# A COMBINATION METHOD OF THEORY AND EXPERIMENT IN DETERMINATION OF MACHINE-TOOL DYNAMIC STRUCTURE PARAMETERS

PHƯƠNG PHÁP KẾT HỢP LÝ THUYẾT VÀ THỰC NGHIỆM TRONG XÁC ĐỊNH CÁC THÔNG SỐ ĐỘNG LỰC HỌC CỦA HỆ DAO ĐỘNG MÁY - CÔNG CỤ

#### ABSTRACT

In this paper, a combination method of the theory and experiment in determination of dynamic parameters of machine-tool system was proposed. By experimental research, the data of action force and the vibrations was collected in time domain. By the theoretical research, using Fourier Transform, the data in time domain was transformed and analysed in frequency domain. And then, the parameters of the machine-tool dynamic structure were determined. The proposed method was verified by the comparison of the calculated results and the analysed results from analysis software.

Keywords: Dynamic parameters, vibration, machine-tool dynamic.

### TÓM TẮT

Trong nghiên cứu này, một phương pháp kết hợp giữ lý thuyết và thực nghiệm trong xác định thông số hệ dao động của hệ thống máy - công cụ được đề xuất và thử nghiệm. Thông qua nghiên cứu thực nghiệm, dữ liệu về lực tác động và dao động sinh ra được ghi lại trong miền thời gian. Bằng mô hình lý thuyết với phép biến đổi Fourier, dữ liệu trong miền thời gian được biến đổi và phân tích trong miền tần số. Từ đó, thông số của hệ thống dao động máy - công cụ được tính toán, xác định. Phương pháp đề xuất đã được kiểm tra thông qua việc so sánh kết quả tính toán với kết quả phân tích từ phần mềm.

Từ khóa: Thông số động lực học, dao động, hệ thống máy - công cụ.

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#### **1. INTRODUCTION**

During machine operations, the machine tool experiences vibrations. The unbalance in turning and boring, nonsymmetric teeth in drilling can produce periodically varying cutting forces. In the milling process,

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the tool, workpiece, and machine tool structures are subject to periodic vibrations due to the intermittent engagement of tool teeth and periodically varying milling forces. The forced vibrations can simply be solved by applying the predicted cutting forces on the transfer function of the dynamic structure by using the solution of ordinary differential equations in the time domain.

The fundamentals of vibration were explained, including free vibration system and force vibration system. These fundamentals were applied in solving the vibrations in milling machining process. CUTPRO software and its devices was proposed to determine the machine tool dynamic structure. By using this measurement system, the dynamic structure of milling machine tool systems can be investigated and analysed. The experiments were conducted to determine the machine tool dynamic structure in machine-tool systems [1].

The phenomenon of vibration is an inextricable part of any machining process and modern machine shops are well aware of its detrimental effects. The machine tool vibration can destabilize a machining process and in extreme situation lead to chatter with severe implications for quality, tool life and process capability. The vibration plays an important role in limiting the afore-mentioned productivity parameters. Reducing the vibration for a stable machining process may reduce the number of time consuming operations, etc., to obtain the desired surface finish and consequently reducing the machining lead time [2].

There are both forced and self-excited vibrations of machine tool in machining processes. However, the self-excited vibration is the most detrimental for the safety and quality of machining operations [2]. During machining, the machine tool vibrations play an important role in hindering productivity. The poor finished surface and damage of spindle bearing may be caused by excessive vibrations [2, 3, 4]. Vibrations are very important in machining processes; so, investigating and controlling the machine tool vibrations is necessary in the improvement of machining quality.

This study focuses only on the application of theory and experiment mothed in the investigation of machine-tool dynamic structure. By this proposed method machine-tool dynamic structure parameters such as nature frequency, mass (m), spring (k), and damping (c) in x and y directions were determined.

