha_H \\ \Delta\varphi - \alpha_H \text{ if } \varphi > \alpha_H \\ \Delta\varphi + \alpha_H \text{ if } \varphi < \text{-}\alpha_H \end{cases};\quad g_i(\Delta\omega) = \Delta\omega.\dot{f}(\Delta\varphi) = \begin{cases} 0 \text{ if } |\Delta\varphi| \le \alpha_H \\ 1 \text{ if } |\Delta\varphi| > \alpha_H \end{cases}$$

$k_c = \dfrac{c\omega_{dm}}{i^2T_{dm}}$; $k_b = \dfrac{b\omega_{dm}}{i^2T_{dm}}$ are coefficient depending on the mechanical stiffness and friction.

The state variable:

$$\underline{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{bmatrix}^T = \begin{bmatrix} \Delta\varphi_1 & \Delta\varphi_2 & \omega_{L1} & \omega_{L2} & F_{21} & F_{12} \end{bmatrix}^T;$$

The control variable: $\underline{u} = \begin{bmatrix} u_1 & u_2 \end{bmatrix}^T = \begin{bmatrix} \omega_{r1} & \omega_{r2} \end{bmatrix}^T$;

We have:

$$\begin{cases} \dot{x}_1 = u_1 - x_3;\ \dot{x}_2 = u_2 - x_4 \\ \dot{x}_3 = \dfrac{1}{K_{L1}}\left[k_{c1}.f_1(x_1) + k_{b1}.g_1(\dot{x}_1) - (T_{L1} + r_1x_5)\right] \\ \dot{x}_4 = \dfrac{1}{K_{L2}}\left[k_{c2}.f_2(x_2) + k_{b2}.g_2(\dot{x}_2) - (T_{L2} - r_2x_6)\right] \\ \dot{x}_5 = C_{12}\left[r_1x_3 - r_2x_4(1 + \dfrac{1}{C_{12}.l}x_5)\right] \\ \dot{x}_6 = C_{12}\left[r_1x_3 - r_2x_4(1 + \dfrac{1}{C_{12}.l}x_6)\right] \end{cases} \tag{2}$$

The model is described by equation (2) belongs to the class of nonlinear systems as follows:

$$\frac{d}{dt}\underline{x} = A\underline{x} + B\left(\underline{u} + \underline{u}_d(\underline{x}, t)\right) \tag{3}$$

Set $\underline{e} = \underline{x}^{ref} - \underline{x}$ then there is an error-driven model:

$$\frac{d}{dt}\underline{e} = A\underline{e} + B\left(\underline{u} + \underline{u}_d(\underline{e}, t)\right) \tag{4}$$

Where:

$$A = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{-r_1}{K_{L1}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{r_2}{K_{L2}} \\ 0 & 0 & C_{12}.r_1 & -C_{12}.r_2 & 0 & 0 \\ 0 & 0 & C_{12}.r_1 & -C_{12}.r_2 & 0 & 0 \end{bmatrix};$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/K_{L1} & 0 \\ 0 & 0 & 0 & 1/K_{L2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$\underline{u}_d(\underline{e}, t) = \begin{bmatrix} 0 \\ 0 \\ k_{c1}f_1(e_1) - k_{b1}g_1(\dot{e}_1) - T_{L1} \\ k_{c2}f_2(e_2) - k_{b2}g_2(\dot{e}_2) - T_{L2} \end{bmatrix};\ \underline{u} = \begin{bmatrix} \omega_{r1} \\ \omega_{r2} \\ 0 \\ 0 \end{bmatrix};\ \underline{e} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{bmatrix}$$

3. ADAPTIVE SLIDING MODE CONTROL DESIGN

In this section, the main work is to find a state feedback control law based on the adaptive sliding mode control technique for the class of multi motor systems.

Selecting a sliding surface of the form:

$$s = \alpha e + \xi \tag{5}$$

with $\alpha, \xi > 0$ are the design constants.