# 2. THEORETICAL OF MACHINE-TOOL DYNAMIC STRUCTURE

#### 2.1. Theoretical of Forced-Vibration System

A simple structure with a single-degree of freedom (SDOF) system can be modelled by a combination of mass (m), spring (k), and damping (c) elements as shown in Fig. 1. When an external force F(t) is exerted on the structure, its motion is described by Eq. (1) and Eq. (2), [1, 2].



Fig. 1. Vibration system

$$m\ddot{x} + c\dot{x} + kx = F(t) \tag{1}$$
 or

$$\ddot{\mathbf{x}} + 2\zeta\omega_{n}\dot{\mathbf{x}} + \omega_{n}^{2}\mathbf{x} = \frac{\omega_{n}^{2}}{k}F(t)$$
<sup>(2)</sup>

where  $\omega_n$  is the natural frequency and  $\zeta$  is the damping ratio of system, and x is displacement of system in x direction.

If the system receives a hammer blow for a very short duration, or when it is at rest and statically deviates from its equilibrium, the system experiences free vibrations. This means no external forces (outside forces), F(t) = 0, but only internal force controlled the motion. The internal forces are forces within the system including the force of inertia (mx), the damping force (cx, (if c > 0)), and the spring force (kx), a restoring force [1, 2].

# **2.2. Fundamentals of Forced Vibrations and frequency** response function (FRF)

When an external force F(t) is not equal to zero,  $F(t) \neq 0$ , the system experiences forced vibrations. When a constant force  $F(t) = F_0$  is applied to the structure, the system experiences a short-lived free or transient vibration and then stabilizes at a static deflection  $x_{\rm st} = F_0/k$ .

The general response of the structure can be evaluated by solving the differential equation of the motion. The Laplace transform of the equation of motion with initial displacement x(0) and vibration velocity x'(0) under externally applied force F(t) is expressed in Eq. (3) to Eq. (5), [2].

$$\mathcal{L}(\ddot{\mathbf{x}} + 2\zeta\omega_{n}\dot{\mathbf{x}} + \omega_{n}^{2}\mathbf{x}) = \mathcal{L}\left(\frac{\omega_{n}^{2}}{k}F(t)\right)$$
(3)

$$\Rightarrow s^{2}x(s) - sx(0) - x(0) + 2\zeta\omega_{n}sx(s) - 2\zeta\omega_{n}x(0) + \omega_{n}^{2}x(s) = \frac{\omega_{n}^{2}}{2}F(s)$$
(4)

$$\Rightarrow (s^{2} + 2\zeta\omega_{n}s + \omega_{n}^{2})x(s) - (s + 2\zeta\omega_{n})x(0) - x'(0) = \frac{\omega_{n}^{2}}{k}F(s)$$
(5)

The system's general response, the vibrations of the structure with a SDOF dynamics, can be expressed by Eq. (6).

$$x(s) = \frac{\omega_n^2}{k} \frac{1}{s^2 + 2\zeta \omega_n s + \omega_n^2} F(s) + \frac{(s + 2\zeta \omega_n) x(0) + x'(0)}{s^2 + 2\zeta \omega_n s + \omega_n^2}$$
(6)

The frequency response function of the system is represented by Eq. (7) by neglecting the effect of initial conditions that will eventually disappear as transient vibrations.

$$\Phi(s) = \frac{x(s)}{F(s)} = \frac{\omega_n^2}{k} \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$
(7)

Assuming that the external force is harmonic (it can be represented by sin or cosine function or their combinations). The forced vibration system can be rewritten by Eq. (8).

$$\ddot{\mathbf{x}} + 2\zeta \omega_{n} \dot{\mathbf{x}} + \omega_{n}^{2} \mathbf{x} = \frac{\omega_{n}^{2}}{k} F_{0} \sin(\omega t)$$
(8)

where  $\omega$  is the frequency of external force F(t).

The system experiences forced vibrations at the same frequency  $\omega$  of the external force, but with the time or phase delay ( $\varphi$ ). It is assumed that the transient vibrations caused by initial loading have diminished and the system is at steady-state operation. Then the motion can be rewritten by Eq. (9).

$$x(t) = Xsin(\omega t + \phi)$$
(9)

where X is the amplitude of vibration.