A(x, t), B(x, t) are nonlinear functions that can be approximated by a perceptron neural network as follows:

$$\begin{aligned} A(x) \text{ » } \hat{A}(x, q_a^t) &= q_a y_a(x) = \sum_{i=1}^{p_a} q_{ai} y_{ai}(x); \\ B(x) \text{ » } \hat{B}(x, q_b^t) &= q_b y_b(x) = \sum_{i=1}^{p_b} q_{bi} y_{bi}(x). \end{aligned} \tag{6}$$

Where θ_{ai}, θ_{bi} are variable coefficients; $\psi_{ai}(x)$, $\psi_{bi}(x)$ are the components selected based on the basis functions and p_a, p_b are the corresponding node number.

All signals of closed loop will be bounded and θ_a, θ_b converge to:

$$\begin{aligned}\theta_a^* &= \underset{\theta_{ai}}{\operatorname{argmin}}\left\{\sup_{x\in\Omega}\left|A(x,t)+D(x,t)-\hat{A}(x,\theta_{ai})\right|\right\}\\ \theta_b^* &= \underset{\theta_{bi}}{\operatorname{argmin}}\left\{\sup_{x\in\Omega}\left|B(x,t)-\hat{A}(x,\theta_{ai})\right|\right\}\end{aligned} \tag{7}$$

$$\hat{B}^{-1}(x,\theta_b^t)=\hat{B}^T(x,\theta_b^t)\left[\varepsilon_0 I_m+\hat{B}\ (x,\theta_b^t)\hat{B}^T(x,\theta_b^t)\right]^{-1} \tag{8}$$

Inside ε_0 is the coefficient of determination; I_m is a homogenous matrix mxm.

In order to approximate $\hat{A}(x,t);\hat{B}(x,t)$ an updated adaptation rule is defined as follows:

$$\begin{cases}\dot{\theta}_a^t=\eta_a\alpha\psi_a(x)s\\ \dot{\theta}_b^t=\eta_b\alpha\psi_b(x)su_{eq}\end{cases} \tag{9}$$

Minimum estimated error:

$$\begin{aligned}\varepsilon_a(x,t)&=A(x,t)+D(t)-A^*(x,\theta_a^*)\\ \varepsilon_b(x,t)&=B(x,t)-B^*(x,\theta_b^*)\end{aligned} \tag{10}$$

The control signal u(t) ensures the stability of the closed-loop control system and compensates for the approximate errors as:

$$u(t)=u_{eq}(t)-u_c(t) \tag{11}$$

$$u_{eq}=\hat{B}^T(x,\theta_b^t)\begin{bmatrix}\varepsilon_0 I_m\\ +\hat{B}\ (x,\theta_b^t)\hat{B}^T(x,\theta_b^t)\end{bmatrix}^{-1}\begin{pmatrix}-\hat{A}(x,\theta_a^t)\\ +\dot{x}_d-\beta\,\text{sgn}(s)\end{pmatrix} \tag{12}$$

$$u_c(t)=B^{-1}s\left(\bar{\varepsilon}_b\left|u_{eq}\right|+\bar{\varepsilon}_a\right) \tag{13}$$

Prove that the system is stable

The Lyapunov candidate function is selected as follows:

$$V=\frac{1}{2}s^2+\frac{1}{2}\frac{1}{\eta_a}\tilde{\theta}_a^2+\frac{1}{2}\frac{1}{\eta_b}\tilde{\theta}_b^2 \tag{14}$$

Where $\tilde{\theta}_a=\theta_a-\theta^*{}_a;\tilde{\theta}_b=\theta_b-\theta^*{}_b$ are the deviations of the adaptive law. We have:

$$\dot{V}=s\dot{s}+\tilde{\theta}_a\dot{\tilde{\theta}}_a+\tilde{\theta}_b\dot{\tilde{\theta}}_b \tag{15}$$