Using the complex harmonic functions of external force and vibration, the harmonic force and the corresponding harmonic response can be expressed by Eq. (10).

$$\begin{cases} F(t) = F_0 \sin(\omega t) = F_0 e^{j\omega t} \\ x(t) = X \sin(\omega t + \phi) = X e^{j(\omega t + \phi)} \end{cases}$$
(10)

The integration of motion is expressed by Eq. (11).

$$\Rightarrow \begin{cases} \dot{x}(t) = j\omega X e^{j(\omega t + \phi)} \\ \ddot{x}(t) = -\omega^2 X e^{j(\omega t + \phi)} \end{cases}$$
(11)

Substituting  $\dot{x}(t)$  and  $\ddot{x}(t)$  into Eq. (8), the forced vibration can be written by Eq. (12).

$$-\omega^{2} X e^{j(\omega t + \phi)} + 2\zeta \omega_{n} j \omega X e^{j(\omega t + \phi)} + \omega_{n}^{2} X e^{j(\omega t + \phi)} = \frac{\omega_{n}^{2}}{k} F_{0} e^{j\omega t}$$
(12)

$$\Rightarrow (\omega_n^2 - \omega^2 + 2\zeta \omega_n j\omega) X e^{j(\omega t + \phi)} = \frac{\omega_n^2}{k} F_0 e^{j\omega t}$$
(13)

$$\Rightarrow (\omega_n^2 - \omega^2 + 2\zeta \omega_n j\omega) e^{j\phi} X e^{j\omega t} = \frac{\omega_n^2}{k} F_0 e^{j\omega t}$$
(14)  
so,

$$\Rightarrow (\omega_n^2 - \omega^2 + 2\zeta \omega_n j \omega) X e^{j\omega t} = \frac{\omega_n^2}{k} F_0 e^{j\omega t}$$
(15)

The frequency response function (FRF) of the system can be expressed by Eq. (16).

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$$\Phi(j\omega) = \frac{X(j\omega)}{F_0(j\omega)} = \frac{\omega_n^2}{k} \frac{1}{\omega_n^2 - \omega^2 + 2\zeta\omega_n j\omega}$$
(16)

The excitation to natural frequency ratio is  $r = \omega/\omega_n$ . So, the FRF can be expressed by Eq. (17) and Eq. (18).

$$\Phi(j\omega) = \frac{1}{k} \cdot \frac{1}{1 - r^2 + 2j\zeta r}$$
(17)

$$\Phi(j\omega) = \frac{1}{k} \cdot \frac{1}{(1-r^2)+2j\zeta r}$$
(18)

The resulting amplitude (Gain) and phase of the harmonic vibration are expressed by Eq. (19) to Eq. (20).

$$|\Phi(j\omega)| = \frac{X(j\omega)}{F_0(j\omega)} = \frac{1}{k} \cdot \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$
(19)

$$\phi = \tan^{-1} \frac{x}{F} (j\omega) = \tan^{-1} \frac{-2\zeta r}{1 - r^2}$$
(20)

The FRF ( $\Phi(j\omega)$ ) can be separated into real (G( $\omega$ )) and imaginary H( $\omega$ )) parts of  $\frac{x}{F_0}(e^{j(\alpha-\phi)})$  as in Eq. (21) and Eq. (23).

$$G(\omega) = \frac{1 - r^2}{k[(1 - r^2)^2 + (2\zeta r)^2]}$$
(21)

$$H(\omega) = \frac{-2\zeta r}{k[(1-r^2)^2 + (2\zeta r)^2]}$$
(22)

or

$$\Phi(j\omega) = G(\omega) + jH(\omega)$$
(23)

# **2.3.** Determination of machine-tool dynamic structure from FRF

When the input and the natural frequency of the forced vibration system are the same ( $\omega = \omega_n$ ), the amplitude of the vibration becomes larger and larger. Practically, the systems with very little damping may undergo large vibrations that can destroy the system. This phenomenon is called resonance. With the frequency response function, when the input force frequency is equal to the natural frequency of vibration system, the real part of FRF is equal to zero, and the imaginary part of FRF is equal to  $\frac{-1}{2k\zeta}$ , or  $(H(\omega_n) = \frac{-1}{2k\zeta})$ . And so, the natural frequency of a vibration system (dynamic structure) can be determined at points that the real part of FRF is equal to zero.

The maximum magnitude of FRF occurs at  $\omega = \omega_n \sqrt{1 - 2\zeta^2}$ . And the real part G( $\omega_n$ ) of FRF has two extrema at frequency  $\omega_1$  and  $\omega_2$  as in Eq. (25).

$$\begin{cases} \omega_1 = \omega_n \sqrt{1 - 2\zeta} \longrightarrow G_{\max} = \frac{1}{4k\zeta(1-\zeta)} \\ \omega_2 = \omega_n \sqrt{1 + 2\zeta} \longrightarrow G_{\min} = -\frac{1}{4k\zeta(1-\zeta)} \end{cases}$$
(24)

so, the damping ratio can be determined by Eq. (25), [2].

$$\frac{G_{\max}}{G_{\min}} = \frac{1+\zeta}{\zeta-1} \implies \zeta = \frac{G_{\max}+G_{\min}}{G_{\max}-G_{\min}}$$
(25)

Finally, the modal stiffness (k), the modal mass (m), and the modal damping constant can be calculated by Eq. (26).

$$k = \frac{-1}{2\zeta H(\omega_n)}$$

$$m = \frac{k}{\omega_n^2}$$

$$c = 2\zeta \sqrt{km}$$
(26)

The simple dynamic structure modal of the machinetool dynamic structure is described in Fig. 2. This system can be modelled by a combination of mass (m), spring (k), and damping (c) elements in x and y directions.



Fig. 2. Machine tool dynamic structure modal

Milling process is a dynamic process; so, by the effect of machine-tool dynamic structure, the machine-tool vibrations in x and y directions were calculated by Eq. (27), [5-9].

$$\begin{cases} m_{x}\ddot{x}(t) + c_{x}\dot{x}(t) + k_{x}x = F_{x}(t) \\ m_{v}\ddot{y}(t) + c_{v}\dot{y}(t) + k_{v}y = F_{v}(t) \end{cases}$$
(27)

Where: x,  $(m_x)$ ,  $(k_x)$ ,  $(c_x)$ ,  $F_x(t)$  are the displacement, mass, stiffness, damping ratio, and external force in x direction. And, y,  $(m_y)$ ,  $(k_y)$ ,  $(c_y)$ ,  $F_y(t)$  are the displacement, mass, stiffness, damping ratio, and external force in y direction.

By analysis of the Frequency Response Function of each forced vibration system (x and y directions), the parameters of machine-tool dynamic structure such as natural frequency, modal stiffness, damping ratio, modal mass are determined.

#### **3. EXPERIMENTAL METHOD**

The setup of the experiments in this paper includes tool, CNC machine, FRF measurement. The description of the setup is as the followings:

#### 3.1. Tool, and CNC machine

In order to investigate of machine-tool dynamic structure, the tool and machine were chosen as follows. Tool: a new carbide flat-end mill with number of flutes N = 2, a helix angle  $\beta = 30^{\circ}$ , a rake angle  $\alpha_r = 5^{\circ}$ , and a diameter of 10mm. The experiments were performed at a three-axis vertical machining center (DECKEL MAHO - DMC70V hi-dyn).

# **3.2. Setup for determination of Frequency Response Function**

In order to determine the frequency response function and the dynamic structure of machine-tool, an integrated device system that consisted of the acceleration sensor (ENDEVCO-25B-10668), hammer (KISTLER-9722A2000), signal processing box (NI 9234), and a PC was used. The detail setting of the measurement experiment is illustrated in Fig. 3. The experiments were performed with the assistance of CUTPRO<sup>™</sup> software to measure the force and response displacement, [10].