Substitute equation (12) into (5) and take the derivative:

$$\begin{aligned}\dot{s}&=\alpha\dot{e}=\alpha(\dot{x}-\dot{x}_d)\\ &=\alpha\begin{pmatrix}A(x,t)+B(x,t)u(t)+D(x,t)\\ -\hat{A}(x,t)-\hat{B}(x,t)u_{eq}(t)-\beta\,\text{sgn}(s)\end{pmatrix}\\ &=\alpha\begin{pmatrix}\left(A(x,t)+D(x,t)-\hat{A}(x,t)\right)\\ +\left(B(x,t)-\hat{B}(x,t)\right)u_{eq}(t)-B(x,t)u_c(t)-\beta\,\text{sgn}(s)\end{pmatrix}\end{aligned} \tag{16}$$

Transform equation (10) to get:

$$\begin{aligned}\varepsilon_a(x,t)+A^*(x,\theta_a^*)-\hat{A}(x,\theta_a)&=A(x,t)+D(t)-\hat{A}(x,\theta_a)\\ \varepsilon_b(x,t)+B^*(x,\theta_b^*)-\hat{B}(x,\theta_b)&=B(x,t)-\hat{B}(x,\theta_b)\end{aligned} \tag{17}$$

Replace (17) into (16):

$$\begin{aligned}\dot{s}=\alpha[&\left(\varepsilon_a(x,t)+A^*(x,\theta_a^*)-\hat{A}(x,\theta_a)\right)\\ &+\left(\varepsilon_b(x,t)+B^*(x,\theta_b^*)-\hat{B}(x,\theta_b)\right)u_{eq}(t)\\ &-B(x,t)u_c(t)-\beta\,\text{sgn}(s)]\end{aligned} \tag{18}$$

have again

$$\begin{aligned}A^*(x,\theta_a^*)-\hat{A}(x,\theta_a)&=\psi_a(\theta_a^*-\theta_a)=-\psi_a\tilde{\theta}_a\\ B^*(x,\theta_b^*)-\hat{B}(x,\theta_b)&=\psi_b(\theta_b^*-\theta_b)=-\psi_b\tilde{\theta}_b\end{aligned} \tag{19}$$

Replace (19) into (18):

$$\dot{s}=\alpha\begin{bmatrix}\left(\varepsilon_a(x,t)-\psi_a\tilde{\theta}_a\right)+\left(\varepsilon_b(x,t)-\psi_b\tilde{\theta}_b\right)u_{eq}(t)\\ -B(x,t)u_c(t)-\beta\,\text{sgn}(s)\end{bmatrix} \tag{20}$$

From (20), (15) and (5) we have:

$$\begin{aligned}\dot{V}&=s\alpha\left(\varepsilon_a(x,t)+\varepsilon_b(x,t)u_{eq}(t)-B(x,t)u_c(t)-\beta\,\text{sgn}(s)\right)\\ &=s\alpha\left(\varepsilon_a(x,t)+\varepsilon_b(x,t)u_{eq}(t)-\bar{\varepsilon}_a-\bar{\varepsilon}_b\left|u_{eq}\right|-\beta\,\text{sgn}(s)\right)\\ &\le-\alpha s\beta\,\text{sgn}(s)\end{aligned} \tag{21}$$

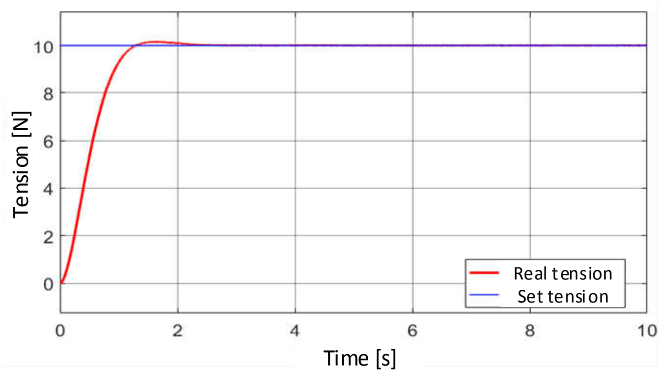
with $\alpha>0;\text{sgn}(s)\ge 0$ infer $\dot{V}\le 0\ \forall x\in\Omega$. Therefore, all signals of the closed system are blocked and θ_a, θ_b goes to θ_a^*,θ_b^* when $t\to\infty$. So the closed system is stable.