Fig. 3. Setup of FRF measurement (Tap testing) [10]

a. Tool; b. Acceleration sensor; c. Force sensor; d. Signal processing box; e. PC and CUTPROTM software

# 4. RESULTS AND DISCUSSIONS

#### 4.1. Force and Response Displacement in Tapper test





Fig. 4. The force and response displacement in X and Y directions

The signal of hammer force and the displacement values obtained from the force and displacement sensors are shown in the time domain, as shown in Fig. 4. It seems that by Tapper test, in each direction (x or y direction), the impact force was the single peak force. Besides, in each direction, the response displacement decreased from maximum value to zero. So, these are damped oscillation systems. The tapper test results can be transformed form time domain to frequency domain to determine the frequency response function (FRF) of machine-tool dynamic structure.

# 4.2. Determination of Frequency Response Function (FRF)



Fig. 5. The real part and imaginary part of FRF in X and Y directions

In this study, Fourier transform was use to transform the measured results of the force and response displacement from time domain to frequency domain. The FRF was determined by Eq. (16) that the FRF was separated into two parts (dash line): the real part and the imaginary part as, shown in Fig. 5 for both x and y directions. The analysed results of FRF was compared with the analysed results of CUTPRO software (solid line) as described in Fig. 5. This figure is shown that the analysed results were quite close to the results of CUTPRO software in both x and y directions. So, the proposed method in this study can be used the determine the frequency response function of a machinetool dynamic structure.

#### 4.3. Determination of machine-tool dynamic structure

Using the determined results of the frequency response function (FRF) as expressed in Section 4.2, the machine-tool dynamic parameter such as nature frequency ( $\omega_n$ ), mass (m), spring (k), and damping (c) were calculated and listed in Table 1. The calculated results of machine-tool dynamic structure parameter were compared with the analysed results of CUTPRO software. The calculated results between proposed method and CUTPRO software are quite close together. The average difference of two methods is about 12.4 %.

Direction	Mode No.	ω <sub>n</sub> [Hz]	ω <sub>1</sub> [Hz]	ω <sub>2</sub> [Hz]	H(ω <sub>n</sub> ) [m/N]	ζ	k [N/m]	m [kg]
X direction	Research	2776	2742	2832	-4.47E-06	0.016	6.91E+06	0.896
	CUTPRO	2772	2734	2832	-5.02E-06	0.018	5.64E+06	0.734
	Different (%)	0.14	0.29	0.00	11.00	8.296	22.53	22.174
Y direction	Research	2780	2724	2842	-4.18E-06	0.021	5.64E+06	0.730
	CUTPRO	2778	2722	2836	-5.39E-06	0.021	4.52E+06	0.586
	Different (%)	0.07	0.07	0.21	22.52	3.434	24.79	24.606

Table 1. Machine-tool dynamic structure parameters

The obtained results showed that the machine-tool dynamic structure parameters of the different directions are different. Besides, in each direction, the machine-tool dynamic structure parameters are quite close to each other but not the same when determining by proposed method and by CUTPRO software. So, the proposed method in this study can be used as a convenient method to determine the machine-tool dynamic structure parameters.

#### **5. CONCLUSIONS**

In this study, a combination method of theoretical and experimental method was performed to investigate the machine-tool dynamic structure. Depending on the analysis of experimental results, the conclusions of this study can be drawn as follows.

1. The tapper test results can be transformed form time domain to frequency domain to determine the frequency response function (FRF) of machine-tool dynamic structure.

2. In each x or y direction, the determined FRF was separated into two parts: the real part and the imaginary

part. The analysed results of FRF were quite close to the results from CUTPRO software. So, the proposed method in this study can be used the determine the frequency response function of a machine-tool dynamic structure.

3. The calculated results of machine-tool dynamic structure parameters between proposed method and CUTPRO software are quite close together. The difference of two methods is about 7.6 %. So, the proposed method in this study can be used as a convenient method to determine the machine-tool dynamic structure parameters.

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