4. SIMULATION RESULTS

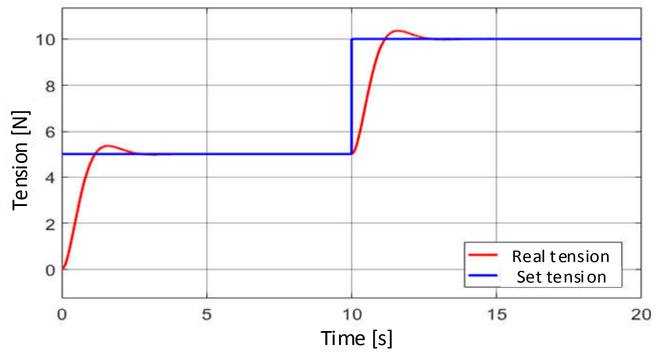
Consider two Siemens three-phase asynchronous motors with the same parameters, specifically as follows: $P_{đm}$ = 4kW; L_m = 0,1958(H); L_s = 0,202(H) ; L_r = 0,2065(H); R_r = 1,275(Ω); R_s =1,663(Ω); p = 2; $n_{đm}$ = 1400v/p; $J_{1,2}$ = 7,47.10^{-5}Kgm^2; $J_{L1,2}$ = 8.258.10^{-5}Kgm^2; Roller radius 1, 2 r_1, r_2 = 0,03m; C_{12} = 0,4N/m; c = 360Nm/rad; b = 0,02; Material ribbon length l = 0,6m.

To clearly validate the efficacy of the adaptive sliding mode control scheme, we consider the following cases:

Case 1: Tension response when gear clearance α_H = 0,08 rad ($4{,}58^0$)



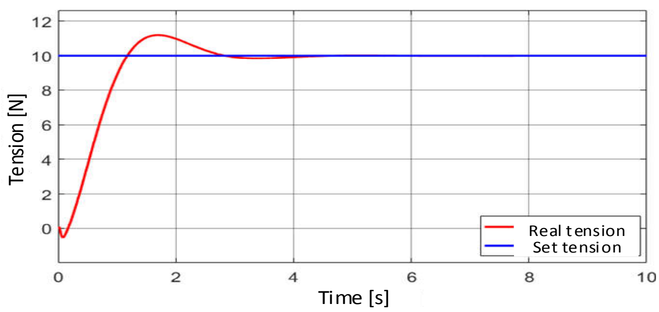
(a)



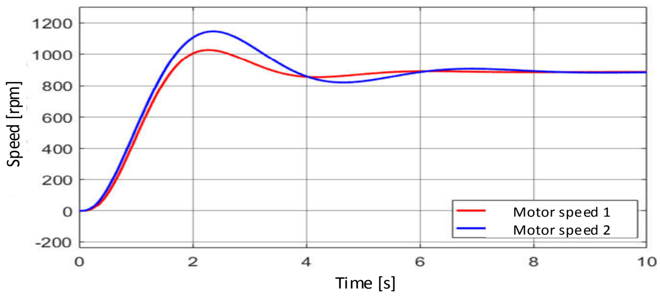
(b)

Figure 2. Tension response when gear clearance $\alpha_H = 0{,}08$rad, the input signal is constant (a)and step function (b)

Case 2: Tensile response of the system considering the influence of the moment of inertia of the load



(a)



(b)

Figure 3. Tensile response (a) and speed response (b) to the influence of moment of inertia $J_{L1} = 4J$; $J_{L2} = J$

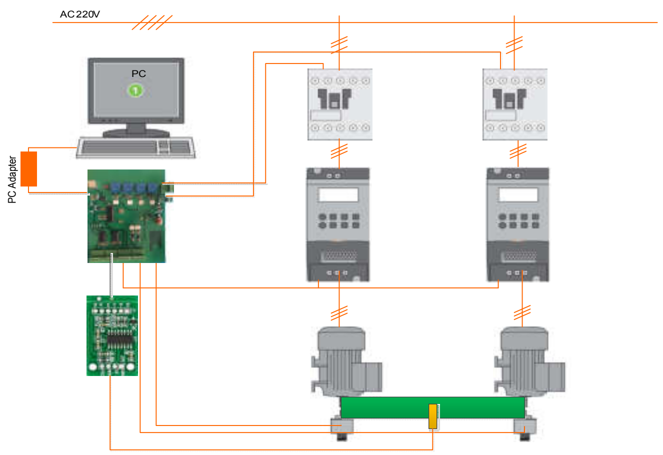


Figure 4. Experimental system model of two-motor drive system

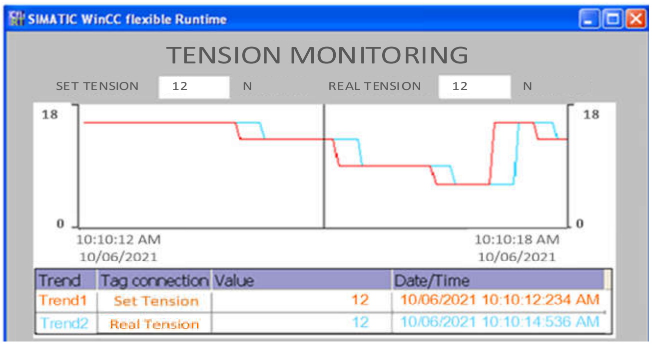


Figure 5. Set signal and real-time tracking signal

Comment: From the simulation results, it is found that the adaptive sliding controller gives good control quality. Under the influence of nonlinear factors such as gear clearance, elastic coefficient, frictional moment and moment of inertia on the load, the system still ensures accuracy in dynamic mode. and static mode, the error in static mode is always zero in different cases.

5. CONCLUSION

This paper described an adaptive sliding mode control law the two-motor system in presence of elastic and backlash, friction. The effectiveness of the proposed control scheme was pointed out by theory analysis, simulation and experimental results.

REFERENCES

[1]. Dao Phuong Nam, Tran Xuan Tinh, Pham Tuan Thanh, Nguyen Hong Quang, Dao Sy Luat, 2021. *On Finite-Time Output Feedback Sliding Mode Control of a Elastic Multi-Motor System.* International Journal of Power Electronics and Drive System (IJPEDS), Vol. 12, No.1.

[2]. Bousmaha Bouchiba, Ismail Khalil Bousserhane, Mohammed Karim Fellah, Hazzab, 2017. *Neural Network Sliding Mode Control for Multi-Machine Web Winding System.* Roum. Sci. Techn. Électrotechn. et Énerg, vol. 62, pp. 109-113.

[3]. Hakan Koc, Dominique Knittel, Michel de Mathelin, Gabriel Abba, 2002. *Modeling and Robust Control of Winding Systems for Elastic Webs.* IEEE Transaction on Control Systems Technology, vol. 10, no. 2, pp. 197-208.

[4]. Dominique Knittel, Edouard Laroche, Daniel Gigan, Hakan Koç, 2003. *Tension Control for Winding Systems With Two-Degrees-of-Freedom Hinf Controllers.* IEEE Transactions on Industry Application, vol. 39, no. 1, pp. 113- 120.

[5]. Jie-Shiou Lu, Ming-Yang Cheng, Ke-Han Su, Mi-Chi Tsai, 2018. *Wire tension control of an automatic motor winding machine an iterative learning sliding mode control approach.* Robotics and Computer-Integrated Manufacturing, vol. 50, pp. 50-62.

[6] Pham Tam Thanh, Dao Phuong Nam, Tran Xuan Tinh and Luong Cong Nho, 2018. *High-Gain Observer–Based Sliding Mode Control of Multimotor Drive Systems.* Adaptive Robust Control Systems, Published by InTech Janeza Trdine 9, 51000 Rijeka, Croatia.

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